Cyber-Physical Systems

Deadline based Scheduling

ICEN 553/453– Fall 2022

Prof. Dola Saha

Real-Time Systems

- The operating system, and in particular the scheduler, is perhaps the most important component

Examples:

- Control of laboratory experiments
- Process control in industrial plants
- Robotics
- Air traffic control
- Telecommunications
- Military command and control systems

- Correctness of the system depends not only on the logical result of the computation but also on the time at which the results are produced
- Tasks attempt to react to events that take place in the outside world
- These events occur in “real time” and tasks must be able to keep up with them

Hard and Soft Real-Time Tasks

- Hard
  - One that must meet its deadline
  - Otherwise it will cause unacceptable damage or a fatal error to the system
- Soft
  - Has an associated deadline that is desirable but not mandatory
  - It still makes sense to schedule and complete the task even if it has passed its deadline

Periodic and Aperiodic Tasks

- **Periodic tasks**
  - Requirement may be stated as:
    - Once per period *T*
    - Exactly *T* units apart
- **Aperiodic tasks**
  - Has a deadline by which it must finish or start
  - May have a constraint on both start and finish time

Characteristics of Real Time Systems

Real-time operating systems have requirements in five general areas:

- Determinism
- Responsiveness
- User control
- Reliability
- Fail-soft operation

Determinism

- Concerned with how long an operating system delays before acknowledging an interrupt
- Operations are performed at fixed, predetermined times or within predetermined time intervals
  - When multiple processes are competing for resources and processor time, no system will be fully deterministic

The extent to which an operating system can deterministically satisfy requests depends on:

- The speed with which it can respond to interrupts
- Whether the system has sufficient capacity to handle all requests within the required time

Responsiveness

- Together with determinism make up the response time to external events
  - Critical for real-time systems that must meet timing requirements imposed by individuals, devices, and data flows external to the system
- Concerned with how long, after acknowledgment, it takes an operating system to service the interrupt

Responsiveness includes:

- Amount of time required to initially handle the interrupt and begin execution of the interrupt service routine
- Amount of time required to perform the ISR
- Effect of interrupt nesting

User Control

- Generally much broader in a real-time operating system than in ordinary operating systems
- It is essential to allow the user fine-grained control over task priority
- User should be able to distinguish between hard and soft tasks and to specify relative priorities within each class
- May allow user to specify such characteristics as:

Paging or process swapping → What processes must always be resident in main memory → What disk transfer algorithms are to be used → What rights the processes in various priority bands have

Reliability

- More important for real-time systems than non-real time systems
- Real-time systems respond to and control events in real time so loss or degradation of performance may have catastrophic consequences such as:
  - Financial loss
  - Major equipment damage
  - Loss of life

Fail-Soft Operation

- A characteristic that refers to the ability of a system to fail in such a way as to preserve as much capability and data as possible
- Important aspect is stability
  - A real-time system is stable if the system will meet the deadlines of its most critical, highest-priority tasks even if some less critical task deadlines are not always met

Features common to Most RTOSs

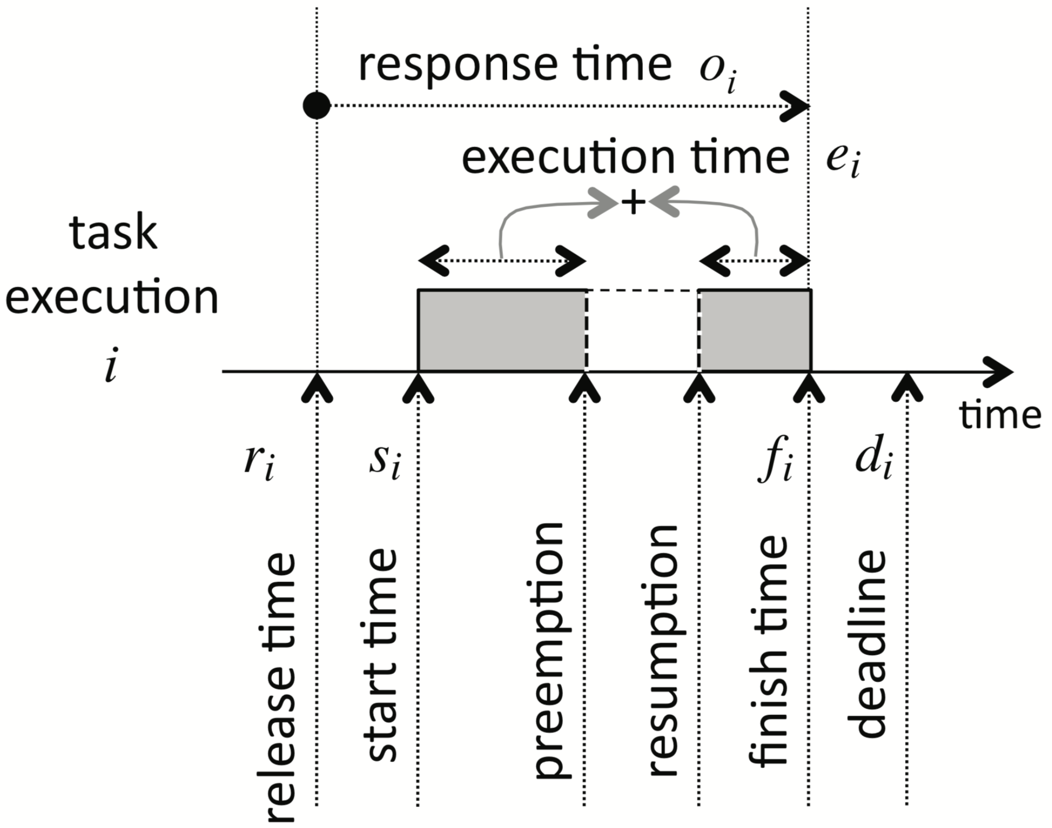
- A stricter use of priorities than in an ordinary OS, with preemptive scheduling that is designed to meet real-time requirements
- Interrupt latency is bounded and relatively short
- More precise and predictable timing characteristics than general purpose OSs

Task Model

$$s_i \geq r_i$$

$$f_i \geq s_i$$

$$o_i = f_i - r_i$$



Scheduling Strategies

- Goal: all task executions meet their deadlines

$$f_i \leq d_i$$

- A schedule that accomplishes this is called a feasible schedule.
- A scheduler that yields a feasible schedule for any task set is said to be optimal with respect to feasibility.

Criteria or Metrices

- Processor Utilization μ
- Maximum Lateness

$$L_{\max} = \max_{i \in T} (f_i - d_i)$$

- Total Completion Time or Makespan

$$M = \max_{i \in T} f_i - \min_{i \in T} r_i$$

- Average Response Time

$$\overline{t_r} = \frac{1}{n} \sum_{i=1}^{n} (f_i - a_i)$$

Rate Monotonic Scheduling

- Simple process model: n tasks invoked periodically with:
  - periods T1, … ,Tn, which equal the deadlines
  - known worst-case execution times (WCET) C1, … ,Cn
    - no mutexes, semaphores, or blocking I/O
  - independent tasks, no precedence constraints
  - fixed priorities
  - preemptive scheduling
- Rate Monotonic Scheduling (RMS): priorities ordered by period (smallest period has the highest priority)

Feasibility for RMS

- Feasibility is defined for RMS to mean that every task executes to completion once within its designated period.

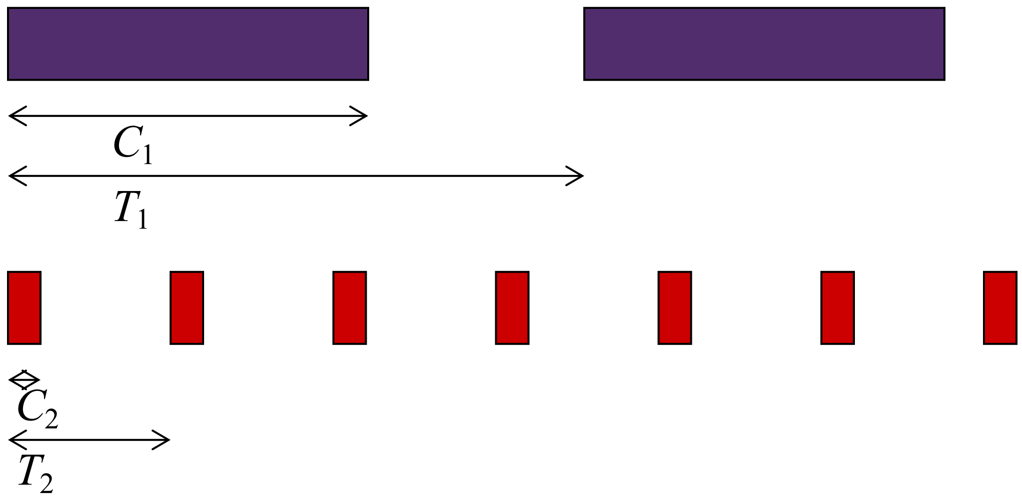
- Theorem: Under the simple process model, if any priority assignment yields a feasible schedule, then RMS also yields a feasible schedule.

- RMS is optimal in the sense of feasibility.

Liu and Layland, "Scheduling algorithms for multiprogramming in a hard-real-time environment," J. ACM, 1973.

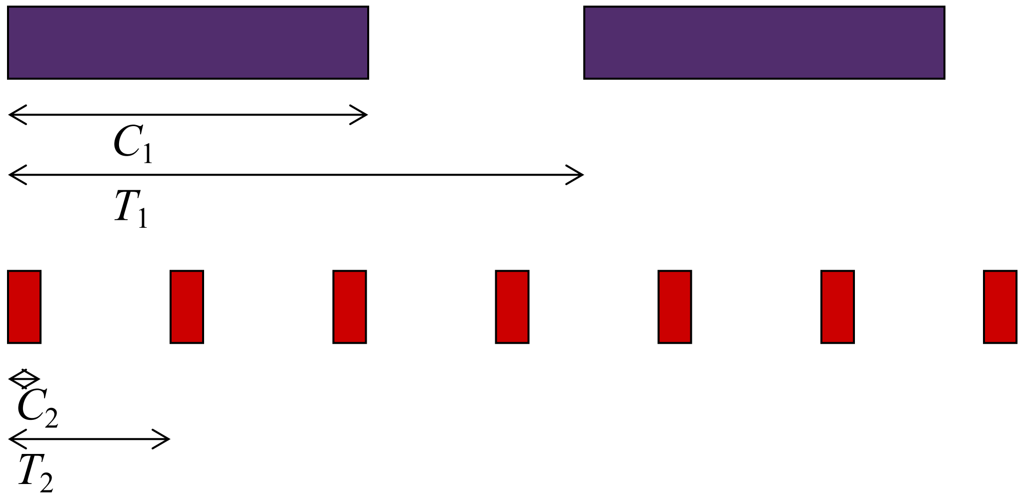
Showing Optimality of RMS:

- Consider two tasks with different periods.
- Is a non-preemptive schedule feasible?



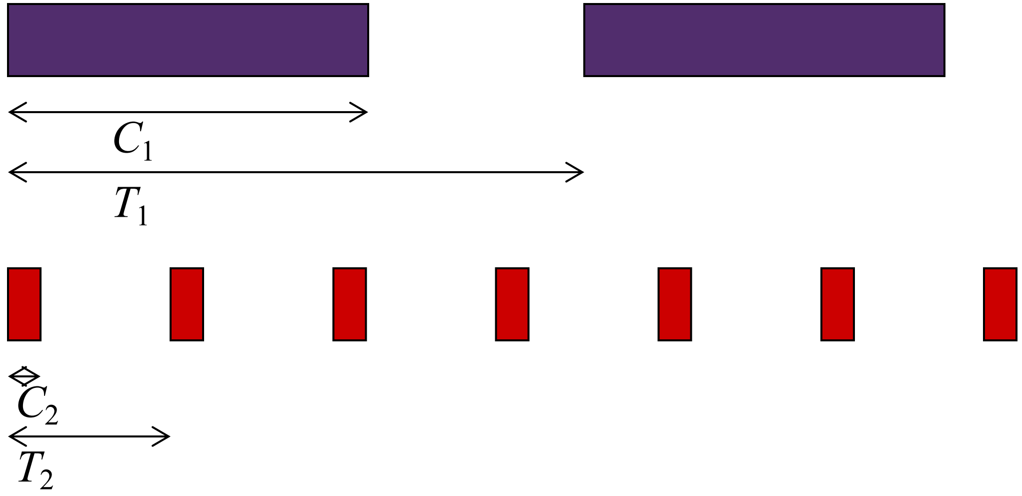
Showing Optimality of RMS:

- Non-preemptive schedule is not feasible. Some instance of the Red Task (2) will not finish within its period if we do non-preemptive scheduling.



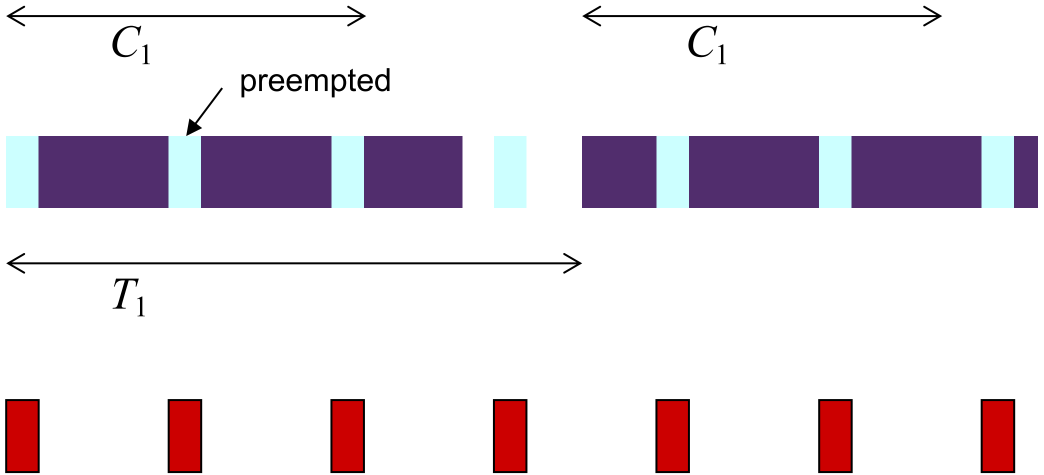
Showing Optimality of RMS:

- What if we had a preemptive scheduling with higher priority for red task?



Showing Optimality of RMS:

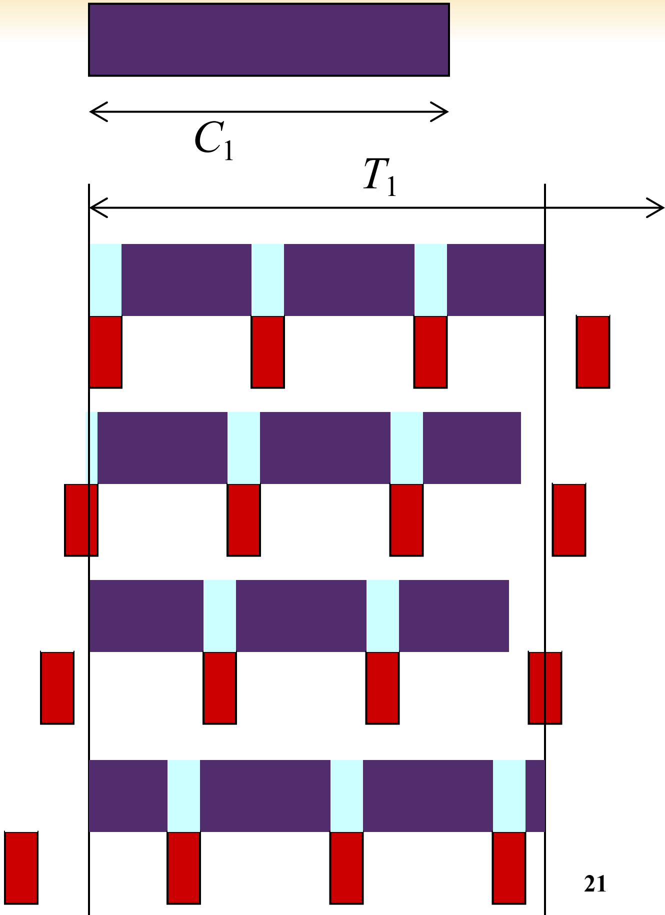
➢ Preemptive schedule with the red task having higher priority is feasible. Note that preemption of the purple task extends its completion time.



Alignment of tasks

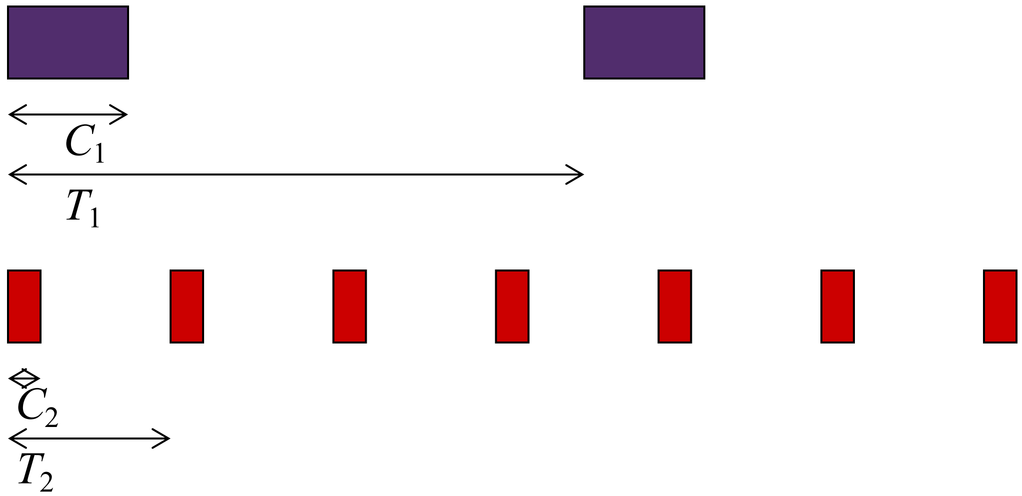
- Completion time of the lower priority task is worst when its *starting phase* matches that of higher priority tasks.

- Thus, when checking schedule feasibility, it is sufficient to consider only the worst case: All tasks start their cycles at the same time.



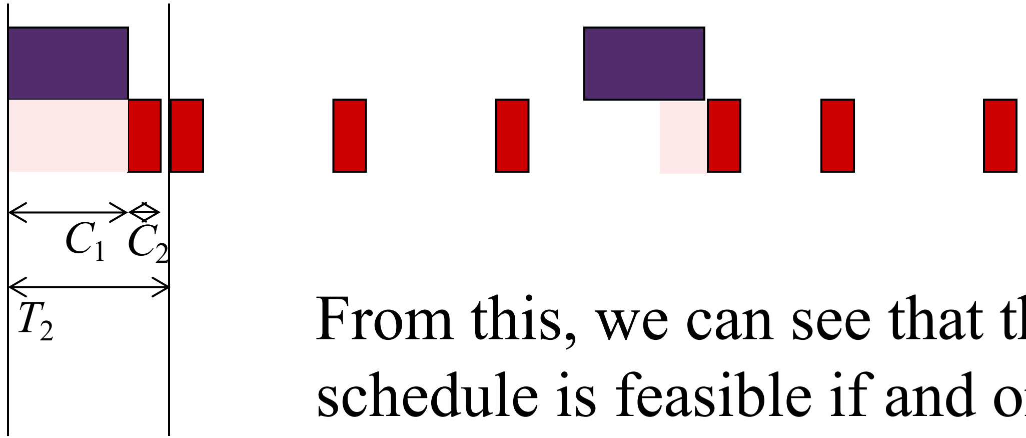
Showing Optimality of RMS: (two tasks)

- It is sufficient to show that if a non-RMS schedule is feasible, then the RMS schedule is feasible.
- Consider two tasks as follows:



Showing Optimality of RMS: (two tasks)

The non-RMS, fixed priority schedule looks like this:



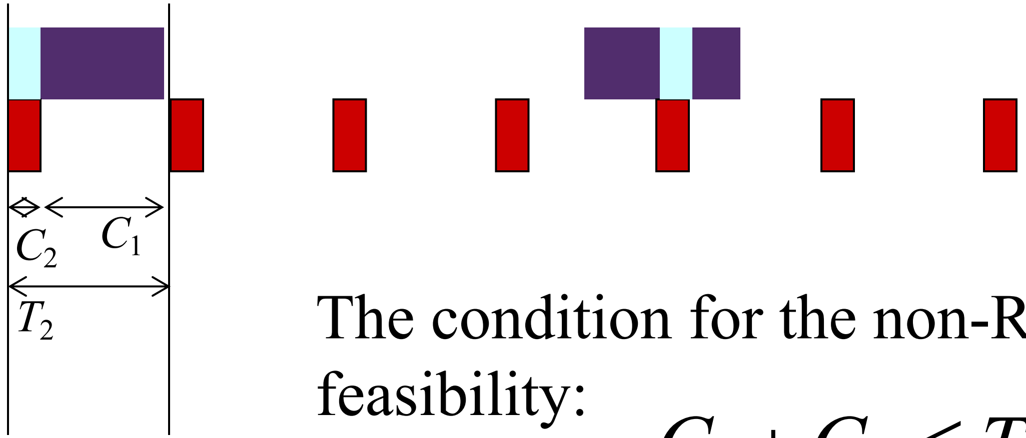
From this, we can see that the non-RMS schedule is feasible if and only if

$$C_1 + C_2 \leq T_2$$

We can then show that this condition implies that the RMS schedule is feasible.

Showing Optimality of RMS: (two tasks)

The RMS schedule looks like this: (task with smaller period moves earlier)



The condition for the non-RMS schedule feasibility:

$$C_1 + C_2 \leq T_2$$

is clearly sufficient (though not necessary) for feasibility of the RMS schedule.

Comments

- This proof can be extended to an arbitrary number of tasks (though it gets much more tedious).
- This proof gives optimality only w.r.t. feasibility.
- Practical implementation:
  - Timer interrupt at greatest common divisor of the periods.
  - Multiple timers

RM Scheduler: Processor Utilization

$$\mu = \sum_{i=1}^{n} \frac{e_i}{p_i}$$

- If $\mu > 1$ for any task set, then that task set has no feasible schedule
- Utilization Bound: RMS is feasible $\mu \leq n(2^{1/n} - 1)$
- As n gets large, $\lim_{n \to \infty} n(2^{1/n} - 1) = \ln(2) \approx 0.693.$
- If a task set with any number of tasks does not attempt to use more than 69.3% of the available processor time, then the RM schedule will meet all deadlines.

Liu and Layland, "Scheduling algorithms for multiprogramming in a hard-real-time environment," J. ACM, 1973.

Jackson's Algorithm: EDD (1955)

- Given n independent one-time tasks with deadlines d_1 , … , d_n, schedule them to minimize the maximum lateness, defined as

$$L_{\max} = \max_{1 \le i \le n} \{ f_i - d_i \}$$

- where f_i is the finishing time of task i. Note that this is negative iff all deadlines are met.
- Earliest Due Date (EDD) algorithm: Execute them in order of non-decreasing deadlines.
- Note that this does not require preemption.

EDD is Optimal

- Optimal in the Sense of Minimizing Maximum Lateness

  - To prove, use an *interchange argument*.

  - Given a schedule S that is not EDD, there must be tasks a and b where a immediately precedes b in the schedule but $d_a > d_b$. Why?

  - We can prove that this schedule can be improved by interchanging a and b. Thus, no non-EDD schedule achieves smaller max lateness than EDD

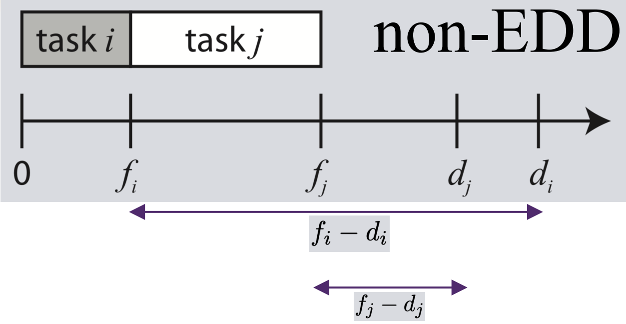
  - So the EDD schedule must be optimal.

Maximum Lateness

- First Schedule (non-EDD)

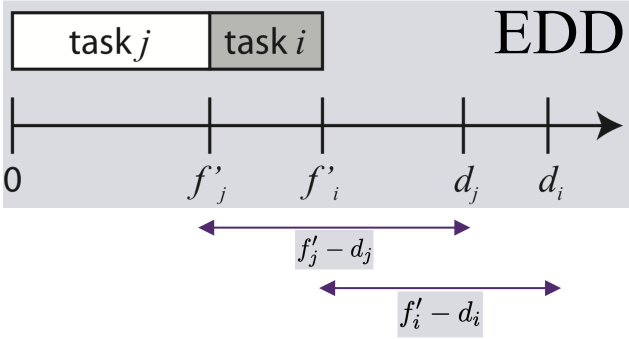
$$L_{\max} = \max(f_i - d_i, f_j - d_j) = f_j - d_j$$

  - where $f_i \leq f_j$ and $d_j < d_i$



- Second Schedule (EDD)

$$L'_{\max} = \max(f'_i - d_i, f'_j - d_j)$$



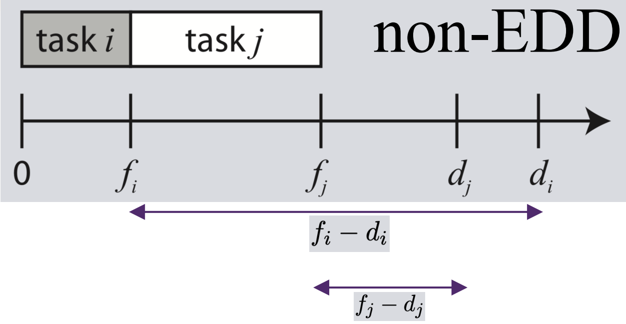
Consider Cases

Case 1: $L'_{\max} = f'_i - d_i$

Since $f'_i = f_j$ $d_j < d_i$

$L'_{\max} = f_j - d_i \leq f_j - d_j$

Hence, $L'_{\max} \leq L_{\max}$

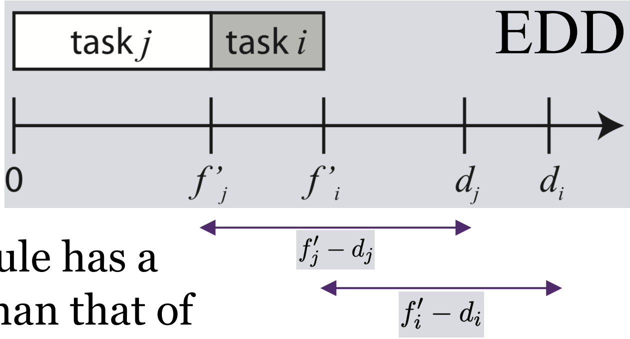


Case 2: $L'_{\max} = f'_j - d_j$

Since $f'_j \leq f_j$

$L'_{\max} \leq f_j - d_j$

Hence, $L'_{\max} \leq L_{\max}$



In both cases, the second schedule has a maximum lateness no greater than that of the first schedule.

EDD minimizes maximum lateness.

Horn's algorithm: EDF (1974)

- Extend EDD by allowing tasks to "arrive" (become ready) at any time.
- Earliest deadline first (EDF): Given a set of *n* independent tasks with *arbitrary arrival times*, any algorithm that at any instant executes the task with the earliest absolute deadline among all arrived tasks is optimal w.r.t. minimizing the maximum lateness.
- Proof uses a similar interchange argument.

Using EDF for Periodic Tasks

- The EDF algorithm can be applied to periodic tasks as well as aperiodic tasks.
  - Simplest use: Deadline is the end of the period.
  - Alternative use: Separately specify deadline (relative to the period start time) and period.

RMS vs. EDF? Which one is better?

- What are the pros and cons of each?

Comparison of EDF and RMS

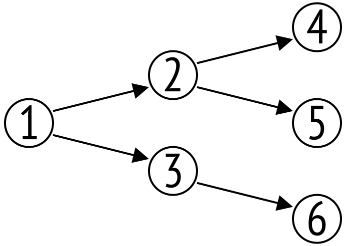
- Favoring RMS
  - Scheduling decisions are simpler (fixed priorities vs. the dynamic priorities required by EDF. EDF scheduler must maintain a list of ready tasks that is sorted by priority.)

Comparison of EDF and RMS

- Favoring EDF
  - Since EDF is optimal w.r.t. maximum lateness, it is also optimal w.r.t. feasibility. RMS is only optimal w.r.t. feasibility.
  - For infeasible schedules, RMS completely blocks lower priority tasks, resulting in unbounded maximum lateness.
  - EDF can achieve full utilization where RMS fails to do that.
  - EDF results in fewer preemptions in practice, and hence less overhead for context switching.
  - Deadlines can be different from the period.

Precedence Constraints

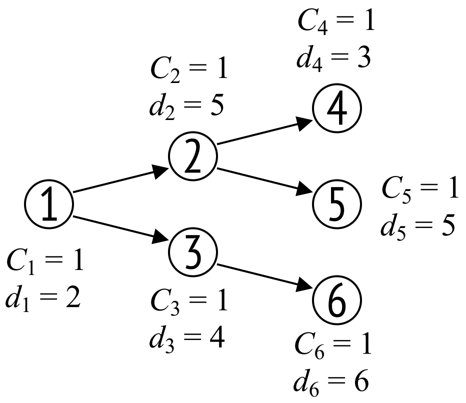
- A directed acyclic graph (DAG) shows precedences, which indicate which tasks must complete before other tasks start.

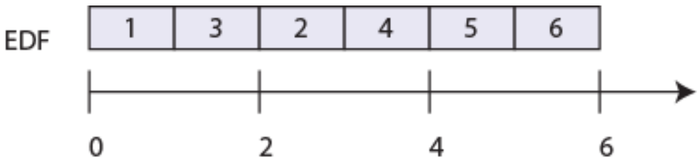


DAG, showing that task 1 must complete before tasks 2 and 3 can be started, etc.

Example: EDF Schedule

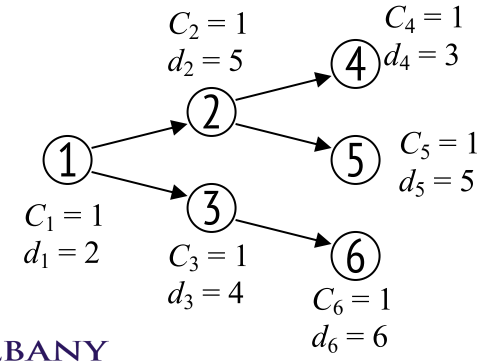
- Is this feasible?

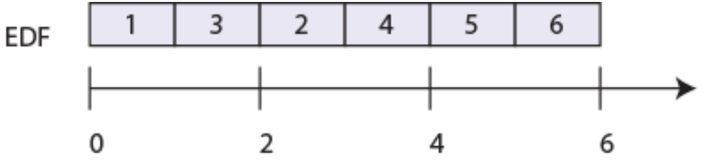




EDF is not optimal under precedence constraints

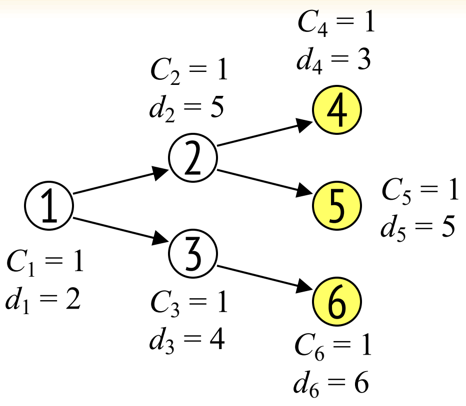
- The EDF schedule chooses task 3 at time 1 because it has an earlier deadline. This choice results in task 4 missing its deadline.

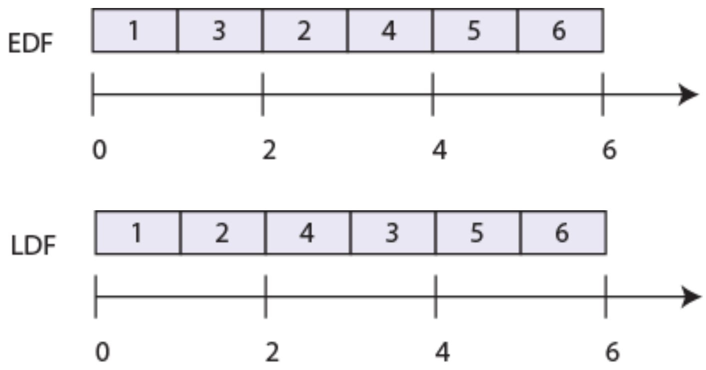




UNIVERSITY AT ALBANY
State University of New York

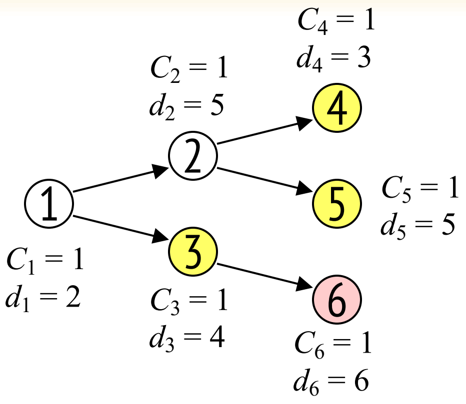
Latest Deadline First (LDF) Lawler 1973

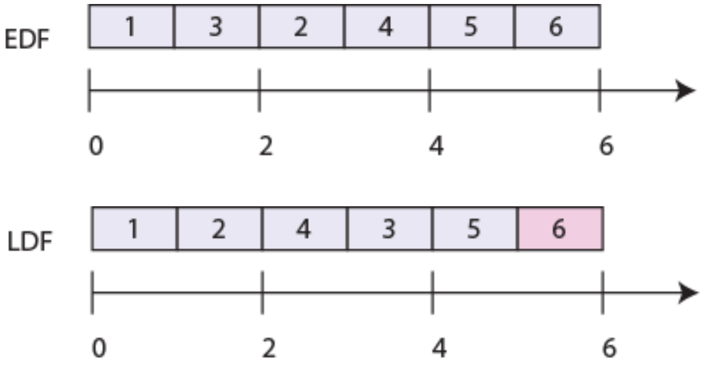




- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

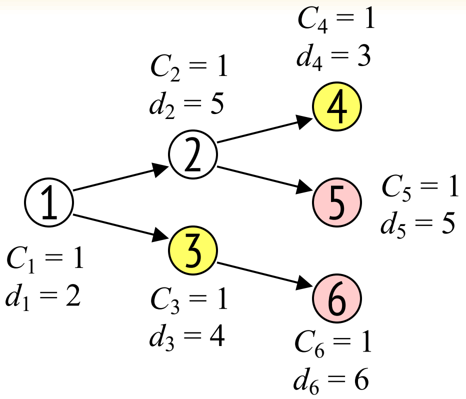
Latest Deadline First (LDF)

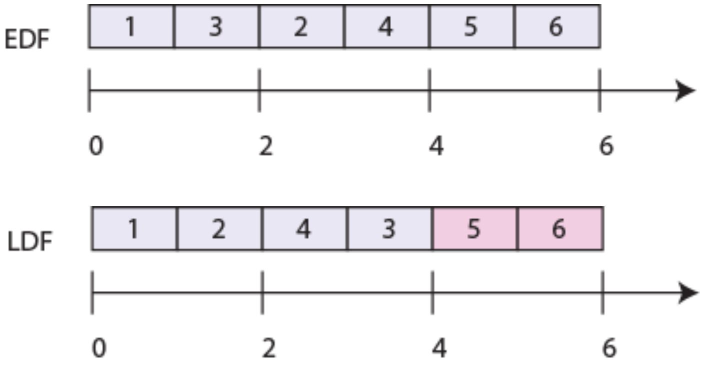




- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

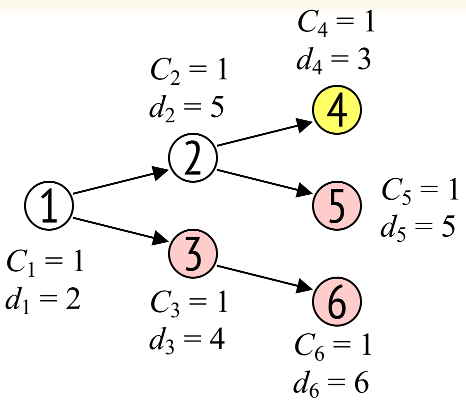
Latest Deadline First (LDF)

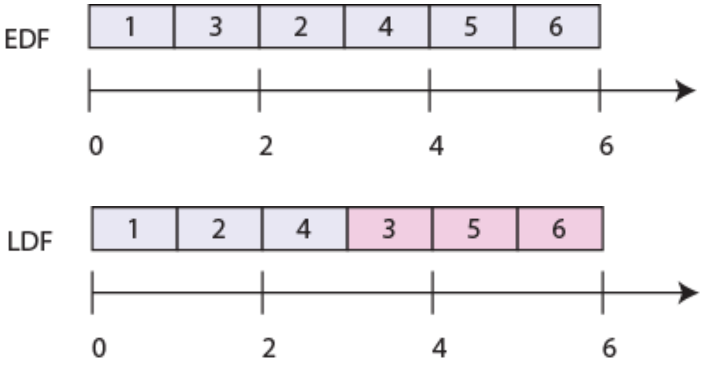




- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

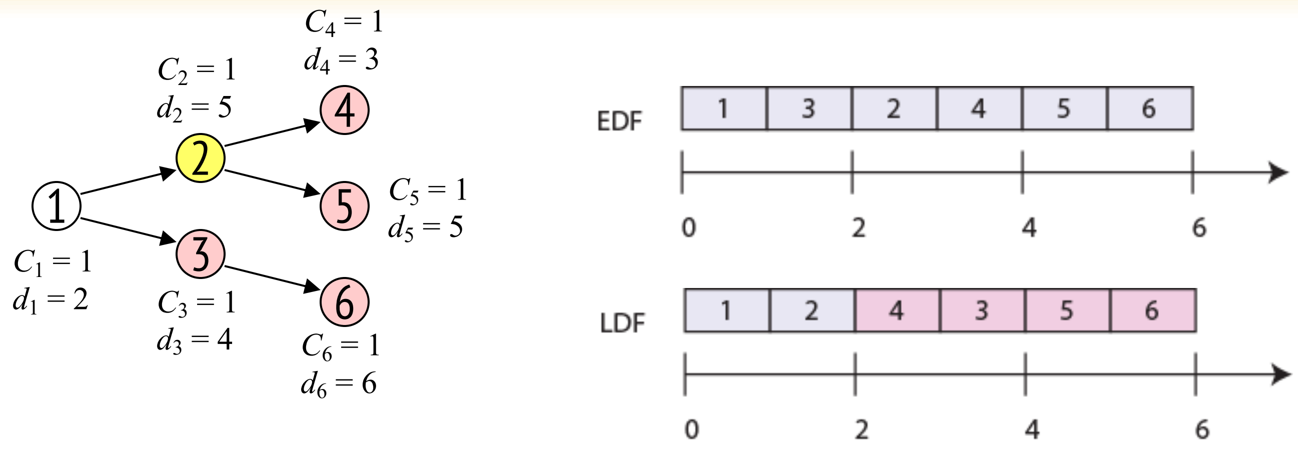
Latest Deadline First (LDF)





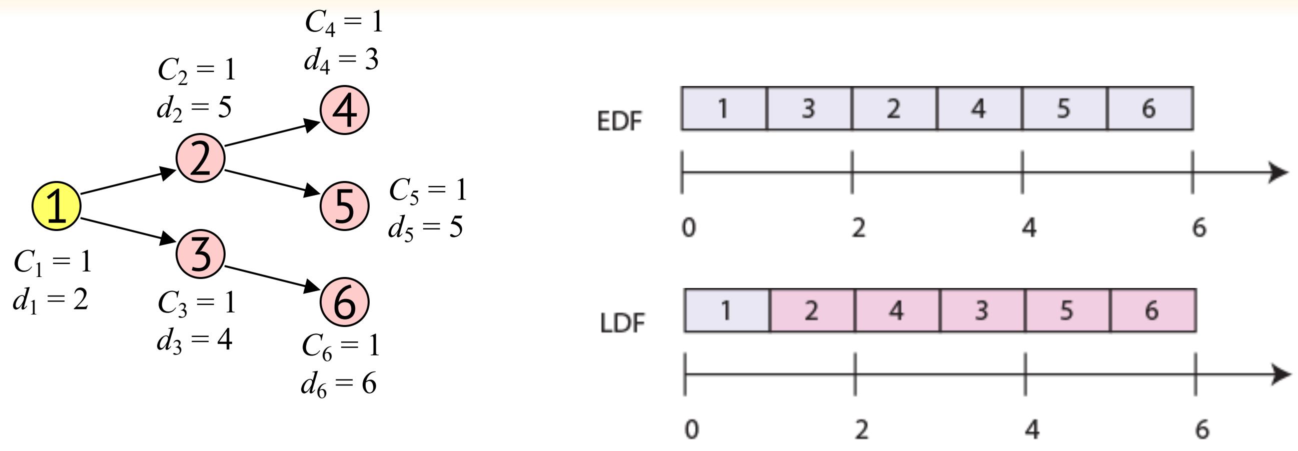
- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

Latest Deadline First (LDF)



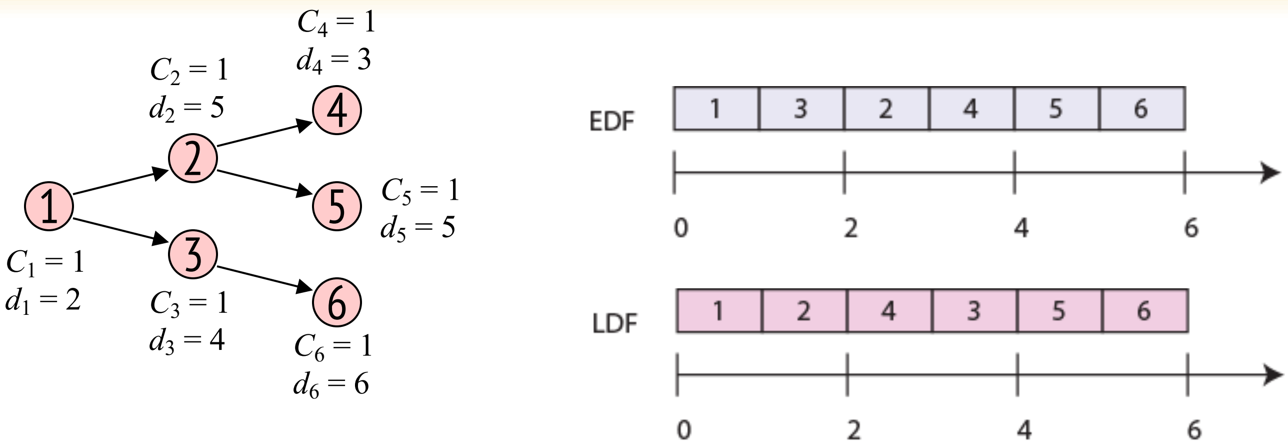
- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

Latest Deadline First (LDF)



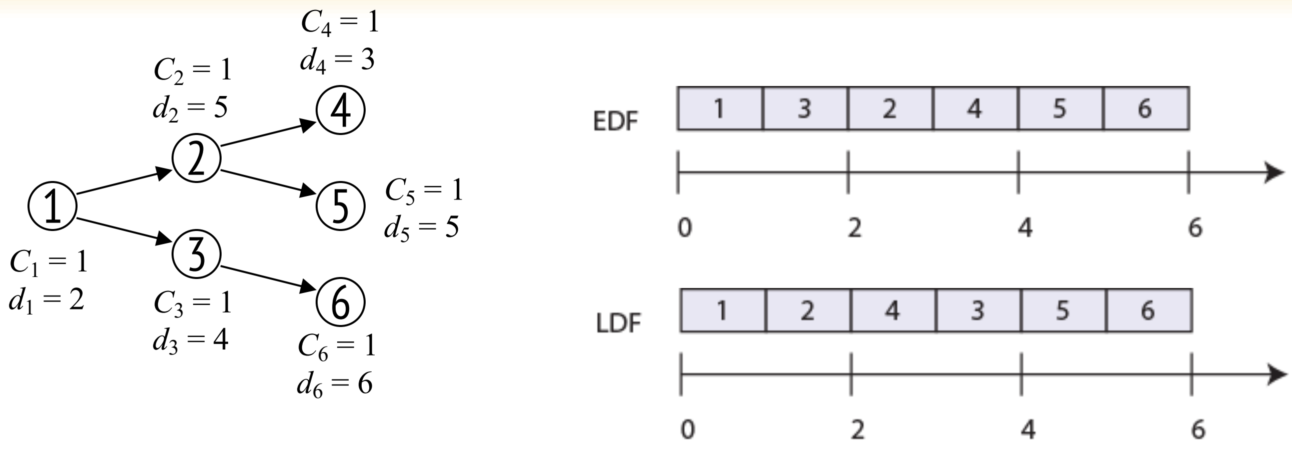
- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

Latest Deadline First (LDF)



- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

LDF is optimal for precedence constraints



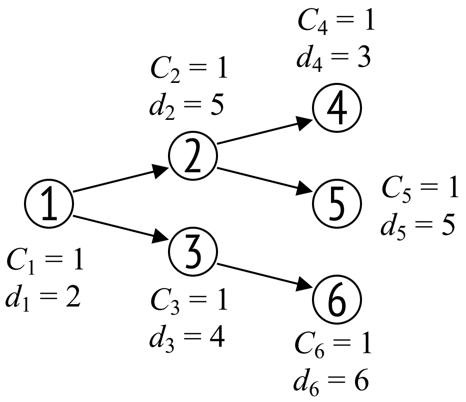
- ➢ The LDF schedule shown at the bottom respects all precedences and meets all deadlines.
- ➢ Also minimizes maximum lateness

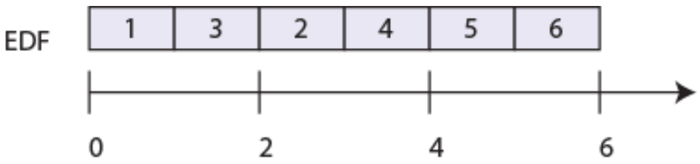
Latest Deadline First (LDF)

- LDF is optimal in the sense that it minimizes the maximum lateness.
- It does not require preemption. (EDF can be made to work with preemption.)
- However, it requires that all tasks be available and their precedences known before any task is executed.

EDF with Precedences or EDF*

- With a **preemptive** scheduler, EDF can be modified to account for precedences and to allow tasks to arrive at arbitrary times.
- Adjust the deadlines and arrival times according to the precedences.





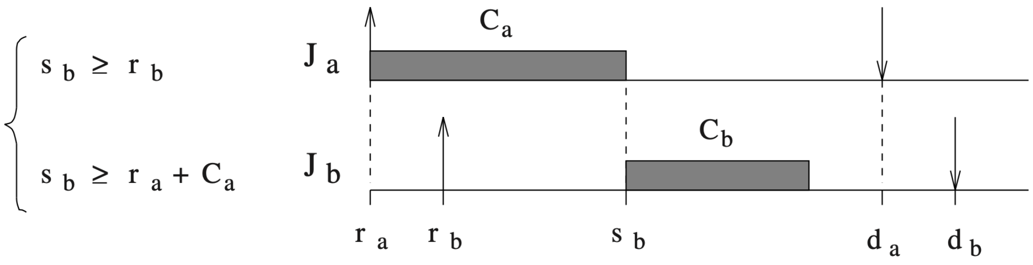
Recall that for the tasks at the left, EDF yields the schedule above, where task 4 misses its deadline.

Modification of Release Times

➢ Observations:

$s_b \geq r_b$ (that is, J_b must start the execution not earlier than its release time);

$s_b \geq r_a + C_a$ (that is, J_b must start the execution not earlier than the minimum finishing time of J_a).

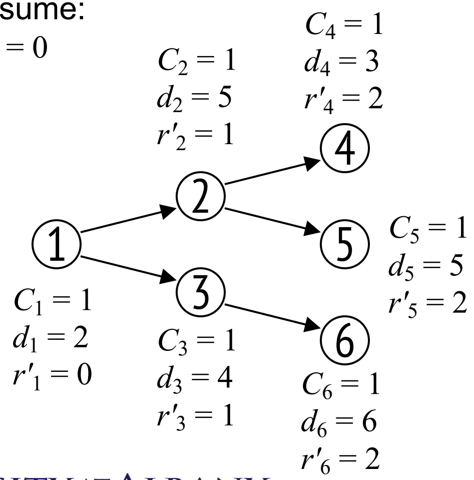


➢ Modification: $r_b^* = \max(r_b, r_a + C_a)$

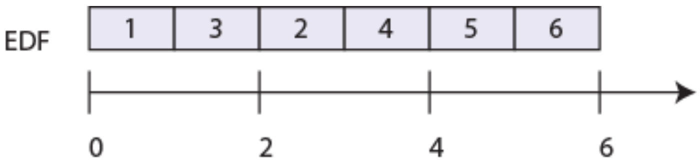
EDF with Precedences: Modifying Release Times

- Given n tasks with precedences and release times r_i, if task i immediately precedes task j, then modify the release times as follows:

assume:
$r_i = 0$



$$r'_j = \max(r_j, r_i + C_i)$$

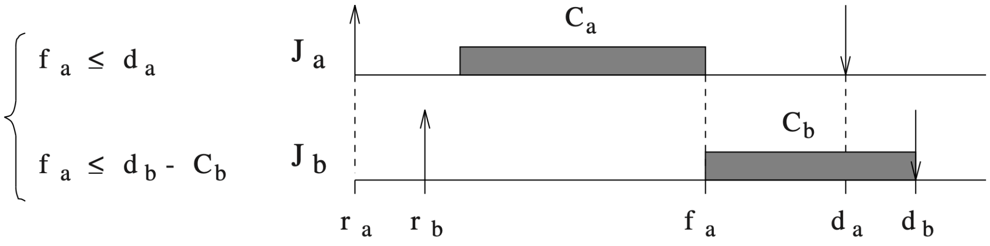


Modification of Deadlines

- Observations:

$f_a \le d_a$ (that is, J_a must finish the execution within its deadline);

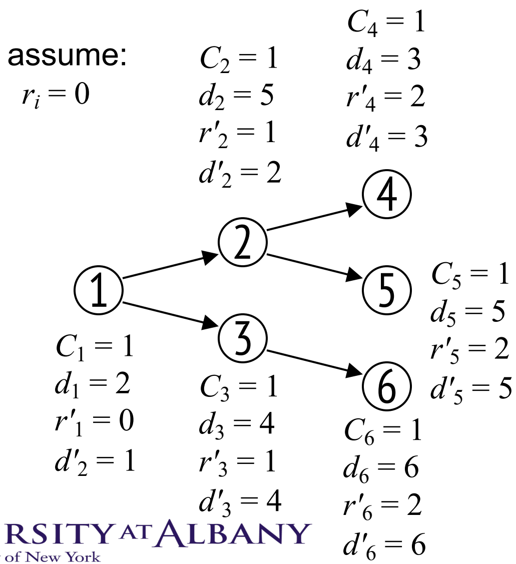
$f_a \le d_b - C_b$ (that is, J_a must finish the execution not later than the maximum start time of J_b).



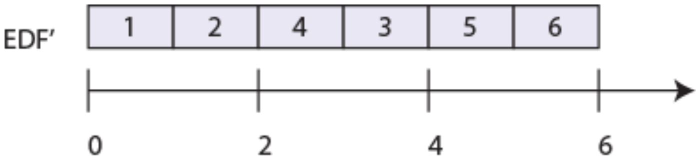
- Modification: $d_a^* = \min(d_a, d_b - C_b)$

EDF with Precedences: Modifying Deadlines

- Given n tasks with precedences and deadlines d_i, if task i immediately precedes task j, then modify the deadlines as follows:



$$d'_i = \min(d_i, d'_j - C_j)$$



Using the revised release times and deadlines, the above EDF schedule is optimal and meets all deadlines.

Optimality

- Generalized modified deadline

$$d'_i = \min(d_i, \min_{j \in D(i)} (d'_j - e_j))$$

- EDF with precedences is **optimal** in the sense of minimizing the maximum lateness.

Classwork Examples

- Create a schedule for the following periodic tasks. Is the schedule feasible?

	C_i	T_i
τ_1	2	6
τ_2	2	8
τ_3	2	12

Classwork Examples

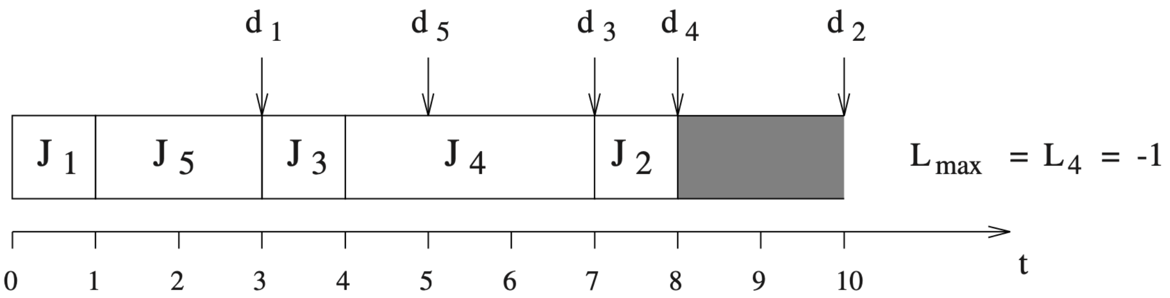
- Create a schedule for Jackson's Algorithm

	J_1	J_2	J_3	J_4	J_5
C_i	1	1	1	3	2
d_i	3	10	7	8	5

Classwork Examples

- Create a schedule for Jackson's Algorithm

	J_1	J_2	J_3	J_4	J_5
C_i	1	1	1	3	2
d_i	3	10	7	8	5



Classwork Examples

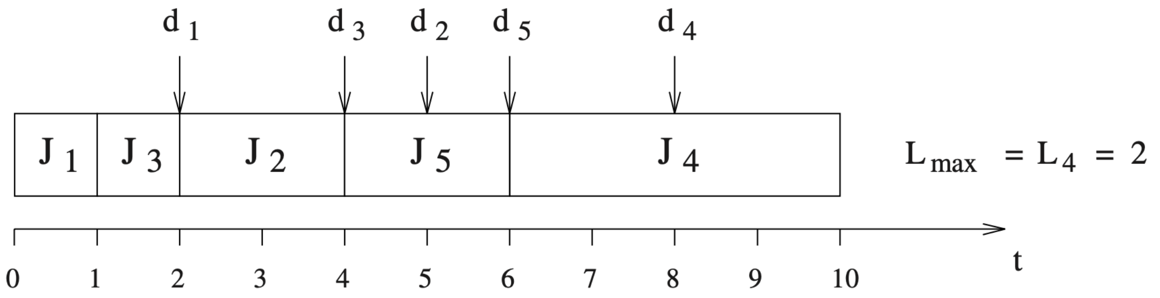
- Create a schedule for Jackson's Algorithm

	J_1	J_2	J_3	J_4	J_5
C_i	1	2	1	4	2
d_i	2	5	4	8	6

Classwork Examples

- Create a schedule for Jackson's Algorithm

	J_1	J_2	J_3	J_4	J_5
C_i	1	2	1	4	2
d_i	2	5	4	8	6



Classwork Examples

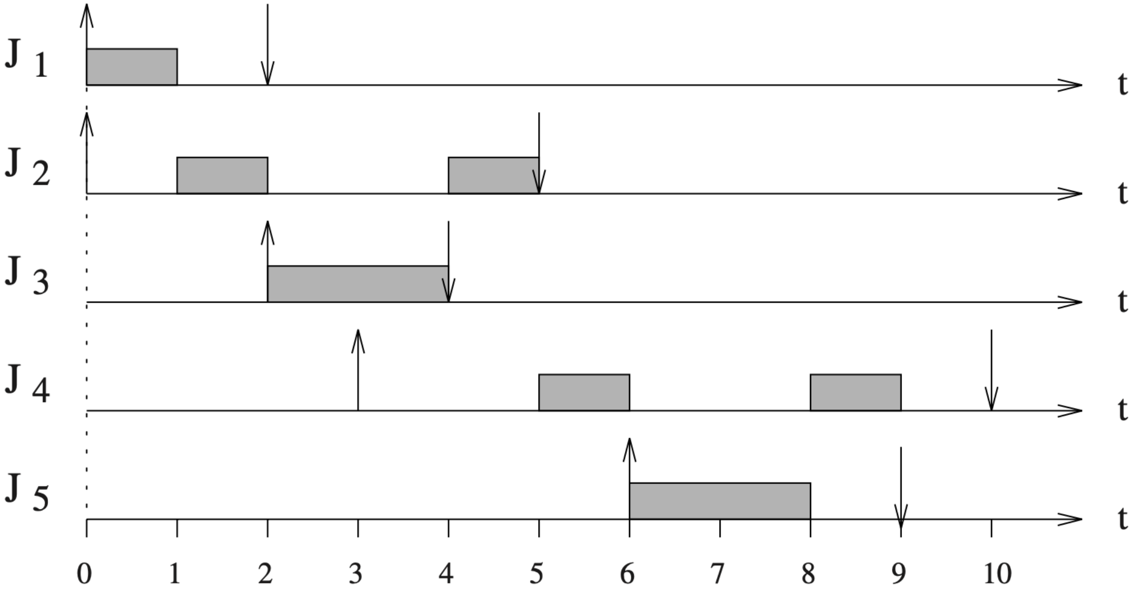
- Create an EDF schedule

	J_1	J_2	J_3	J_4	J_5
a_i	0	0	2	3	6
C_i	1	2	2	2	2
d_i	2	5	4	10	9

Classwork Examples

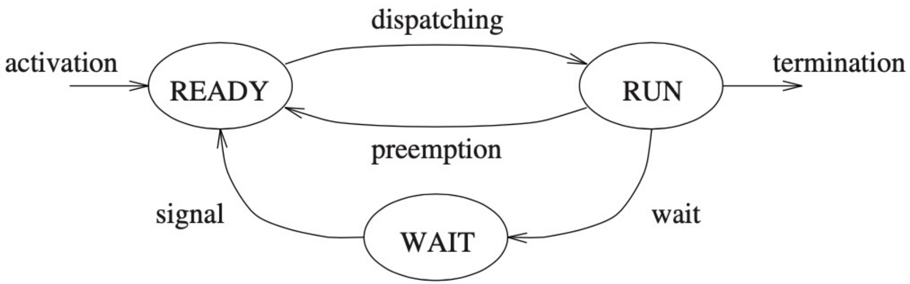
- Create an EDF schedule

	J_1	J_2	J_3	J_4	J_5
a_i	0	0	2	3	6
C_i	1	2	2	2	2
d_i	2	5	4	10	9



Scheduling in Shared Resource

- concurrent tasks use shared resources in exclusive mode
- Recall: critical section and mutexes/semaphores

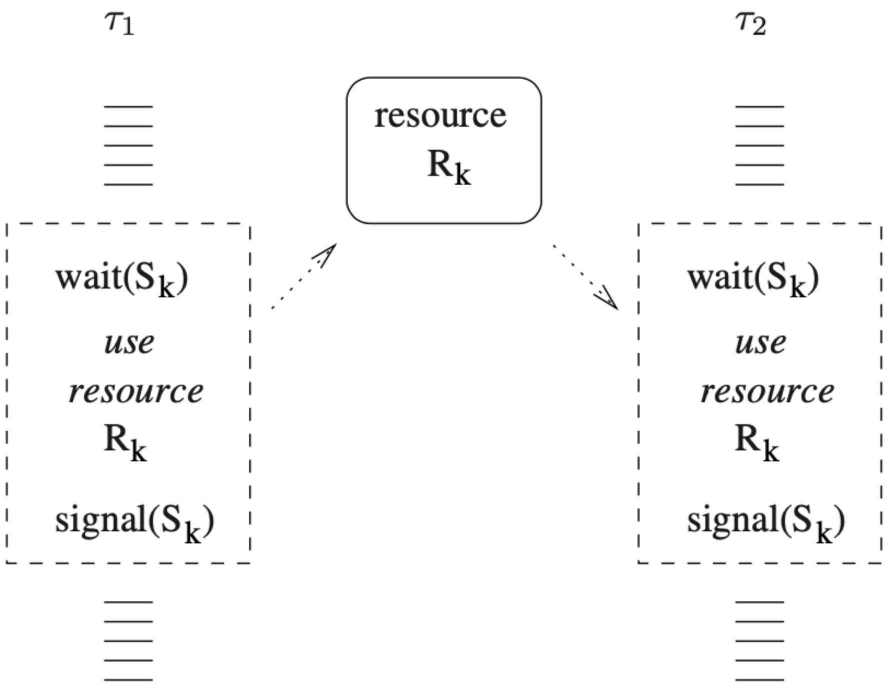


A task waiting for an exclusive resource is said to be *blocked* on that resource

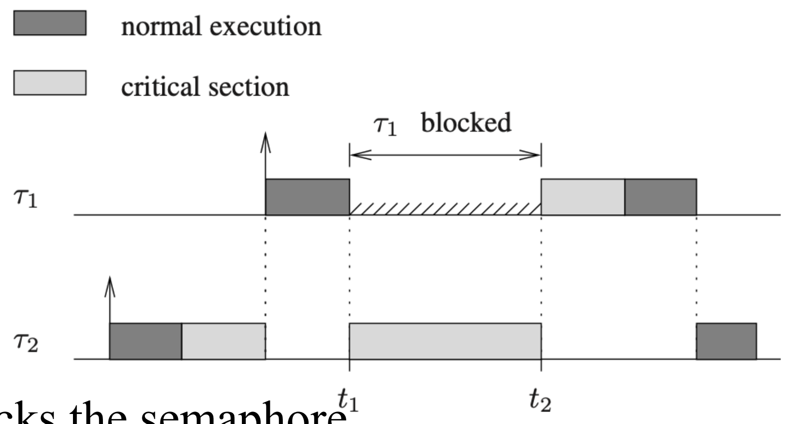
Giorgio C. Buttazzo, *Hard Real-Time Computing Systems,* Springer, 2004.

Two tasks sharing exclusive resources

```
#include <pthread.h>
...
pthread_mutex_t lock;

void* addListener(notify listener) {
  pthread_mutex_lock(&lock);
  ...
  pthread_mutex_unlock(&lock);
}

void* update(int newValue) {
  pthread_mutex_lock(&lock);
  value = newValue;
  elementType* element = head;
  while (element != 0) {
    (*(element->listener))(newValue);
    element = element->next;
  }
  pthread_mutex_unlock(&lock);
}

int main(void) {
  pthread_mutex_init(&lock, NULL);
  ...
}
```

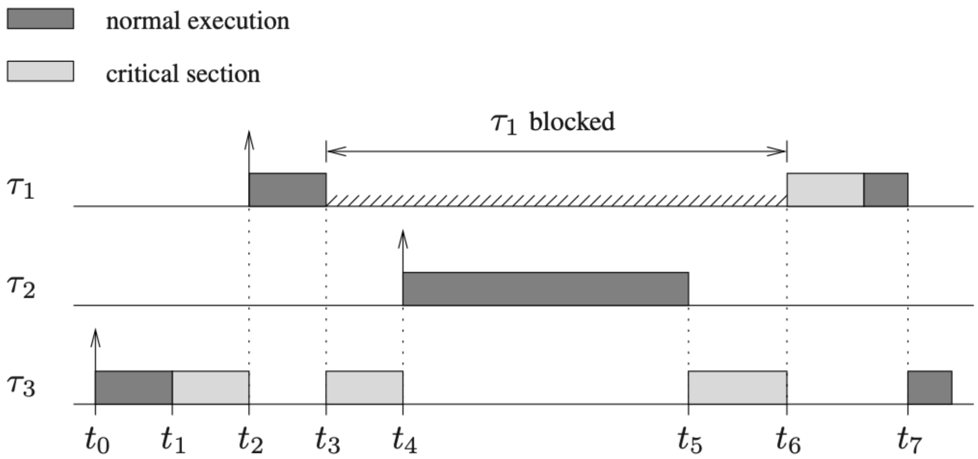


Blocking on critical section



- τ_1 has a higher priority than τ_2
- τ_2 is activated first
  - after a while, it enters the critical section and locks the semaphore.
- While τ_2 is executing the critical section
  - task τ_1 arrives, and it preempts τ_2 as it has higher priority and starts executing.
- At t_1, τ_1 is blocked on the semaphore, so τ_2 resumes
- At t_2, τ_2 releases the critical section
- Maximum blocking time of τ_1 is equal to the time needed by τ_2 to execute its critical section.

Priority Inversion with Mutex



- A *priority inversion* is said to occur in the interval $[t_3, t_6]$, since the highest-priority task τ_1 waits for the execution of lower-priority tasks (τ_2 and τ_3).

Priority Inversion: Why is it a problem?

- Maximum blocking time of τ_1 depends on
  - the length of the critical section executed by τ_3
  - the worst-case execution time of τ_2
- Can lead to uncontrolled blocking (with multiple medium priority tasks)
  - can cause critical deadlines to be missed
- The duration of priority inversion is unbounded

Resource Access Protocols to avoid PI

- Non-Preemptive Protocol (NPP)
- Highest Locker Priority (HLP) or Immediate Priority Ceiling (IPC)
- Priority Inheritance Protocol (PIP)
- Priority Ceiling Protocol (PCP)
- Stack Resource Policy (SRP)

Terminology

- n periodic tasks, $\tau_1,\tau_2,\ldots,\tau_n$
- m shared resources, $R_1,R_2,\ldots,R_m$
- Each task is characterized by
  - a period T_i
  - a worst-case computation time C_i
- Each resource R_k is guarded by a distinct semaphore S_k
- each task is characterized by
  - a fixed *nominal* priority P_i (assigned by the algorithm) and
  - an *active* priority p_i ($p_i \geq P_i$), which is dynamic and initially set to P_i

Terminology

B_i denotes the maximum blocking time task τ_i can experience.

$z_{i,k}$ denotes a generic critical section of task τ_i guarded by semaphore S_k.

$Z_{i,k}$ denotes the longest critical section of task τ_i guarded by semaphore S_k.

$\delta_{i,k}$ denotes the duration of $Z_{i,k}$.

$z_{i,h} \subset z_{i,k}$ indicates that $z_{i,h}$ is entirely contained in $z_{i,k}$.

σ_i denotes the set of semaphores used by τ_i.

$\sigma_{i,j}$ denotes the set of semaphores that can block τ_i, used by the lower-priority task τ_j.

Terminology

$\gamma_{i,j}$ denotes the set of the longest critical sections that can block τ_i, accessed by the lower priority task τ_j. That is,

$$\gamma_{i,j} = \{Z_{j,k} \mid (P_j < P_i) \text{ and } (S_k \in \sigma_{i,j})\} \tag{7.1}$$

γ_i denotes the set of all the longest critical sections that can block τ_i. That is,

$$\gamma_i = \bigcup_{j:P_j<P_i} \gamma_{i,j} \tag{7.2}$$

Assumptions

- Priorities:
  - Tasks τ_1 , τ_2 , . . . , τ_n have different priorities
  - They are listed in descending order of nominal priority
  - τ_1 has the highest nominal priority
- Tasks do not suspend themselves on I/O
- The critical sections used by any task are *properly* nested
  - given any pair $z_{i,h}$ and $z_{i,k}$

    either $z_{i,h} \subset z_{i,k}$, $z_{i,k} \subset z_{i,h}$, or $z_{i,h} \cap z_{i,k} = \emptyset$.
- Critical sections are guarded by binary semaphores

Non-Preemptive Protocol

- Disallow preemption during the execution of any critical section

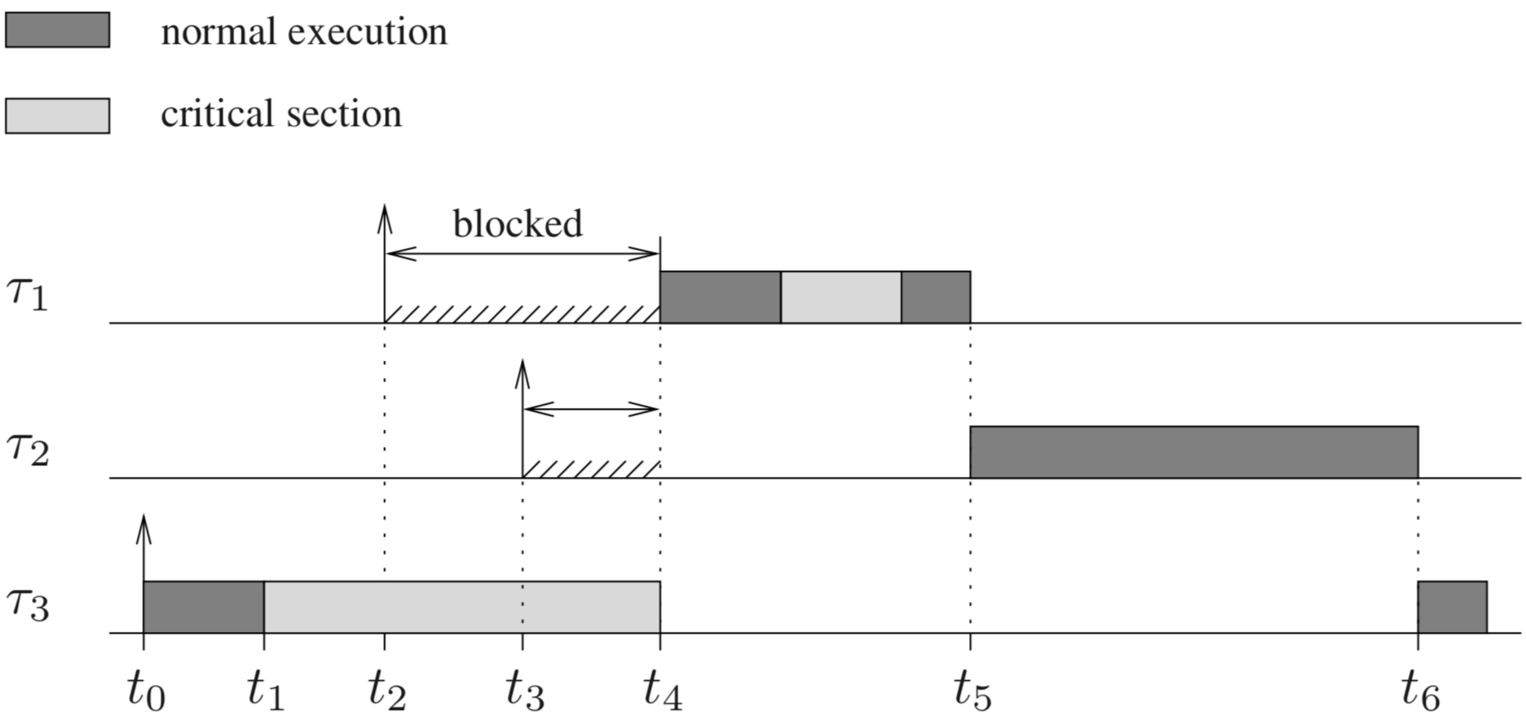
- Raise the priority of a task to the highest priority level whenever it enters a shared resource

as a task τ_i enters a resource R_k, its dynamic priority is raised to the level:

$$p_i(R_k) = \max_h \{P_h\}.$$

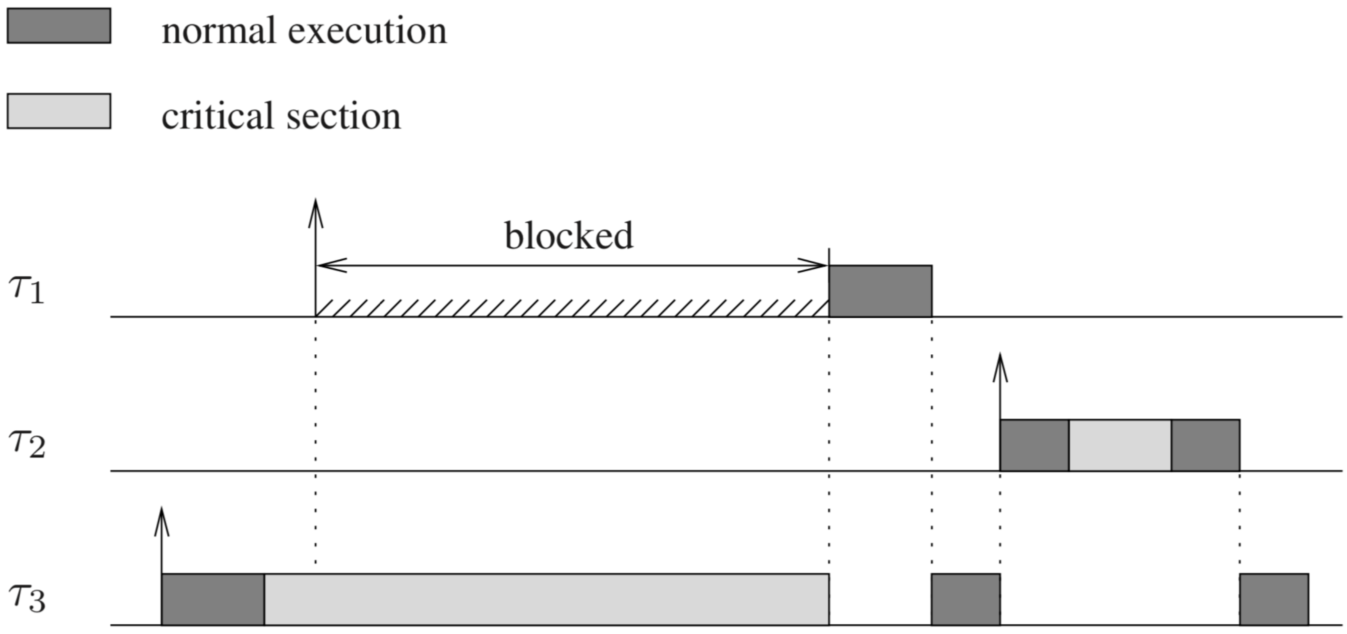
- The dynamic priority is then reset to the nominal value P_i when the task exits the critical section

Example (NPP preventing priority inversion)



NPP causes unnecessary blocking

τ_1 is the highest-priority task that does not use any resource



Blocking Time Computation (NPP)

- task τ_i cannot preempt a lower priority task τ_j if τ_j is inside a critical section

$$\gamma_i = \{Z_{j,k} \mid P_j < P_i,\ k = 1, \ldots, m\}$$

- a task inside a resource *R* cannot be preempted, only one resource can be locked at any time *t*
- a task τ_i can be blocked at most for the length of a single critical section belonging to lower priority tasks
- maximum blocking time τ_i is the duration of the longest critical section of lower priority tasks

$$B_i = \max_{j,k}\{\delta_{j,k} - 1 \mid Z_{j,k} \in \gamma_i\}$$

- one unit of time is subtracted from $\delta_{j,k}$ since $Z_{j,k}$ must start before the arrival of τ_i to block it

Highest Locker Priority (HLP)

- Raises the priority of a task that enters a resource R_k to the highest priority among the tasks sharing that resource
- as soon as a task τ_i enters a resource R_k, its dynamic priority is raised to the level

$$p_i(R_k) = \max_h \{P_h \mid \tau_h \text{ uses } R_k\}$$

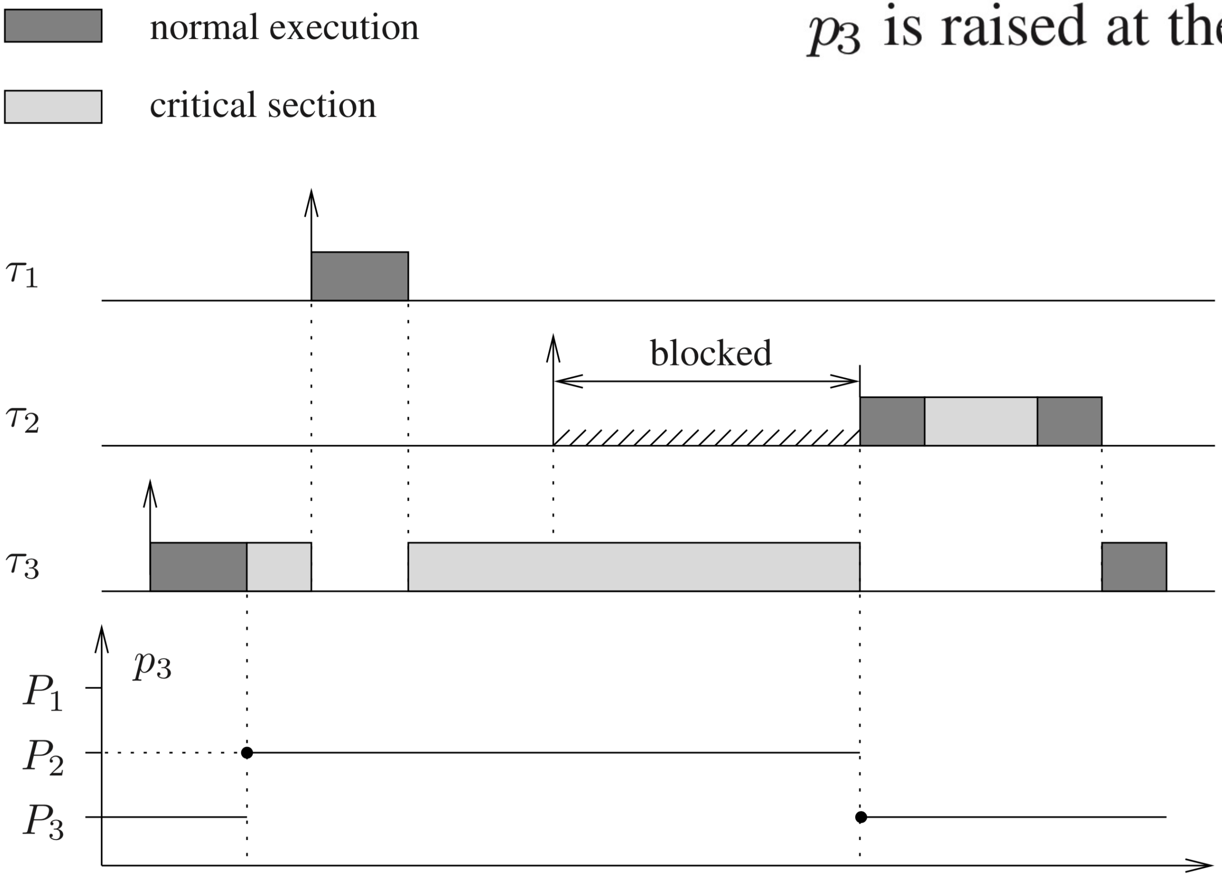
- each resource R_k is assigned a priority ceiling $C(R_k)$ (computed off-line) equal to the maximum priority of the tasks sharing R_k

$$C(R_k) \stackrel{\text{def}}{=} \max_h \{P_h \mid \tau_h \text{ uses } R_k\}$$

- Also termed Immediate Priority Ceiling

HLP Example

p_3 is raised at the level $C(R) = P_2$



Blocking Time (HLP)

- a task τ_i can only be blocked by critical sections belonging to lower priority tasks with a resource ceiling higher than or equal to P_i
- a task can be blocked at most once (Proof in the book)
- the maximum blocking time of τ_i is given by the duration of the longest critical section among those that can block τ_i

$$B_i = \max_{j,k}\{\delta_{j,k} - 1 \mid Z_{j,k} \in \gamma_i\}$$

Priority Inheritance Protocol (PIP)

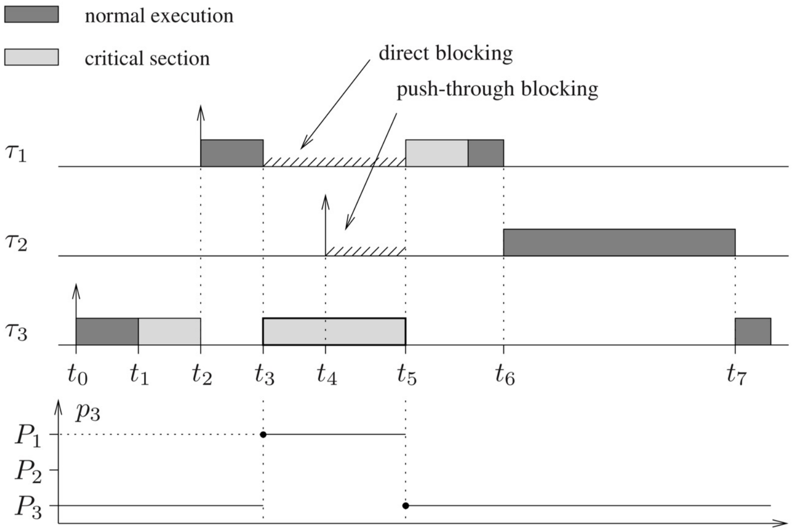
- When a task τ_i blocks one or more higher-priority tasks, it temporarily assumes (*inherits*) the highest priority of the blocked tasks
- When a task τ_i is blocked on a semaphore, it transmits its active priority to the task τ_j, that holds that semaphore
- τ_j executes the rest of its critical section with a priority $p_j = p_i$.

$$p_j(R_k) = \max\{P_j, \max_h\{P_h|\tau_h \text{ is blocked on } R_k\}\}$$

- When τ_j exits a critical section the active priority of τ_j is updated
  - if no other tasks are blocked by τ_j, p_j is set to P_j
  - otherwise it is set to the highest priority of the tasks blocked by τ_j
- Priority inheritance is transitive
  - if a task τ_3 blocks a task τ_2, and τ_2 blocks a task τ_1, then τ_3 inherits the priority of τ_1 via τ_2

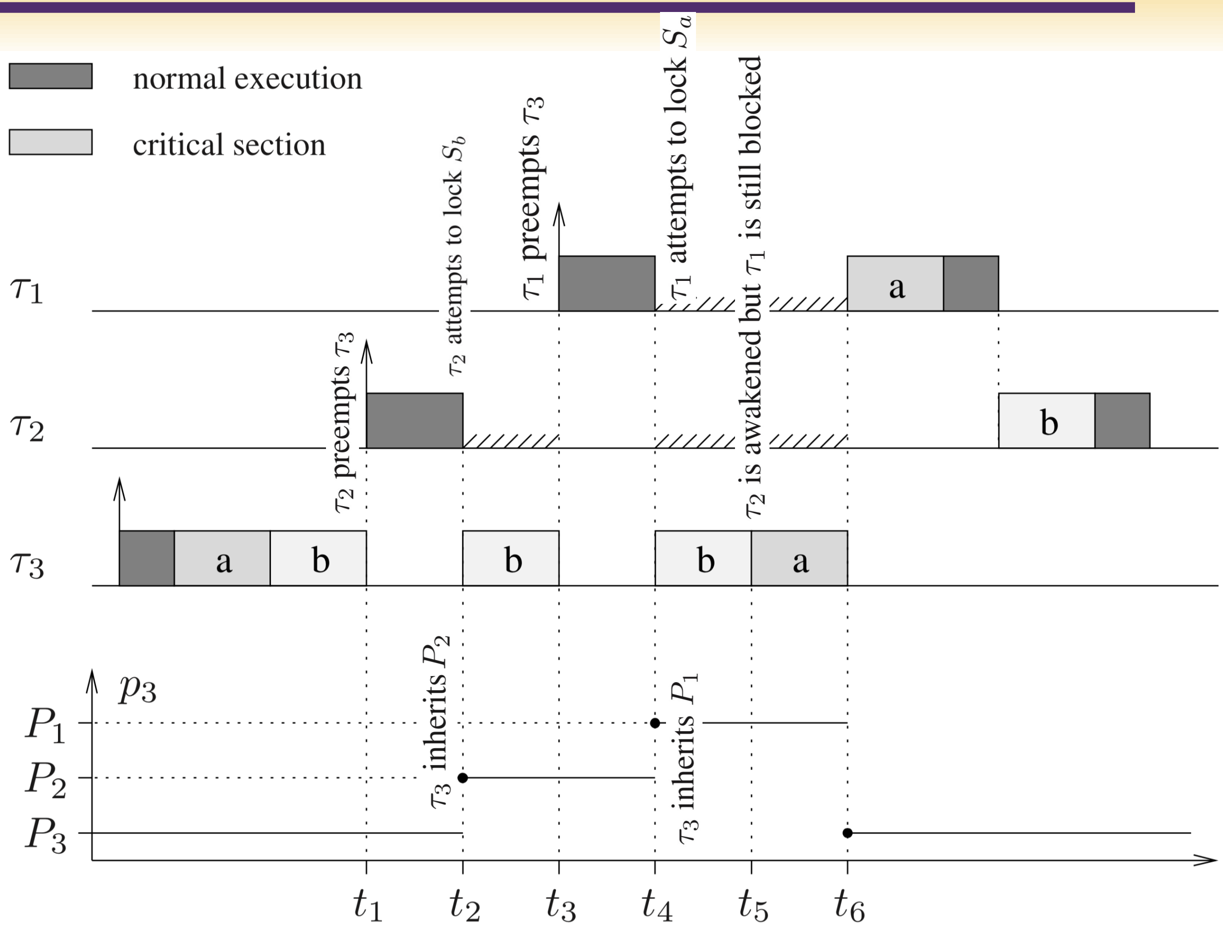
Types of Blocking in PIP

- Direct
  - a higher-priority task tries to acquire a resource held by a lower-priority task
  - Required to ensure consistency of shared resource
- Push-through
  - a medium-priority task is blocked by a low-priority task that has inherited a higher priority from a task it directly blocks
  - Required to void unbounded priority inversion



Nested Critical Section (PIP)

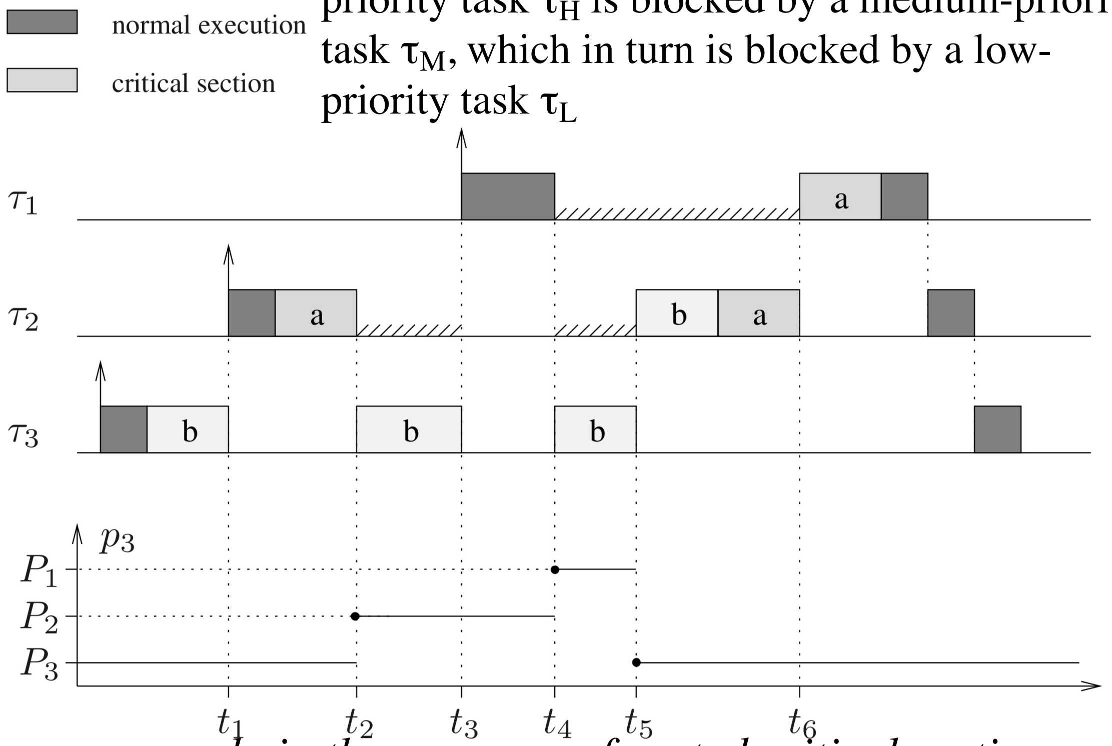
- task τ_1 uses a resource R_a guarded by a semaphore S_a,
- task τ_2 uses a resource R_b guarded by a semaphore S_b
- task τ_3 uses both resources in a nested fashion (S_a is locked first)



Transitive Priority Inheritance

- task τ_1 uses a resource R_a guarded by a semaphore S_a
- task τ_3 uses a resource R_b guarded by a semaphore S_b
- task τ_2 uses both resources in a nested fashion (S_a protects the external critical section and S_b the internal one)

A transitive inheritance occurs when a high-priority task τ_H is blocked by a medium-priority task τ_M, which in turn is blocked by a low-priority task τ_L



Transitive priority inheritance can occur only in the presence of nested critical sections

Blocking Time (PIP)

- a task τ_i can be blocked at most once for each of the l_i lower priority tasks. Hence, for each lower priority task τ_j that can block τ_i, sum the duration of the longest critical section among those that can block τ_i

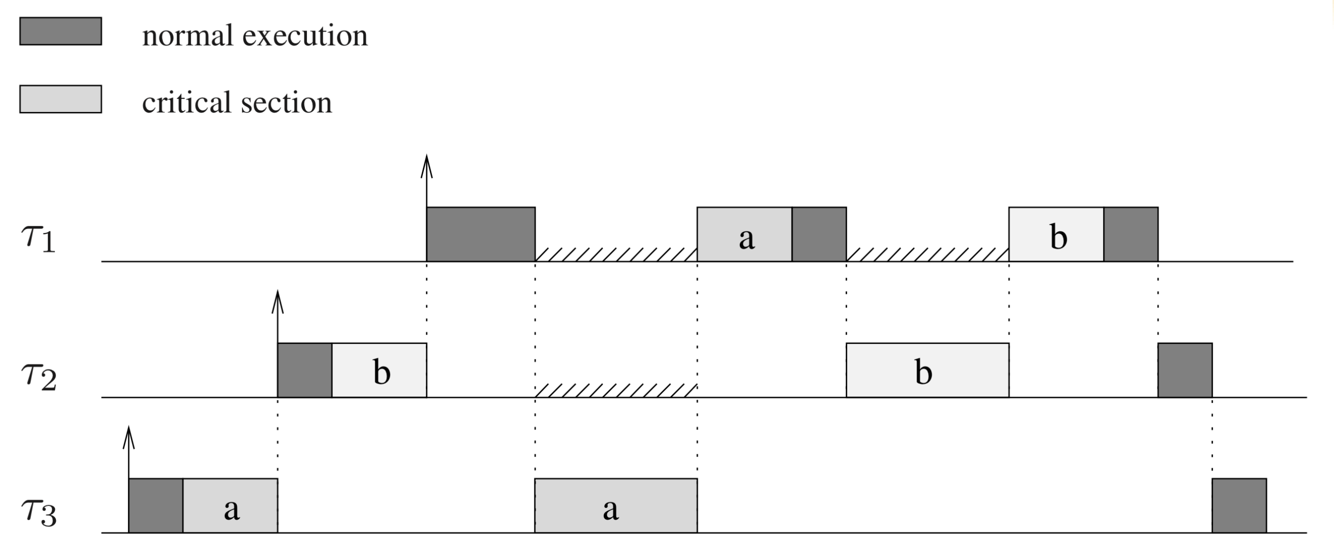
$$B_i^l = \sum_{j:P_j<P_i} \max_k \{\delta_{j,k} - 1 \mid Z_{j,k} \in \gamma_i\}$$

- a task τ_i can be blocked at most once for each of the s_i semaphores that can block τ_i. Hence, for each semaphore S_k that can block τ_i, sum the duration of the longest critical section among those that can block τ_i

$$B_i^s = \sum_{k=1}^{m} \max_j \{\delta_{j,k} - 1 \mid Z_{j,k} \in \gamma_i\}$$

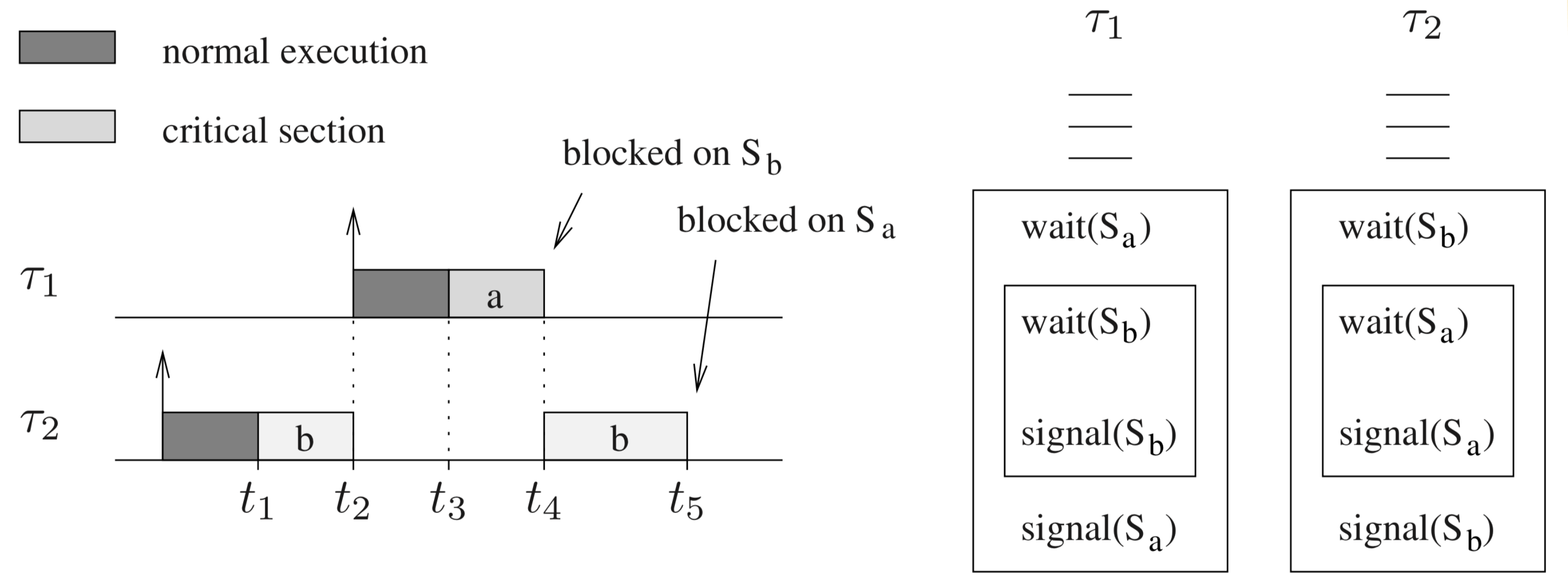
- a task τ_i can be blocked for minimum of the critical sections $B_i = \min(B_i^l, B_i^s)$

Chained Blocking



- τ_1 is blocked for the duration of two critical sections, once to wait for τ_3 to release S_a and then to wait for τ_2 to release S_b
- In the worst case, if τ_1 accesses n distinct semaphores that have been locked by n lower-priority tasks, τ_1 will be blocked for the duration of n critical sections.

Deadlock

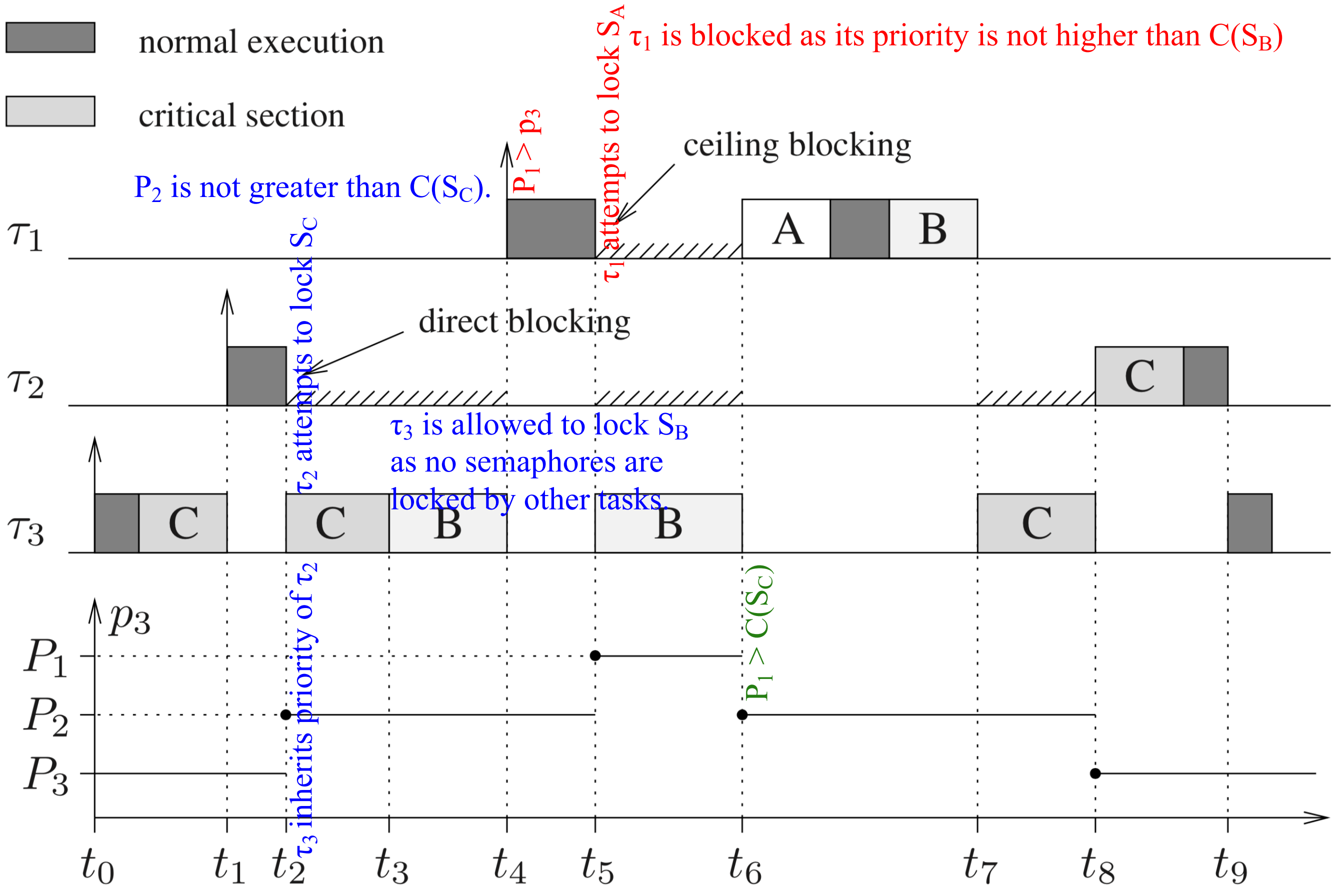


- the deadlock does not depend on the Priority Inheritance Protocol but is caused by an erroneous use of semaphores

Priority Ceiling Protocol (PCP)

- The Priority Ceiling Protocol (PCP)
  - bound the priority inversion phenomenon
  - prevent the formation of deadlocks and chained blocking
- Once a task enters its first critical section, it can never be blocked by lower-priority tasks until its completion
- Each semaphore is assigned a *priority ceiling* equal to the highest priority of the tasks that can lock it

Example Priority Ceiling Protocol



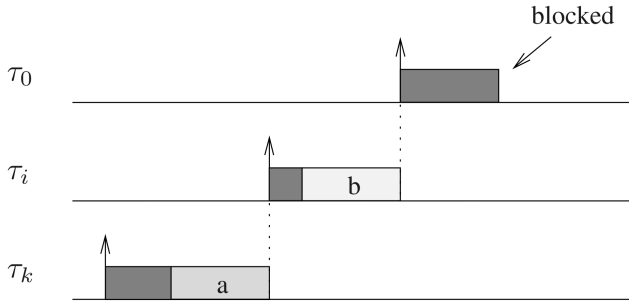
$$\begin{cases} C(S_A) = P_1 \\ C(S_B) = P_1 \\ C(S_C) = P_2. \end{cases}$$

Ceiling Blocking is necessary for avoiding deadlock and chained blocking

Lemma and Proof

If a task τ_k is preempted within a critical section Z_a by a task τ_i that enters a critical section Z_b, then, under the Priority Ceiling Protocol, τ_k cannot inherit a priority higher than or equal to that of task τ_i until τ_i completes.

- If τ_k inherits a priority higher than or equal to that of task τ_i before τ_i completes, there must exist a task τ_0 blocked by τ_k, such that $P_0 \geq P_i$.
- This leads to the contradiction that τ_0 cannot be blocked by τ_k.
- Since τ_i enters its critical section, its priority must be higher than the maximum ceiling C^* of the semaphores currently locked by all lower-priority tasks.
- Hence, $P_0 \geq P_i > C^*$.
- But since $P_0 > C^*$, τ_0 cannot be blocked by τ_k



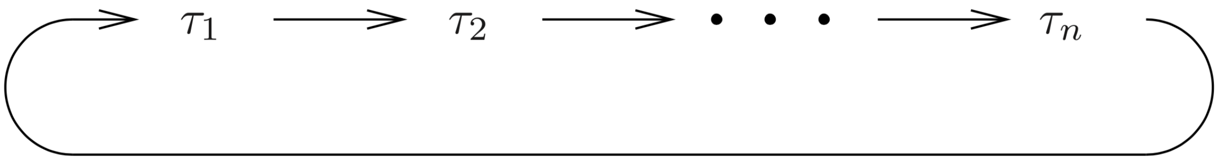
Lemma and Proof

The Priority Ceiling Protocol prevents transitive blocking

- Suppose that a transitive block occurs
  - that is, there exist three tasks τ_1, τ_2, and τ_3, with decreasing priorities, such that τ_3 blocks τ_2 and τ_2 blocks τ_1.
- By the transitivity of the protocol, τ_3 will inherit the priority of τ_1.
- This contradicts the Lemma, which shows that τ_3 cannot inherit a priority higher than or equal to P_2.
- Thus, PCP prevents transitive blocking.

Lemma and Proof

The Priority Ceiling Protocol prevents deadlocks



- Assume that a task cannot deadlock by itself, a deadlock can only be formed by a cycle of tasks waiting for each other
- By the transitivity of the protocol, task τ_n would inherit the priority of τ_1, which is assumed to be higher than P_n.
- This contradicts prior Lemma.
- Hence PCP prevents deadlock.

Blocking Time Computation

A task τ_i can only be blocked by critical sections belonging to lower priority tasks with a resource ceiling higher than or equal to P_i.

$$\gamma_i = \{Z_{j,k} \mid (P_j < P_i) \text{ and } C(R_k) \geq P_i\}.$$

Since τ_i can be blocked at most once, the maximum blocking time τ_i can suffer is given by the duration of the longest critical section among those that can block τ_i

$$B_i = \max_{j,k}\{\delta_{j,k} - 1 \mid Z_{j,k} \in \gamma_i\}$$