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Inflation, the Interest Rate, and the Required Return on Equity

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Abstract

Miller has analyzed capital structure in the presence of both corporate and personal taxes. The present work investigates the effect of inflation on both interest rates and equity returns when the Miller equilibrium condition is employed in a loanable funds model. Both an interest rate effect and a redistribution effect are derived. The interest rate effect forces the responsiveness of the interest rate to the inflation rate to be below that hypothesized by Darby. However, the redistribution effect may change this responsiveness in either direction.

I. Introduction

The number of academic studies concerning inflation has grown in tandem with the price level. Researchers have considered the effect of the rate of inflation on the interest rate. The simplest models show that, without taxes, a 1 percent rise in the rate of anticipated inflation increases nominal interest rates by 1 percent. Recently, the effect of taxation on the relationship between interest rates and inflation has been investigated. For example, Darby [5] has pointed out that a 1 percent increase in the inflation rate should increase the after-tax interest rate by 1 percent, implying a greater movement in pre-tax interest rate. This result has been confirmed and extended by Gandolfi [13], Feldstein [8], and others.

However, empirical evidence has not supported the theoretical work mentioned above. Beginning with Irving Fisher, many researchers have found that a 1 percent increase in the inflation rate yields less than a 1 percent increase in the pre-tax interest rate. The results of Feldstein-Eckstein [9] and Fama [7] suggest a one-to-one relationship. To our knowledge, there is no convincing evidence of a greater than one-to-one relationship.

In addition to the lack of empirical validation, there is at least one conceptual shortcoming in past studies of inflation and interest rates. Most theories of corporate capital structure are not suitable to macroeconomic models. For

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many years, the basic paradigms here have been the Modigliani-Miller theorems in [20] and [21]. These papers yield the unrealistic conclusion that, in the presence of corporate taxes, a firm should issue only debt. An interior optimum to the debt to equity ratio can occur when bankruptcy costs, agency costs, and signaling behavior are considered. Unfortunately, due to their qualitative nature, the results received are not applied easily to macroeconomic models. However, Miller [17] has recently provided an elegant theory of capital structure in the presence of both personal and corporate taxes. Our paper shows that this framework can be readily integrated with a macroeconomic model.

To study the relationship between inflation and interest rates in the presence of taxation, a model that considers the portfolio choices of investors in different tax brackets in order to determine the relative returns on securities subject to different tax treatments is needed. One such model is that of Miller [1977]. Our paper shows that his framework can be readily integrated with a macroeconomic model.

This paper yields at least three developments. The first is a set of circumstances in which the theoretical relationship between interest rates and inflation is near that observed empirically. We do not wish to overplay this result because another set of circumstances in the model produces a responsiveness of the rate of interest to inflation above that found by Darby. Also, we are by no means the first to develop a theory to explain the empirical results concerning the Fisher effect. For example, Mundell [22] argues that a cash-balance effect can lower the responsiveness of interest rates to inflation-rate changes. Gandolfi [12] obtains similar results by considering depreciation charges and capital gains taxes.

More importantly, we posit a macroeconomic model that is consistent with both the theory and practice of corporate finance. Many previous works posit only debt financing, an assumption not consistent with real world practice. Others, such as Feldstein, Green, and Sheshinski [10], assume mixed financing but do not ground their treatment in corporate financial theory.

Third, the effect of inflation on income distribution is considered. Some previous works have posited that any change in taxes due to inflation is immediately restored to the public in such a way that the income of each individual, his saving behavior, and his utility curves are unaltered. This assumption seems acceptable in these models because the results are obtained in a world of essentially identical individuals. However, individuals are clearly not identical in Miller's system of progressive taxation. Results concerning income redistribution are useful additions to the paper.

The paper is organized as follows. A review of the Miller model is presented in Section II. In Section III, we analyze the impact of changes in the rate of inflation on interest rates in the basic Miller model. A generalization of the results to a world with capital gains taxes appears in Section IV. Concluding remarks are presented in Section V.

II. Review of the Miller Model

Miller [17] posits a certainty model with a flat corporate tax rate of tc and a system of graduated personal taxes. Because interest costs are tax deductible and payments to equity holders are not deductible, firms are indifferent between issuing equity and debt when

$$i_D(1 - tc) = i_E \tag{1}$$

where i_D is the pre-tax cost of debt and i_E is the required return on equity.

Under the assumption that individuals are taxed on interest income but do not pay taxes on equity returns,[1] the individual prefers an investment in equity (debt) when

$$i_D(1 - tpi) < (>) i_E \tag{2}$$

where tpi is the marginal personal tax rate for individual i.

Miller (Fig. 1, p. 269 in [17]) determines equilibrium from a demand and a supply schedule. For given i_E, the demand for bonds is a positive function of i_D. Only tax-exempt investors desire bonds when $i_D = i_E$. As i_D rises, more investors choose bonds. For given i_E, the supply of bonds outstanding is infinitely elastic at the value of i_D such that (1) holds.

Because (1) must hold in equilibrium, we can use (2) to classify investors into three categories:

(A) Any high-tax-bracket investor ($tpi > tc$) invests only in stocks.

(B) Any low-tax-bracket investor ($tpi < tc$) invests only in bonds.

(C) Any marginal investor ($tpi = tc$) is indifferent between stocks and bonds.

Because (1) implies that the after-tax cost of debt is equal to the after-tax cost of equity, the value of an individual firm is unaffected by its debt to equity ratio. Thus, the amount of debt issued by a given firm is indeterminant. However, total debt in the economy must be equal to the aggregate holdings in financial assets by all individuals with $tpi < tc$.

III. The Effect of Inflation on Interest Rates

We consider a world of perfect certainty with income taxes but without capital gains taxes. A model where the interest rate is determined by the supply and demand of loanable funds is posited.[2] We specify the saving and investment functions below. The two functions are equal at equilibrium. Inflation impacts the equilibrium since the after-tax rates of interest applicable to saving and investment are affected by the rate of change in prices.

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[1] We employ this assumption initially because Miller uses it in the major example illustrating his model. However, we relax the assumption in a later section of the paper.

[2] This type of model has been used frequently in the analysis of interest rates, inflation, and taxes. For example, see Cross [4] and Gandolfi [13], [12].

A. The Investment Function

In a world with neither taxes nor inflation, the firm invests until

$$MP_K = i$$

where MP_K is the marginal physical product of capital and i is the interest rate. At this point we need no distinction between the cost of equity and the cost of debt.

With both inflation and corporate taxes, the firm invests until

$$MP_K(1 - tc) + \pi = bi_D(1 - tc) + (1 - b)i_E \tag{3}$$

where π is the rate of inflation and b is the proportion of the firm financed with bonds. Both i_D and i_E are nominal quantities.

Under inflation, the two returns to capital are: (1) the marginal physical product of capital; and (2) the price increase in the asset. The first return is taxed at the corporate rate. The second return escapes taxation under our assumption of no capital gains tax. This latter return is equal to π if all assets increase at the same rate and there is no depreciation. Equation (3) reflects the fact that firms can deduct interest payments but cannot deduct payments to equity holders.

Using (1), we can rewrite (3) as

$$MP_K(1 - tc) + \pi = i_E = i_D(1 - tc) . \tag{4}$$

In accordance with Miller's theory, b is the indeterminant in (3) and, therefore, does not appear in (4). We can now solve for MP_K as

$$MP_K = \frac{i_E - \pi}{1 - tc} = i_D - \frac{\pi}{1 - tc} . \tag{5}$$

As mentioned above, investment is carried out until (3) is reached. In terms of (5), the rate of investment can be written as

$$f\left(i_D - \frac{\pi}{1 - tc}\right) = f\left(\frac{i_E - \pi}{1 - tc}\right) . \tag{6}$$

We employ the traditional assumption that $f' < 0$.

B. The Saving Function

1. The After-Tax Rate of Return

Next is the treatment of the saving function. For simplicity, we assume that there are two groups of individuals.[3] Each individual in group 1 has personal tax

[3] The equilibrium condition of (1) still holds when there are only two groups. Our results could be derived for any arbitrarily large number of groups, though the mathematics would be more involved. Because little understanding would be added by this complex approach, we choose the assumption of two groups.

bracket, $tp1 < tc$, and each member in group 2 has tax bracket $tp2 > tc$. In keeping with the basic Miller model, assume that interest payments are taxed at the individual's personal tax rate while returns on equity are completely untaxed. Thus, given (1), all individuals in group 1 prefer bonds while all individuals in group 2 prefer stocks. Each member of group 1 has both identical income and the same saving function. Similarly, each member of group 2 has an identical income (that is presumed to be higher than the income of those in group 1) and possesses the same saving function (that need not be the same as the saving function of each member of group 1).

The saving of each individual is a function of his real after-tax rate of return and real income. This real after-tax return is $i_D(1 - tp1) - \pi$ for each member of group 1. From (1), we can express this return in terms of i_E as $i_E[(1-tp1)/(1-tc)] - \pi$.[4] The real after-tax return is $i_E - \pi$ for each member in group 2.

2. The Individual's Real Income

While the above calculation of the real after-tax interest rate is straightforward, the determination of an individual's income is more complex. We begin our discussion here with a world of no inflation, where each stockholder earns an after-tax rate of return of i_E^0. Each bondholder earns an after-tax rate of

$$(7) \qquad i_D^0(1 - tp1) = i_E^0\left(\frac{1 - tp1}{1 - tc}\right).$$

With an inflation rate of π, the bondholder now earns a real after-tax rate of return of

$$(8) \qquad i_D(1 - tp1) - \pi = i_E\left(\frac{1 - tp1}{1 - tc}\right) - \pi$$

implying that the change in the real after-tax return to the bondholder due to inflation is

$$(9) \qquad \left(i_E - i_E^0\right)\left(\frac{1 - tp1}{1 - tc}\right) - \pi .$$

With no inflation, total taxes in the economy are

$$(10) \qquad \begin{aligned} C + J + D\left(i_D^0 tp1\right) - D\left(i_D^0 tc\right) &= C + J + Di_D^0(tp1 - tc) \\ &= C + J + Di_E^0\left(\frac{tp1 - tc}{1 - tc}\right) \end{aligned}$$

where:

C is the corporate tax on net income before interest, i.e., the corporate tax if interest were not deductible;

[4] While it is more intuitive to express a bondholder's return as a function of i_D, this term frequently will be expressed as a function of i_E so that both the return to a stockholder and the return to a bondholder can be written as a function of the same parameter.

J is the tax on labor income; and

D is the debt outstanding in the economy.

The third term on the left-hand side (LHS) of (10) is the total tax paid by bondholders on interest. The fourth term on the LHS is the total deduction taken by all corporations on interest paid to bondholders.

Under inflation, total taxes are[5]

$$C + J + D\left(i_E\left(\frac{tp1 - tc}{1 - tc}\right)\right)$$

implying from (10) that the change in taxes due to inflation is

$$D\left(\left(i_E - i_E^0\right)\left(\frac{tp1 - tc}{1 - tc}\right)\right) . \tag{11}$$

This expression is negative because $tp1 < tc$.

With a constant level of real government spending and a balanced budget, a reduction in taxes in one sector of the economy must be offset by an increase in taxes in another sector. We assume that a head tax is used to keep the budget balanced, implying that marginal tax brackets are unaffected. For simplicity, we posit that the fraction K of the tax shortfall represented by (11) is paid by bondholders, and the fraction $1 - K$ of (11) is paid by stockholders.[6] From (9) and (11), we see that the income of bondholders rises (falls) during inflation if

$$D\left[\left(i_E - i_E^0\right)\left(\frac{1 - tp1}{1 - tc}\right) - \pi - K\left(i_E - i_E^0\right)\left(\frac{tc - tp1}{1 - tc}\right)\right] \gtreqless 0 . \tag{12}$$

Because income is a constant in the economy, any increase (decrease) in the income of bondholders is offset by a decrease (increase) in the income of stockholders.

We write the aggregate saving function of the low-tax-bracket individuals during inflation as

$$L\left(r_{AT}^L, Y_L\right) \tag{13A}$$

[5] We assume that C, J, and D are invariant to the rate of inflation over a small interval of time. The constancy of C and J follows from the traditional assumption that investment over that interval is insignificant relative to the initial capital stock. Thus, the pre-tax incomes to capital and labor, both of which are functions of capital, can be treated as constants.

We assume that all debt has infinitesimal maturity so that capital gains and losses do not accrue to bondholders when the interest rate changes. In addition, no new debt is issued because the capital stock is a constant. Furthermore, if marginal tax brackets are unaffected by inflation, no existing equity (debt) is converted to debt (equity). Thus, debt is a constant.

[6] We achieve similar results if the extra tax collected from each group is a more complex function of (11).

where r_{AT}^{L} is the real after-tax interest rate and Y_L is the aggregate real income of these individuals. The arguments of the function L in (13A) are[7]

$$(13B) \qquad r_{AT}^{L} = i_E\left(\frac{1 - tp1}{1 - tc}\right) - \pi$$

$$(13C) \quad Y_L = \overline{Y}_L + D\left[\left(i_E - i_E^0\right)\left(\frac{1 - tp1}{1 - tc}\right) - \pi\right] - KD\left[\left(i_E - i_E^0\right)\left(\frac{tc - tp1}{1 - tc}\right)\right].$$

Some explanation of equation (13C) is needed. First $\overline{Y}_L$ is the total income from both labor and capital accruing to low-tax-bracket individuals in a no-inflation world. The remainder of the expression is the left-hand side of (12), the change in the bondholder's income from inflation.

The aggregate saving function of the high-tax-bracket individuals is

$$(14A) \qquad H\left(r_{AT}^{H}, Y_H\right)$$

where r_{AT}^{H} is the real after-tax interest rate for the high-tax-bracket individuals and Y_H is the aggregate real income of these individuals.

The arguments of the function H in (14A) are

$$(14B) \qquad r_{AT}^{H} = i_E - \pi$$

$$(14C) \quad Y_H = \overline{Y}_H - D\left[\left(i_E - i_E^0\right)\left(\frac{1 - tp1}{1 - tc}\right) - \pi\right] + KD\left[\left(i_E - i_E^0\right)\left(\frac{tc - tp1}{1 - tc}\right)\right].$$

As with (13C), some explanation of equation (14C) is needed. First, $\overline{Y}_H$ is the total income from both labor and capital accruing to high-tax-bracket individuals in a no-inflation world. The second and third terms, which also appear in (13C), are the gains to the bondholders from inflation. Because we are in a zero-sum situation, any increase in the wealth of bondholders is a decrease in the wealth of stockholders. We assume that the partial derivatives L_1, L_2, H_1, and H_2 are all positive in (13) and (14).

[7] For simplicity, ignore the effect of real cash balances on the saving function. While a full treatment of this effect is algebraically complex, it is relatively straightforward conceptually. Following the work of Santomero [23], we can include the cash balance effect by writing income as

$$(13C') \qquad Y_L^* = Y_L - l^D\left(Y_L, i_E\left(\frac{1 - tp1}{1 - tc}\right), \pi\right)\pi$$

where money demanded, l^D, is a function of income, the nominal after-tax interest rate, and the inflation rate. (Using Cagan [3], we treat the interest rate and the inflation rate as separate arguments in (13C').) Given that money held is l^D, the loss of income due to inflation is $l^D\pi$.

C. Equilibrium

Equilibrium occurs when saving equals investment, i.e., when (6) equals (13A) plus (14A). The full derivative of this equality with respect to i_E and π is

$$\frac{f'}{1-tc}\left(di_E - d\pi\right) = H_1 \cdot \left(di_E - d\pi\right) - H_2 \cdot D\left(\left(\frac{1-tp1}{1-tc}\right) - K\left(\frac{tc-tp1}{1-tc}\right)\right)di_E$$

$$(15) \qquad + H_2 \cdot Dd\pi + L_1 \cdot \left(\frac{1-tp1}{1-tc}\right)di_E - L_1 d\pi$$

$$+ L_2 D\left(\frac{1-tp1}{1-tc} - K\left(\frac{tc-tp1}{1-tc}\right)\right)di_E - L_2 Dd\pi \ .$$

From (15), we derive

$$(16) \qquad \frac{di_E}{d\pi} = \frac{\frac{f'}{1-tc} - H_1 - L_1 + \left(H_2 - L_2\right)D}{\frac{f'}{1-tc} - H_1 - L_1 \cdot \left(\frac{1-tp1}{1-tc}\right) + \left(H_2 - L_2\right)D\left(\frac{1-tp1}{1-tc} - K\left(\frac{tc-tp1}{1-tc}\right)\right)} \ .$$

1. Interest Rate Effect

An increase in the rate of inflation has two basic effects. First, the change in the real after-tax rate of interest for the high-tax-bracket individuals differs from this change for the low-tax-bracket individuals. Call this the interest rate effect. Second, income is redistributed across the two groups. Equation (16) can be interpreted intuitively by examining these two effects. The terms f', H_1, and L_1 are derivatives of investment and saving with respect to particular interest rates. The terms H_2 and L_2 are derivatives with respect to income. The interest rate effect is highlighted by focusing on f', H_1, and L_1, while the income redistribution effect is illuminated by following H_2 and L_2.

To focus on the interest rate effect, we initially posit that $H_2 = L_2$,[8] causing the two terms in (16) involving $H_2 - L_2$ to be equal to zero. Here, any redistribution of income does not affect our results since the marginal propensity to save for the group gaining income is equal to this propensity for the group losing income. Alternatively, one could focus on the interest rate effect by setting $K = 1$. This assumption implies that the head tax offsetting lost taxes caused by inflation is borne totally by the bondholders. Here, since the same term, $(H_2 - L_2)D$, appears in both the numerator and the denominator of (16), the interest effect to be described below is at most mitigated.[9]

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[8] This assumption will be relaxed later in the paper.

[9] As a third possibility we could have rewritten (13C) and (14C) so that additional taxes paid by group 1 would be $Dk\{(i_E - i_E^0)\ [(1-tp1)\ /\ (1-tc)] - \pi\}$. That is, additional taxes would be a proportion of the increase in bondholders' income from inflation and *not* a proportion of the tax shortfall. Here, all income transfer effects would be eliminated if $k = 1$, so that the interest rate effect would be observed easily.

In this third possibility, no redistribution of income occurs. Conversely, if $H_2 = L_2$, income can be redistributed, though any redistribution does not affect our results.

Given that either $H_2 = L_2$ or $K = 1$, equation (16) equals 1 if $tp1 = tc$. However, Miller's work implies that the tax bracket of each bondholder is below the corporate tax rate ($tp1 < tc$ in our model). Thus, the absolute value of the term involving L_1 in the numerator is less than the absolute value of the term involving L_1 in the denominator. Because $f' < 0$, $H_1 > 0$, and $L_1 > 0$, it follows that $di_E/d\pi < 1$. From equation (1), we know that $di_D/di_E = 1/(1 - tc)$, implying that

$$(17) \qquad \frac{di_D}{d\pi} < \frac{1}{1 - tc}.$$

The inequality in (17) is interesting because other models, such as those of Darby [5], Feldstein [8], and Gandolfi [12], yield the result that $di_D/di_\pi = 1/(1 - tc)$.[10] Thus, if either L_2 is nearly equal to H_2 or K is nearly equal to 1, the Miller model may help explain why the interest rate has not been found empirically to be as responsive to the inflation rate as previous theoretical models suggested.

We can achieve some additional understanding of the result by examining finite changes in (6), (13), and (14) in a world where saving is not a function of income. Let us imagine that $\Delta(1 - tc)i_D \equiv \Delta i_E = \Delta\pi > 0$. From (6), we see that f remains unchanged. Also (14) remains unchanged when the second argument of H is ignored. However, (13) rises when the second argument of L is ignored because, with $tp1 < tc$, $\Delta i_E[(1 - tp1)/(1 - tc)] \equiv \Delta i_E(1 - tp1) > \Delta\pi$. Thus, an equilibrium cannot be maintained when $\Delta i_D/\Delta\pi = [1/(1 - tc)]$ or, equivalently, when $\Delta i_E/\Delta\pi = 1$. However, an equilibrium can be maintained if $\Delta i_E/\Delta\pi \equiv (1 - tc)(\Delta i_D/\Delta\pi) < 1$. Here, f rises and H falls, potentially offsetting the rise in L.

The above finding can be explained intuitively by focusing on the asymmetry in the Miller model. Low-tax-bracket individuals increase their saving if $\Delta i_E \equiv (1 - tc)\Delta i_D = \Delta\pi > 0$. High-tax-bracket individuals would *decrease* their saving if (1) $\Delta i_E \equiv (1 - tc)\Delta i_D = \Delta\pi > 0$ and (2) they were *required* to invest in bonds. However, because these high-tax-bracket individuals are allowed to invest in equities, their saving function is unchanged. In summary, this asymmetry holds because of *both* (1) the differing tax treatment of the returns on the two financial instruments, and (2) the graduated tax system.[11]

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[10] Gandolfi finds that $di_D / d\pi = 1 / (1 - tc)$ in his basic model, though he reaches the inequality of (17) when depreciation and capital gains are considered. In addition, Modigliani [19] finds that the equality relationship, which he calls the "Super Fisher's Law," should hold approximately, but need not hold precisely, in his model. Feldstein, Green and Sheshinski (FGS) [10] estimate that $di_D / d\pi = 1.45$.

The papers of Darby, Feldstein and Gandolfi treat only debt financing while Modigliani and FGS consider both debt and equity financing. Thus, these latter two papers are closer to ours in this regard. However, both Modigliani and FGS restrict the supply of financial instruments and solve for an optimal debt-equity ratio. Conversely, Miller considers a completely elastic supply and finds that leverage does not matter. Furthermore, both Modigliani and FGS take a long-run growth approach while we use a loanable funds model.

[11] To show this more formally, we separately relax these two conditions. First consider an equilibrium with a graduated tax system but where only bonds are employed. Assume that any income

Now that we have established and interpreted (17), we can focus on the extent of the inequality. If $tp1 = tc$, $di_E/d\pi = 1$. This same relationship holds if $L_1 = 0$, i.e., the saving function of each low-tax-bracket individual is completely inelastic with respect to the real after-tax interest rate, and either $K = 1$ or $H_2 = L_2$. Alternatively, imagine that $H_2 = L_2$ and $f' = H_1 = tp1 = 0$, but $L_1 > 0$. Here $di_E/d\pi = 1 - tc$, implying that $di_D/d\pi = 1$. Thus, given that either $K = 1$ or $H_2 = L_2$, our model predicts that $1 < di_D/d\pi < 1/(1 - tc)$ since we assume that $f' < 0, H_1 > 0$, and $L_1 > 0$.

One often hears from real-world practitioners that there should be a near one-to-one relationship between Δi_D and $\Delta\pi$ because the bond market is almost completely dominated by pension funds and other tax-free investors. Miller's model shows that this analysis is not quite correct. Given the premise that $tp1 = 0$ for all bondholders, the one-to-one relationship would hold only if, in addition, the saving function of the high-tax-bracket individuals and the investment function are both totally inelastic with respect to $i_E - \pi$.

2. Redistribution Effect

Earlier, we explored the effect that the elasticity of saving with respect to the real after-tax interest rate has on the equilibrium relationship between the inflation rate and interest rates. We now wish to investigate the effect of inflation-caused income transfers on the relationship between inflation and interest rates.

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redistribution caused by inflation is offset precisely by a head tax so that the savings of the two groups can be written as a function of the real after-tax interest rate, yielding

$$L\left(i_D(1 - tp1) - \pi\right) \quad \text{and}$$

$$H\left(i_D(1 - tp2) - \pi\right).$$

Given (6), (18), and (19), equilibrium occurs when

$$f\left(i_D - \frac{\pi}{1 - tc}\right) = L\left(i_D(1 - tp1) - \pi\right) + H\left(i_D(1 - tp2) - \pi\right),$$

implying the following condition

$$\frac{di_D}{d\pi} = \frac{\frac{f'}{1 - tc} - H' - L'}{f' - H' \cdot (1 - tp2) - L' \cdot (1 - tp1)}.$$

If $tc = tp2 = tp1$, $di_D / d\pi = 1 / (1 - tc)$, which is the result achieved by Darby, Gandolfi, and Feldstein. However, let $tp2 > tc > tp1$ and, for symmetry, $tp2 - tc = tc - tp1$. Here, $di_D / d\pi \gtreqless 1 / (1 - tc)$ as $H' \gtreqless L'$. Thus, we have a more ambiguous relationship with only one financial instrument than we have with the Miller model. This result occurs because, if $(1 - tc)\Delta i_D = \Delta\pi > 0$, high-tax-bracket individuals reduce saving while low-tax-bracket individuals increase saving. One can compare the extent of the reduction to the extent of the increase by comparing H' and L'.

Next consider an equilibrium with both bonds and stocks but with no graduated tax system; imagine that the personal tax rate is tc for all individuals. Here, $di_E / d\pi = 1$ in equation (16) under the assumption that either $L_2 = H_2$ or $K = 1$. (We do not consider the cases in which either the personal tax rate is greater than tc for *all* individuals or the personal tax rate is less than tc for *all* individuals because Miller's model does not hold in either of these two cases.)

Employing (16), we previously established that $1 - t_c < di_E / d\pi < 1$ if $H_2 = L_2$. This inequality also must hold if $H_2 < L_2$; i.e., the marginal propensity to save is negatively related to income. To see this, consider the term in the denominator of (16), $(1-tp1)/(1-tc) - K[(tc-tp1)/(1-tc)]$. The value of this term lies between 1 and $1/(1-tc)$ under the assumption that $0 \leqslant K \leqslant 1$ because the term equals 1 when $tp1 = t_c$ and equals $1/(1-tc)$ when $tp1 = K = 0$. Given that $f' < 0$, $H_1 > 0$ and $L_1 > 0$, $di_E/d\pi = 1$ only when $tp1 = tc$, a case not considered by Miller. And, $di_E / d\pi$ can never fall to $1 - tc$.

An example where $H_2 < L_2$ in our model is the Friedman [11] saving function

$$S = g(r_{AT})y \tag{21}$$

where r_{AT} is the real after-tax interest rate facing the individual, y is the permanent real income of the individual, and $g' > 0$. The marginal propensity to save out of income, dS/dy, equals $g(r_{AT})$. Given (13B) and (14B), $L_2 > H_2$ since $g' > 0$. Thus, $1 - tc < di_E / d\pi < 1$ in Friedman's model.

The redistribution of income effect, given Friedman's consumption function, is in the same direction as the interest rate effect discussed earlier. This redistribution effect can be explained intuitively. In our model, inflation causes income to be redistributed from the equity holders to the bondholders. Because the bondholders have a greater marginal propensity to save, the saving schedule increases with inflation, putting downward pressure on the interest rate.

However, if $H_2 > L_2$, $di_E/d\pi$ can be greater than 1. To see this, imagine that $L_1 = 0$ and $K < 1$. Here, the coefficient of $(H_2 - L_2)D$ in the denominator of (16) is greater than one. Because $f' < 0$ and $H_1 > 0$, $di_E / d\pi > 1$ in this case.

D. A Note on Possible Extensions

As mentioned earlier, we are primarily interested in applying the Miller equilibrium condition to a basic macroeconomic model. Therefore, in most cases, we do not extend Miller's insight on capital structure to other, more complex economies. However, there are three extensions worth discussing briefly in this section.

Without the imposition of a head tax, real tax revenues decline in our model. Conversely, real taxes may rise with the rate of inflation in practice, because, among other reasons, of tax penalties on tangible assets at the corporate level and tax bracket "creep" at the individual level.

We first examine penalties at the corporate level. Because replacement cost accounting is not allowed, costs of both inventories and depreciable assets are understated in an inflationary economy. A tax on capital gains is an excellent proxy for illusory inventory gains and might be at least a serviceable proxy for understated depreciation.[12] Gandolfi [12] shows that, in the presence of this tax, the investment schedule is lowered as the rate of inflation rises, thereby reducing the responsiveness of the interest rate to the rate of inflation. Because this effect

[12] The relationship of a capital gains tax and a tax on understated inventory costs first was suggested to me by M. Flannery.

on the investment function applies directly to our model, no more will be said concerning it.

In addition, however, these corporate tax penalties are likely to redistribute income from stockholders to bondholders as inflation rises. The extra taxes reduce the value of equity. And, because this tax penalty reduces the tax shortfall caused by inflation (or even leads to a surplus), bondholders will pay a smaller head tax (or even receive a rebate). Thus, this corporate tax penalty accentuates the redistribution effect discussed previously.

Second, we consider penalties at the personal level. Tax bracket "creep" also impacts both the interest rate effect and the redistribution effect. If a rise in the rate of inflation increases the marginal tax bracket of bondholders, their real after-tax rate of return is reduced for a given i_D and π. Thus, if redistribution effects are ignored, the values of both $di_E / d\pi$ and $di_D / d\pi$ are greater here than they were in the previous section.

The impact of bracket creep on the redistribution of income effect is indeterminate. Inflation will cause the labor income of both group 1 and group 2 to be taxed at a higher rate. Political pressures will determine which group suffers more from bracket creep.

Third, we briefly consider uncertainty. Under certainty, all in group 1 choose bonds while all in group 2 choose equity, i.e., full separation is achieved. With uncertainty, group 1 (group 2) individuals may purchase some equity (bonds) as well to achieve the desired level of risk. Here, the asymmetry mentioned earlier is weaker, reducing the interest rate effect and thereby raising $di_E / d\pi$. However, the impact of uncertainty on the interest rate effect is partially offset by flexibility in financial instruments. For example, if tax-exempt bonds are available to high-tax-bracket individuals and if risky bonds and risky preferred stocks are available to low-tax-bracket individuals, separation may obtain.[13] Furthermore, the redistribution effect remains with or without separation. Inflation causes income to be redistributed from group 2 to group 1. If group 1 has a greater marginal propensity to save, e.g., Friedman's model holds, the saving function increases with inflation, putting downward pressure on the interest rate.

IV. Generalization to an Equity Tax

Because Miller [17] derives his results for the case where there is no personal tax on the returns to equity holders, we used this assumption in the previous section. Though Miller's general conclusions also apply in the presence of a tax on equity, our results concerning inflation change in this case. We analyze this situation below.

One can model this additional personal tax in a variety of ways. We posit that the tax is $i_E\, a{\cdot}tpi$, where $a < 1$. This tax can arise for at least two reasons. First, $a{\cdot}tpi$ might be the capital gains tax. Second, if we employ the commonly

[13] In fact, DeAngelo-Masulis [6] posit a states-of-the-world model where both a complete set of elementary debt securities that each pay off only in a specific state and a complete set of similar elementary equity securities are issued. Since separation must occur here, both Miller's results and our results of Section IIIC hold exactly.

used assertion that capital gains are not taxed, the dividend yield is $a \cdot i_E$. The reader can assume either of the two possibilities for much of the following discussion though previous authors have questioned voluntary dividend payments in Miller's model.[14]

For any tax bracket, tpi, the after-tax return to the stockholder is $i_E(1 - a \cdot tpi)$ while the return to the bondholder is $i_D(1 - tpi) = i_E[(1 - tpi)/(1 - tc)]$. By equating these two returns, we find that the individual is indifferent between bonds and stocks when

$$tp^* = \frac{tc}{1 - a + a \cdot tc} . \tag{22}$$

Note that $tp^* > tc$ when $a > 0$.

We now posit that $tp1 < tp^* < tp2$, rather than $tp1 < tc < tp2$. To simplify matters, assume that income is redistributed through taxation in such a manner that income for each of the two groups is unaffected by inflation.[15] Thus, we can express our saving functions in terms of the after-tax real rate only, yielding

$$L\left(i_E\left(\frac{1 - tp1}{1 - tc}\right) - \pi\right) \quad \text{and}$$

$$H\left(i_E(1 - a \cdot tp2) - \pi\right) .$$

Our equilibrium condition is now

$$f\left(\frac{i_E - \pi}{1 - tc}\right) = L\left(i_E\left(\frac{1 - tp1}{1 - tc}\right) - \pi\right) + H\left(i_E(1 - a \cdot tp2) - \pi\right) .$$

Taking the total derivative with respect to i_E, π, and a and then rearranging terms, we obtain

$$\frac{di_E}{d\pi} = \frac{\dfrac{f'}{1 - tc} - L' - H' - H' \cdot \left(i_E tp2 \dfrac{da}{d\pi}\right)}{\dfrac{f'}{1 - tc} - L' \cdot \left(\dfrac{1 - tp1}{1 - tc}\right) - H' \cdot (1 - a \cdot tp2)} . \tag{23}$$

To interpret (23), first note that, if $a = da/d\pi = 0$, this equation is merely (16) when $H_2 = L_2$. As in (16), $di_E / d\pi < 1$.

Let us now consider the case where $a > 0$ and $da/d\pi = 0$. Here the relationship between L' and $L' \cdot [(1 - tp1)/(1 - tc)]$ is ambiguous. Since $tp^* > tc$, $tp1$ could be greater than tc. This, of course, differs from the previous section where $tp1 < tc$. In addition, $H'(1 - a \cdot tp2) < H'$.

These two effects allow the possibility that $di_E / d\pi > 1$. To see this, imagine that $tp1 > tc$, implying that $L' > L'[(1 - tp1)/(1 - tc)]$. Given that $da/d\pi = 0, f' < 0, H' > 0$ and $L' > 0$, we find that $di_E / d\pi > 1$. This same inequality can be reached when $tp1 < tc$ if L' is small and H' is large.

[14] For example, see [14].

[15] We employ this assumption purely for simplicity. The interested reader can add our earlier discussion on income redistribution to this section to get a more complete model.

The difference between this section and the previous one can be explained intuitively. In the last section, the saving of equity holders was unaffected when $\Delta i_E = \Delta\pi > 0$. However, the saving of these individuals would be reduced in this section because of the new tax on equity. Furthermore, any bondholder whose tax bracket is above *tc* also would be hurt if $\Delta i_E \equiv \Delta i_D / (1 - tc) = \Delta\pi > 0$. In other words, *all* individuals with personal tax brackets above *tc* must reduce savings if $\Delta i_E = \Delta\pi$. This is in marked contrast to the previous section. There, *all* individuals whose tax rate exceeded *tc* were unaffected when $\Delta i_E = \Delta\pi$ because all of them invested in the then untaxed equity securities.

Now consider (23) when a is endogenous. Because $f' < 0$, $H' > 0$, and $L' > 0$, the effect of π on i_E is reduced (increased) if $da/d\pi < 0$ ($da/d\pi > 0$). While stockholders are hurt by the tax on equity, this tax is reduced as a is lowered.

We believe that a is likely to be a function of inflation. First, stocks are probably sold less frequently during inflationary periods because nominal returns are higher then, implying that $da/d\pi < 0$. Second, inflation may affect dividend policy.

Unfortunately, it is quite difficult to model this second relationship because, even with stable prices, there is no complete theory of dividend policy. On the one hand, many researchers state that a corporate strategy of no dividends is preferred because the tax rate on dividends is above the tax rate on capital gains. On the other hand, some have argued theoretically that dividends may be irrelevant even in a world of differential taxes.[16] Furthermore, though the empirical literature on the subject is impressive, the debate is far from settled on this front either.[17]

We lean toward the assumption that $da/d\pi < 0$, since a zero or positive relationship between dividends and price level changes is difficult to sustain, particularly at high rates of inflation. For example, imagine a no-growth all-equity firm where initially $i_E = 8$ percent, $\pi = 0$, and $a = ½$, implying that the dividend yield is 4 percent. Later, if $\pi = 10$ percent, $i_E = 18$ percent,[18] and $a = ½$, the dividend yield is 9 percent. However, the ratio of cash flow to market value is 8 percent in a no-growth firm, implying that dividends exceed cash flow if $\Delta a = \Delta\pi$.

IV. Conclusions

Recently, Miller [17] has analyzed capital structure in the presence of both corporate and personal taxes. Our work investigates the effect of inflation on interest rates when the Miller equilibrium condition is employed in a loanable-funds model.

We view saving as a function of both the real after-tax interest rate and real income. To nullify any effects from the redistribution of income, we initially assume that the marginal propensity to save is unrelated to one's tax bracket.

[16] See [1] and [18].

[17] For example, see [16] and [2].

[18] We are assuming that $\Delta i_E = \Delta\pi$ in this example. A similar conclusion would still follow if this equality did not hold.

Here $(di_E/d\pi) < 1$, i.e., the change in the return on equity must be less than the change in the inflation rate, a result caused by the asymmetry in the Miller model.

Next, we consider the redistribution of income. The shares of national income held by bondholders and stockholders are affected by the rate of inflation. This income redistribution is not merely a function of after-tax rates of return on stocks and bonds but of the government's taxation policy. Because taxes are reduced with an increase in the rate of inflation in our model, the government must raise taxes by another means. Thus, government cannot easily escape being an agent of redistribution. If (1) income is redistributed from the stockholders to the bondholders when the rate of inflation increases, and (2) the Friedman consumption function holds, $1 - tc < di_E / d\pi \leqslant 1$. However, the Friedman consumption function implies in our model that the marginal propensity of low-tax-bracket individuals to save is above that of the marginal propensity of high-tax-bracket individuals to save. If this relationship is reversed, $di_E / d\pi$ can be greater than 1. In other words, $di_D / d\pi$ can be greater than $1/(1 - tc)$.

We find three additional effects when the Miller model is generalized to include taxation on equities. First, an individual whose personal tax bracket is higher than tc may hold bonds. His real after-tax return is lowered if $(di_E / d\pi) \equiv [di_D(1 - t_c)] / d\pi = 1$. Second, the real after-tax return is lowered for all equity holders if $(di_E/d\pi) = 1$. Ignoring all redistribution effects, we find that these two factors increase the responsiveness of interest rates to changes in the rate of inflation. Third, inflation rate changes may affect a, that portion of equity returns subject to tax. The responsiveness of interest rates is reduced (increased) if $(da / d\pi) < 0$ $[(da / d\pi) > 0]$.

References

[1] Black, Fisher, and Myron Scholes. "The Effect of Dividend Yield and Dividend Policy on Common Stock Prices and Returns." *Journal of Financial Economics,* Vol. 1 (May 1974), pp. 1-22.

[2] Blume, Marshall. "Stock Returns and Dividend Yields: Some More Evidence." Working Paper No. 1-79, Rodney L. White Center for Financial Research, University of Pennsylvania, Wharton School, Philadelphia, PA. (1979).

[3] Cagan, Philip. *Studies in the Quantity Theory of Money.* M. Friedman, ed. Chicago, IL: University of Chicago Press (1956).

[4] Cross, Stephen. "A Note on Inflation, Taxes and Investment Returns." *Journal of Finance,* Vol. 35 (March 1980), pp. 177-180.

[5] Darby, Michael. 'The Financial and Tax Effects of Monetary Policy on Interest Rates." *Economic Inquiry,* Vol. 13 (June 1975), pp. 266-276.

[6] DeAngelo, Harry, and Ronald Masulis. "Optimal Capital Structure under Corporate and Personal Taxation." *Journal of Financial Economics,* Vol. 8 (March 1980), pp. 3-29.

[7] Fama, Eugene. "Short-Term Interest Rates as Predictors of Inflation." *American Economic Review,* Vol. 65 (June 1975), pp. 269-282.

[8] Feldstein, Martin. "Inflation, Taxes, and the Rate of Interest: A Theoretical Analysis." *American Economic Review,* Vol. 66 (December 1976), pp. 889-920.

[9] Feldstein, Martin, and Otto Eckstein. "The Fundamental Determinants of the Interest Rate." *Review of Economics and Statistics,* Vol. 52 (November 1970), pp. 363-376.

[10] Feldstein, Martin; Jerry Green; and Eytan Sheshinski. ''Inflation and Taxes in a Growing Economy with Debt and Equity Finance.'' *Journal of Political Economy,* Vol. 86 (April 1978), pp. 553-570.

[11] Friedman, Milton. *A Theory of the Consumption Function.* Princeton, NJ: Princeton University Press (1957).

[12] Gandolfi, Arthur. ''Inflation, Taxation, and Interest Rates.'' *Journal of Finance,* Vol. 37 (June 1982), pp. 797-807.

[13] ——————. ''Taxation and the 'Fisher Effect.' '' *Journal of Finance,* Vol. 31 (December 1976), pp. 1375-1386.

[14] Hamada, Robert. ''Differential Taxes and the Structure of Equilibrium Rates of Return: Managerial Implications and Remaining Conundrums.'' Working Paper #68, Center for Research in Security Prices, University of Chicago, Chicago, IL. (April 1982).

[15] Jaffe, Jeffrey. ''A Note on Taxation and Investment.'' *Journal of Finance,* Vol. 33 (December 1978), pp. 1439-1446.

[16] Litzenberger, Robert, and K. Ramaswamy. ''The Effect of Personal Taxes and Dividends on Capital Asset Prices: Theory and Empirical Evidence.'' *Journal of Financial Economics,* Vol. 7 (June 1979), pp. 163-195.

[17] Miller, Merton. ''Debt and Taxes.'' *Journal of Finance,* Vol. 32 (May, 1977), pp. 261-275.

[18] Miller, Merton, and Myron Scholes. ''Dividends and Taxes.'' *Journal of Financial Economics,* Vol. 6 (December 1978), pp. 333-364.

[19] Modigliani, Franco. ''Debt, Dividend Policy, Taxes, Inflation and Market Valuation.'' *Journal of Finance,* Vol. 37 (May 1982), pp. 255-273.

[20] Modigliani, Franco, and Merton Miller. ''The Cost of Capital, Corporation Finance, and the Theory of Investment.'' *American Economic Review,* Vol. 48 (June 1958), pp. 261-297.

[21] ——————. ''Corporate Income Taxes and the Cost of Capital: A Correction.'' *American Economic Review,* Vol. 53 (June 1963), pp. 433-443.

[22] Mundell, Robert. ''Inflation and Interest Rates.'' *Journal of Political Economy,* Vol. 71 (June 1963), pp. 280-283.

[23] Santomero, Anthony. ''A Model of the Demand for Money by Households.'' *Journal of Finance,* Vol. 29 (March 1974), pp. 89-102.

[24] Taggert, Robert. ''Taxes and Corporate Capital Structure in an Incomplete Market.'' *Journal of Finance,* Vol. 35 (June 1980), pp. 645-660.