Modern Wireless Networks

OFDM

IECE 574– Spring 2021

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Traditional Modulation

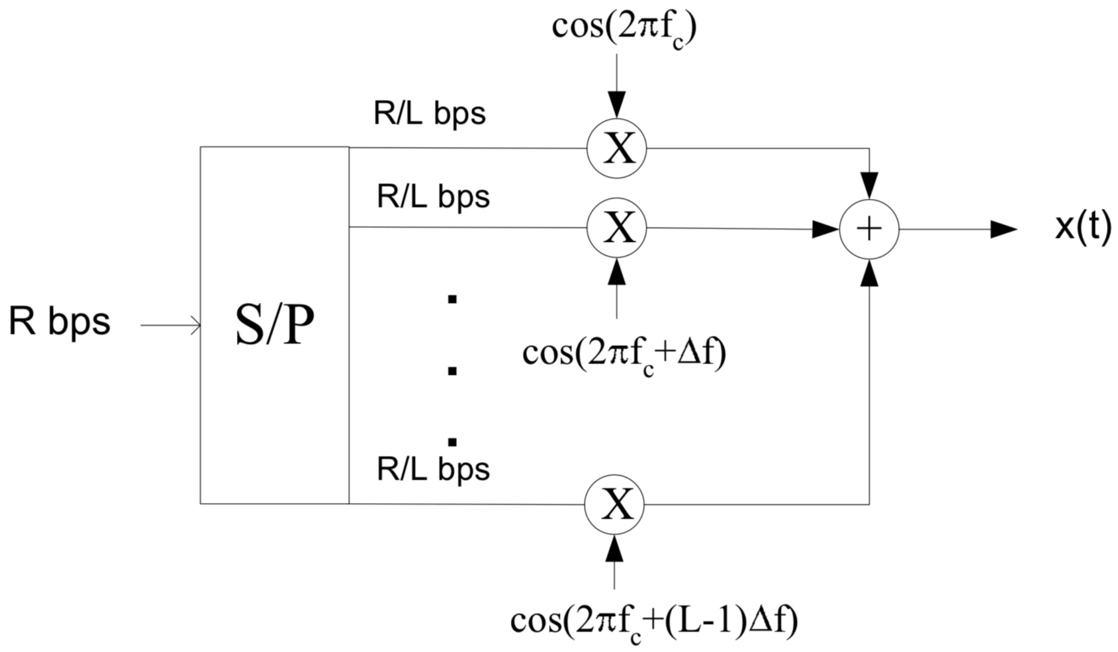
- At high data rate
  - symbol duration T_s has to become very small in order to achieve the required data rate
  - System bandwidth (B) becomes very large
  - Delay dispersion depends on the environment and not on the transmission system
  - $B > B_{coh}$

Multicarrier Modulation to combat ISI

- Divides the wideband incoming data stream into L narrowband substreams
- Each substream is then transmitted over a different orthogonal frequency subchannel
- Number of substreams L is chosen to make the symbol time on each substream much greater than the delay spread of the channel
- Make the substream bandwidth less than the channel coherence bandwidth

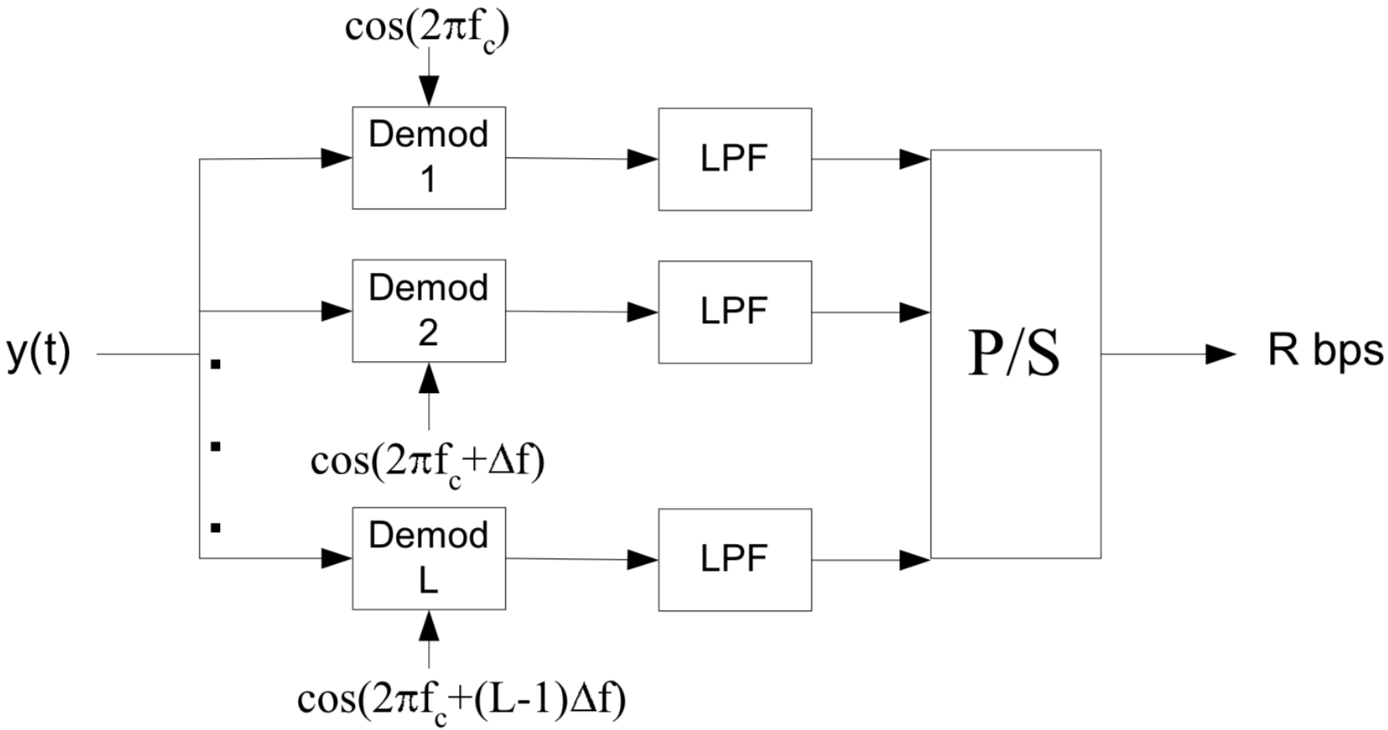
Basic Multicarrier Transmitter

- a high rate stream of R bps is broken into L parallel streams each with rate R/L and then multiplied by a different carrier frequency



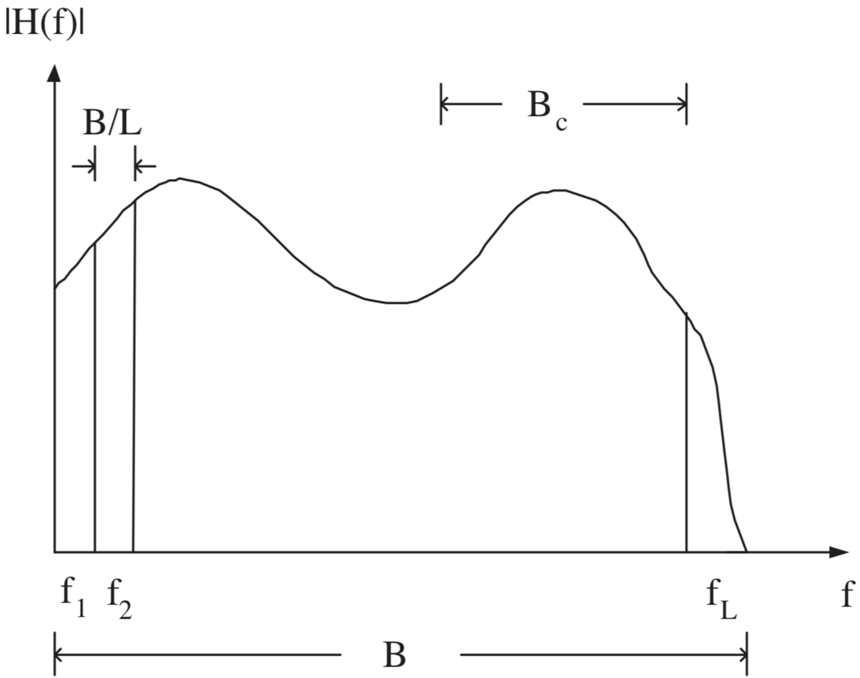
A Basic Multicarrier Receiver

- each subcarrier is decoded separately, requiring L independent receivers



Channel Effects

- flat fading on each subchannel since $B/L \ll B_c$, even though the overall channel experiences frequency selective fading, i.e. $B > B_c$.



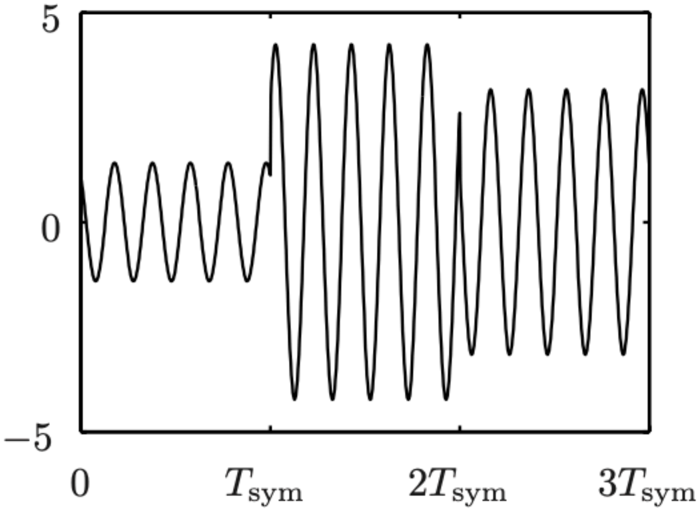
Multicarrier FDMA: impractical

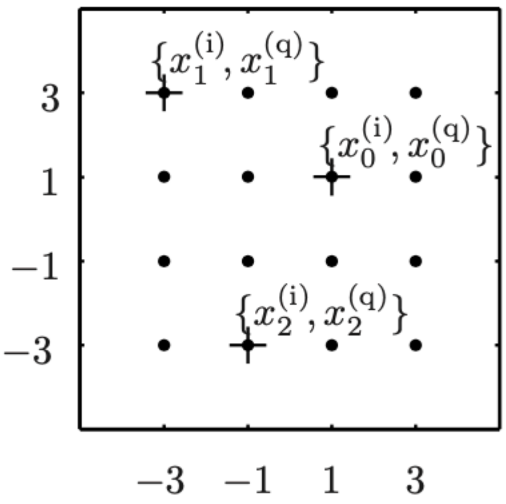
- a large bandwidth penalty will be inflicted since the subcarriers *can't have perfectly rectangular pulse shapes* and still be time-limited
- very high quality (and hence, expensive) *low pass filters* will be required to maintain the orthogonality of the the subcarriers at the receiver.
- this scheme requires *L independent RF units* and demodulation paths

OFDM Basics

- OFDM utilizes an efficient computational technique known as the Discrete Fourier Transform (DFT), more commonly known as the Fast Fourier Transform (FFT)
- No need for multiple radios

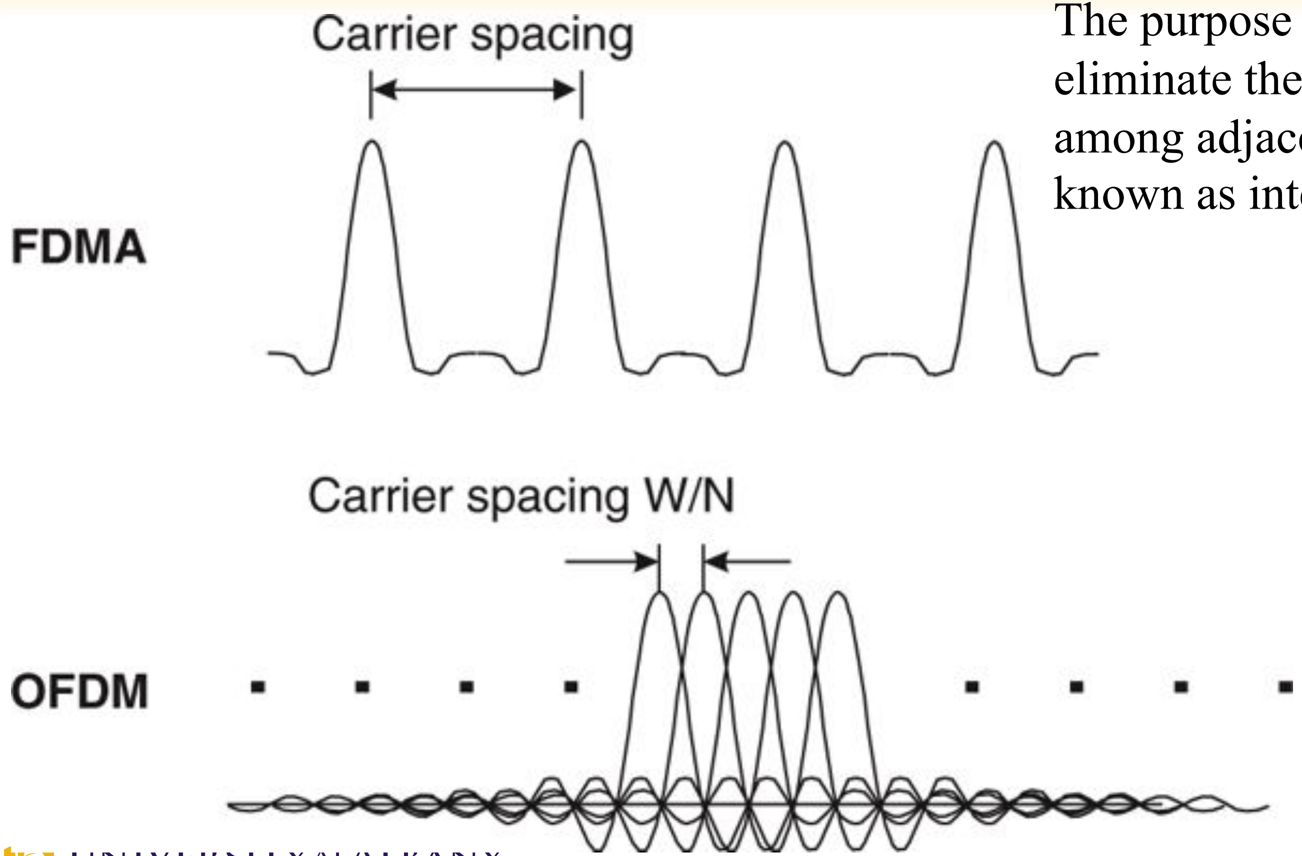
Sinusoid





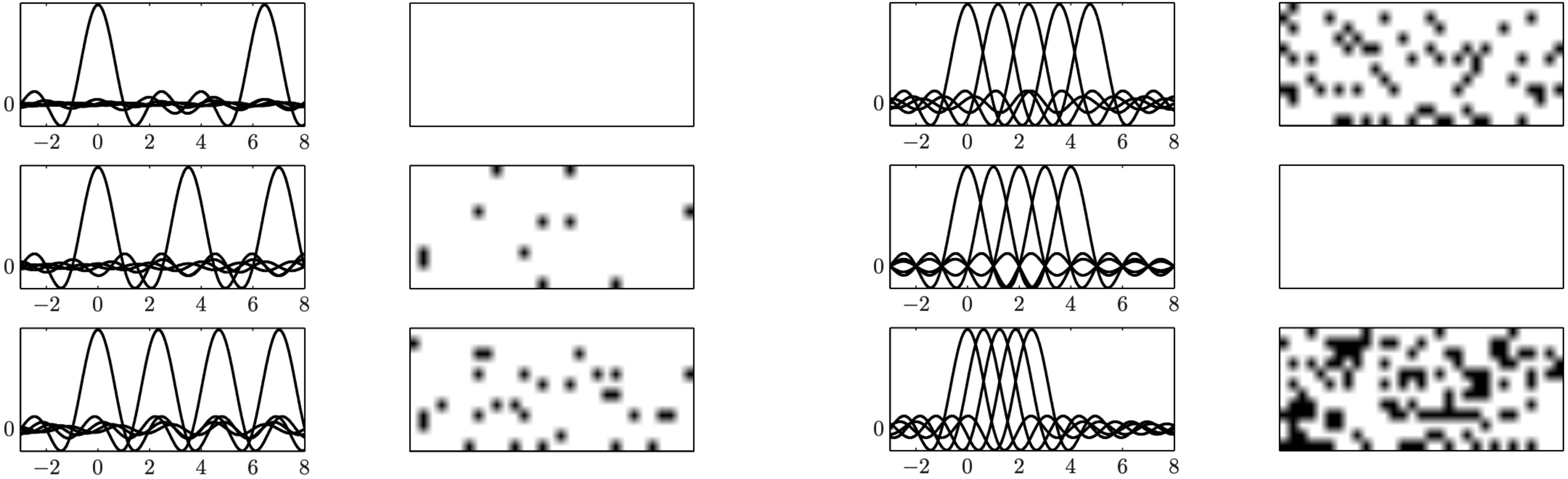
Amplitude/phase modulation of a sinusoid

FDMA and OFDM Comparison

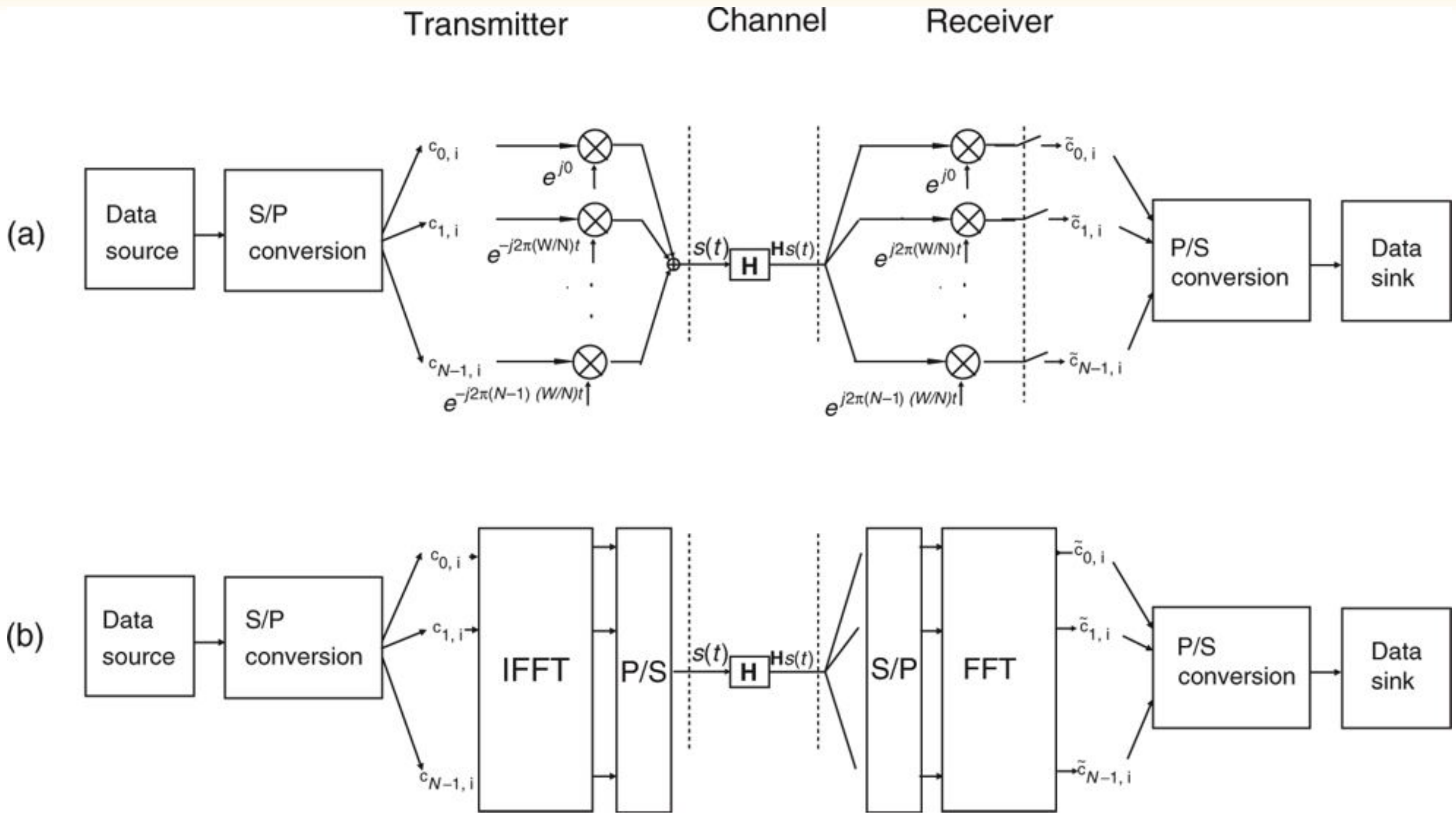


The purpose of the non-overlap is to eliminate the possible interference among adjacent sub-channels, also known as inter-carrier interference (ICI)

Multicarrier Communication



FDMA and OFDM Comparison



Multicarrier to OFDM

- N data to be transmitted are X_k (complex numbers in constellation points)
- kth carrier frequency for X_k is f_k
- Multicarrier transmitter output is $$x(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}$$
- For digital transmission, $t=nT_s$, where T_s is the sampling interval
- Digital multicarrier transmitter output is $$x(nT_s) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k nT_s}$$
- If the carrier frequencies are uniformly spaced by a frequency spacing of f_s or $f_k = kf_s$
- Then $$x(nT_s) = \sum_{k=0}^{N-1} X_k e^{j2\pi k f_S nT_s}$$
- Let $f_s=1/(NT_s)$ = the minimum separation to keep orthogonality among signals
- Then the **OFDM** signal is given by $$x_n = x(nT_s) = \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}$$

These carriers are called *subcarriers.*

Symbol Duration vs Samples

- Moving from baseband to passband

$$m_0(t) = \sum_{n=0}^{N-1} \frac{1}{2}\left\{X_n e^{j2\pi f_0 n t} + {X_n}^* e^{-j2\pi f_0 n t}\right\}$$
$$= \sum_{n=-(N-1)}^{N-1} \frac{1}{2} X_n e^{j2\pi f_0 n t},$$

Fourier coefficients $F_n = \begin{cases} \frac{1}{2}X_n & if \quad 1 \le n \le N-1 \\ \frac{1}{2}X_n^* & if \quad -(N-1) \le n \le -1 \\ 0 & if \quad n = 0 \end{cases}$

Fourier coefficients of a real signal are conjugate complex symmetric

$$m_0(t) = \sum_{n=-(N-1)}^{N-1} F_n \cdot e^{j2\pi f_0 n t}$$

In discrete time $m_0(i\Delta t) = \sum_{n=-(N-1)}^{N-1} F_n e^{j2\pi n f_0 i \Delta t}$

Nyquist Criterion

$$f_s > 2(N-1) f_0$$

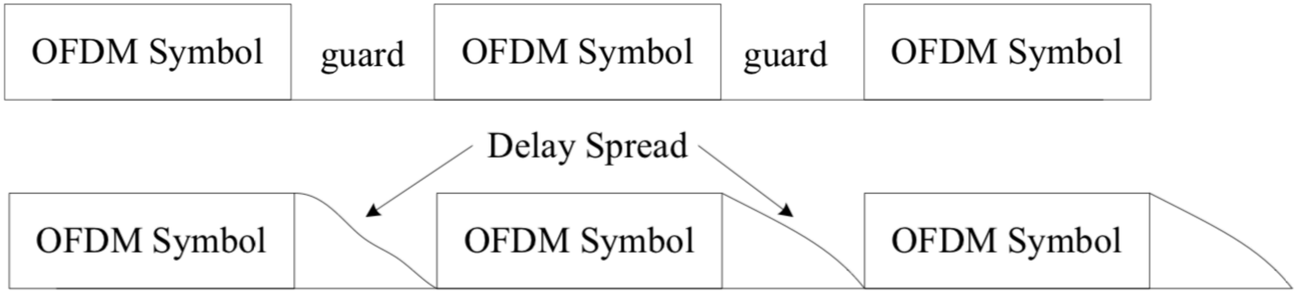
Symbol Duration vs Samples

- WLAN
  - 20MHz Bandwidth with 64 subcarriers
  - Subcarrier Spacing = 20e6/64 = 312.5KHz
  - Sampling Rate = 20MSamples/sec (to meet Nyquist Rate)
  - Each symbol = 64 samples (based on FFT size)
  - Symbol Duration (without guard) = 1/20e6*64 = 3.2microseconds

  - Homework: Calculate symbol duration for10MHz & 5MHz Bandwidth with 64 subcarriers

OFDM Basics

- L data symbols are grouped into a block – OFDM symbol
- T_s = symbol time for each data symbol
- $T = LT_s$ = OFDM symbol duration
- τ = Delay spread of the channel
- If guard time $Tg > \tau$, no interference between subsequent OFDM symbols
- No ISI



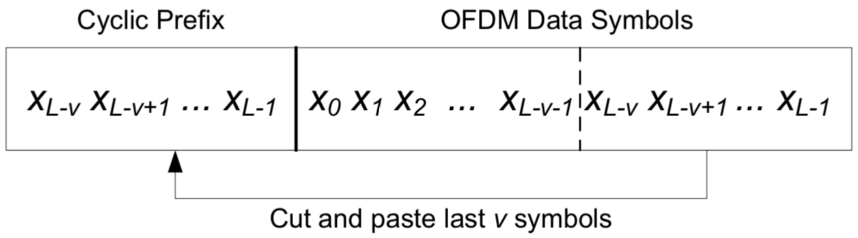
Combat ICI and ISI

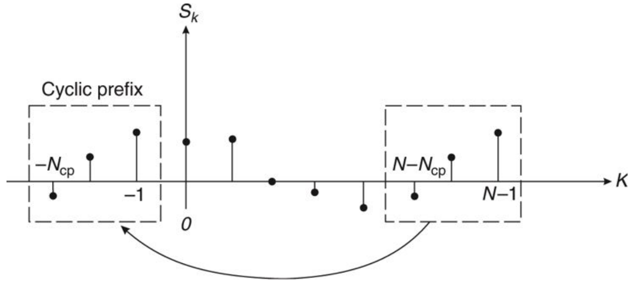
- Inter Carrier Interference (ICI) - delay dispersion leads to loss of orthogonality between the subcarriers
  - Larger subcarrier spacing leads to smaller ICI
  - Larger spacing leads to shorter symbol duration

  - **Cyclic Prefix**: A special type of Guard Interval to combat ISI and ICI
  - Assumption: channel is static for the duration of the OFDM symbol. If this assumption is not fulfilled, interference between the subcarriers can still occur

Circular Convolution in Channel

- Cyclic Prefix





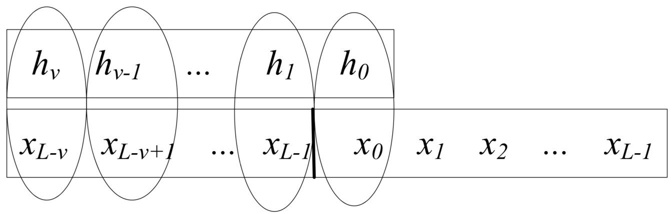
- If max channel delay spread = τ_{max} = v samples
- Then add a guard of v samples
- Time domain representation:

$$\mathbf{x} = [x_1\ x_2\ \dots x_L]$$

$$\mathbf{x}_{cp} = [\underbrace{x_{L-v}\ x_{L-v+1}\ \dots\ x_{L-1}}_{\text{Cyclic Prefix}}\ \underbrace{x_0\ x_1\ \dots\ x_{L-1}}_{\text{Original data}}]$$

CP creates Circular Convolution

- Output of Channel: $y_{CP} = h * x_{CP}$



$$y_0 = h_v x_{L-v} + h_{v-1} x_{L-v+1} \ldots + h_1 x_{L-1} + h_0 x_0$$

- for L ≫ v, the inefficiency due to the cyclic prefix can be made arbitrarily small by increasing the number of subcarriers
- The first v samples of y_{cp} contain interference from the preceding OFDM symbol, and so are discarded
- The last v samples disperse into the subsequent OFDM symbol, so also are discarded
- This leaves exactly L samples that is required to recover the L data symbols embedded in x.

Exploiting Properties of DFT

- Circular Convolution $\mathrm{DFT}\{y[n]\} = \mathrm{DFT}\{h[n] \circledast x[n]\}$
- Frequency domain output $Y[m] = H[m]X[m]$
- It is ISI-free channel in the frequency domain, where each input symbol X[m] is simply scaled by a complex-value H[m]
- Note that the *duality* between circular convolution in the time domain and simple multiplication in the frequency domain is a property unique to the DFT.

Exploiting Properties of DFT

- L point DFT

$$\mathrm{DFT}\{x[n]\} = X[m] \triangleq \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} x[n] e^{-j\frac{2\pi nm}{L}}$$

- Inverse DFT (IDFT)

$$\mathrm{IDFT}\{X[m]\} = x[n] \triangleq \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} X[m] e^{j\frac{2\pi nm}{L}}$$

- At receiver, if channel frequency response H[m] is known, input is derived as

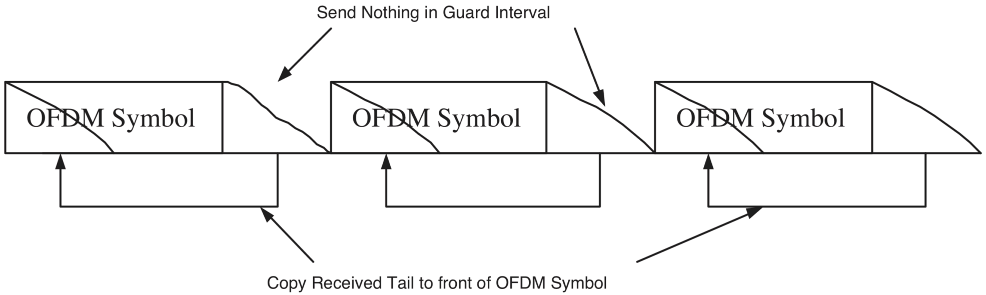
$$\hat{X}[m] = \frac{Y[m]}{H[m]}$$

Penalties of CP

- v redundant symbols are sent
- Required bandwidth increases from B to $\frac{L+v}{L}B$
- Transmit power penalty $10log_{10}\frac{L+v}{L}dB$
- Rate loss = Power loss = $\frac{L}{L+v}$

Zero Prefix

- Null Guard Band
- At the receiver, the "tail" can be added back in
- Recreates the effect of a CP



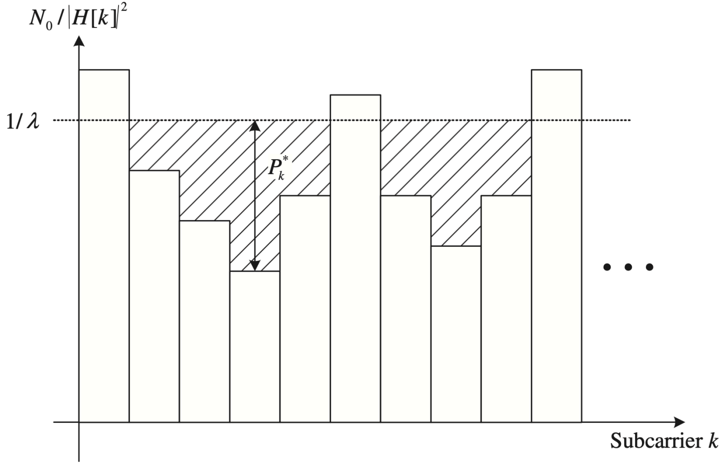
- Reduces Tx power by $10log_{10}\frac{L+v}{L}dB$

Zero Prefix Issues

- Increases the receiver power by $10log_{10}\frac{L+v}{L}dB$
- With CP transmitted, the tail can be ignored
- Additional noise from the received tail symbols is added back into the signal
- Higher noise power compared to transmitted CP
- $\sigma^2 \rightarrow \frac{L+v}{L}\sigma^2$

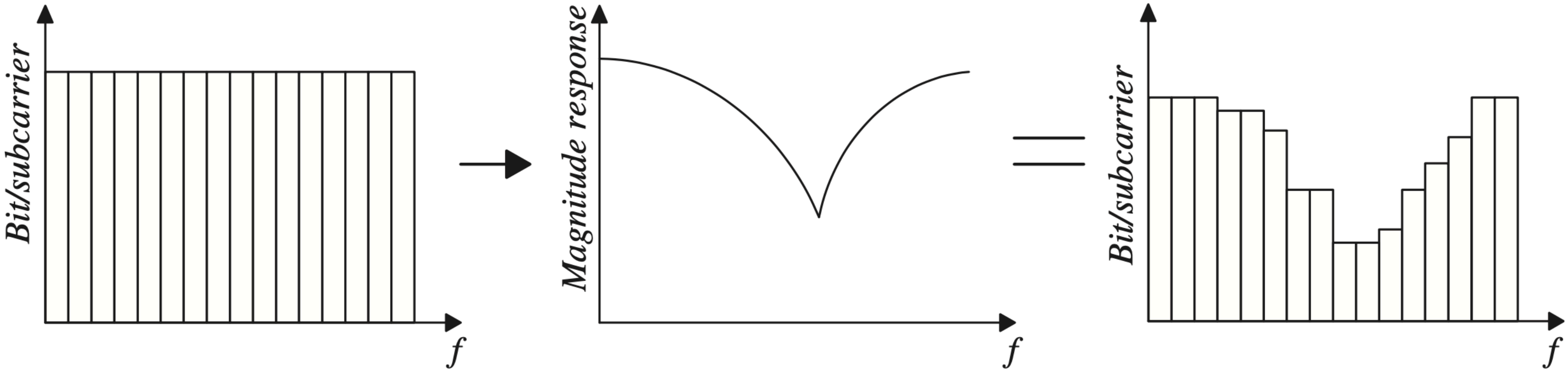
Waterfilling Algorithm

- Allocates more (or less) bits and power to some subcarriers with larger (or smaller) SNR for maximizing the channel capacity



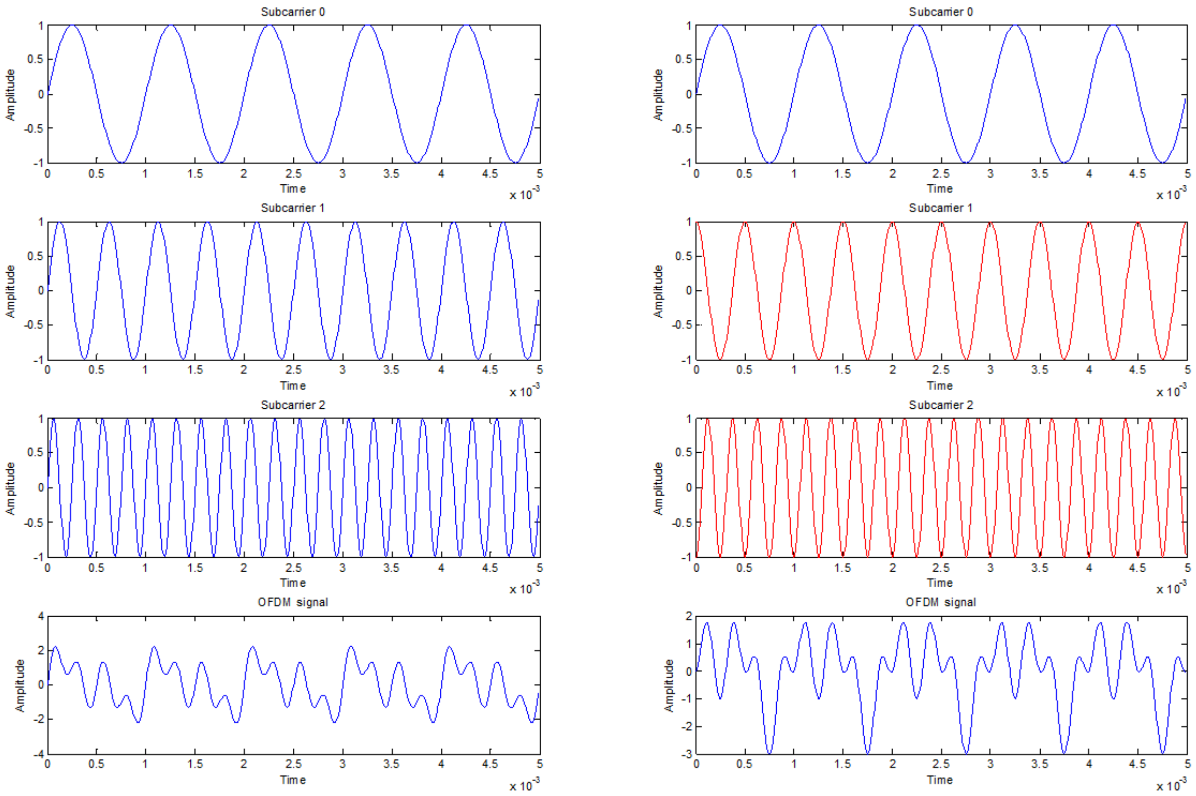
Adaptive Bit Loading

- different numbers of bits are allocated to different subcarriers subject to frequency-selective fading



Peak to Average Power Ratio (PAPR)

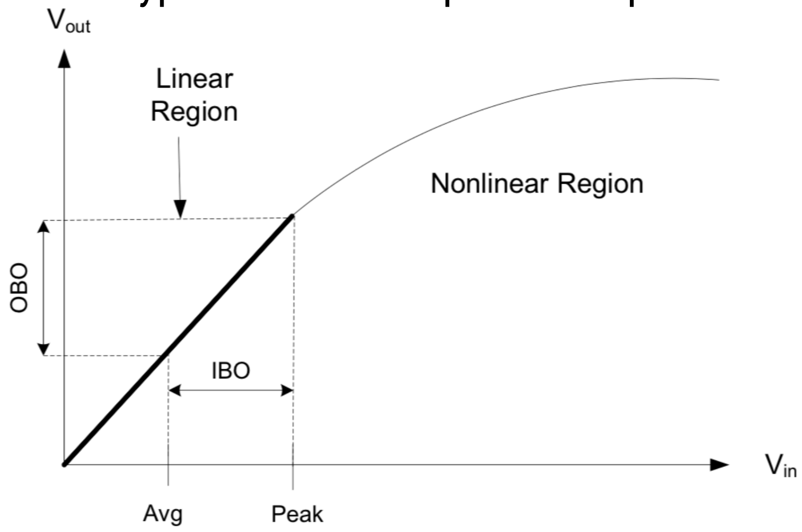
- In time domain, OFDM is a sum of multiple narrowband



High PAPR: Implementation challenge

- $PAPR = 10log_{10}\frac{P_{peak}}{P_{avg}}$
- generates out-of-band energy (spectral regrowth)
- in-band distortion (constellation tilting and scattering)

Typical Power Amplifier Response

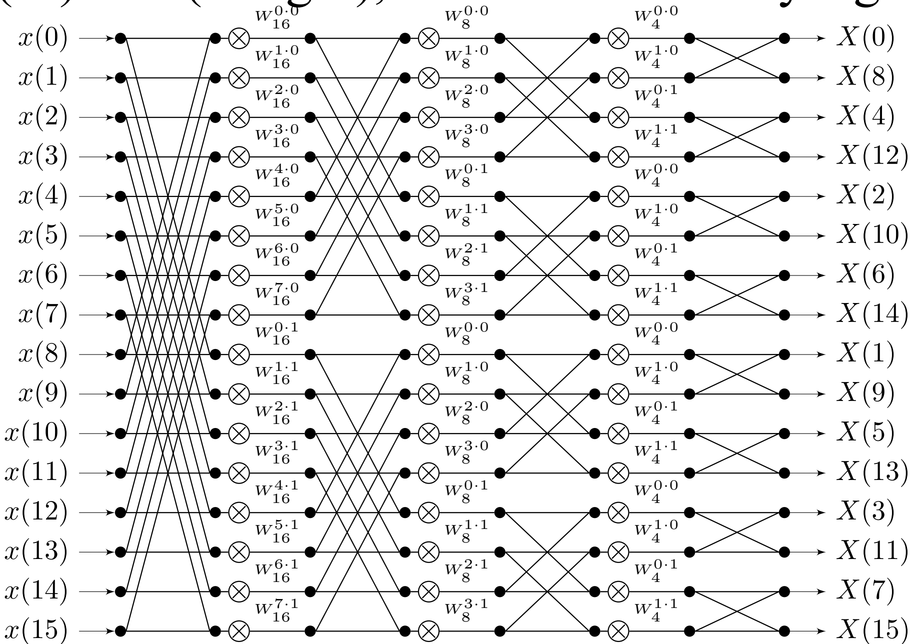


PAPR Reduction Techniques

- clipping and filtering
- selected mapping
- coding techniques

FFT Implementation

- The key to making OFDM realizable in practice is the utilization of the FFT algorithm for computing the DFT and the IFFT algorithm for computing the IDFT, which reduces the number of required multiplications and additions from $O(L^2)$ to $O(L \log L)$, which is extremely significant.



Brief History of OFDM

- Although OFDM has become widely used only recently, the concept dates back some 40 years.
  - **1958:** The "Kineplex" system was developed, which was a multicarrier modem for the HF bands (3 to 30MHz). This is widely considered the first ever multicarrier system—it actually used multiple HF radios as the FFT was not re-discovered[9] until 1954.
  - **1966:** Chang shows in the Bell Labs technical journal that multicarrier modulation can solve the multipath problem without reducing data rate. This is generally considered the first theoretical publication on multicarrier modulation, although there were naturally precursory studies, including Holsinger's 1964 MIT dissertation and some of Gallager's work on waterfilling.

Brief History of OFDM

- **1971:** Weinstein and Ebert show that multicarrier modulation can be accomplished using a "Discrete Fourier Transform" (DFT).
- **1985:** Cimini at Bell Labs identifies many of the key issues in OFDM transmission and does a proof of concept design.
- **1993:** DSL adopts OFDM, also called "Discrete Multitone," following successful field trials/competitions at Bellcore vs. equalizer-based systems.
- **1999:** IEEE 802.11 committee on wireless LANs releases 802.11a standard for OFDM operation in 5GHz UNI band.
- **2002:** IEEE 802.16 committee releases OFDM-based standard for wireless broadband access for metropolitan area networks under revision 802.16a.

Brief History of OFDM

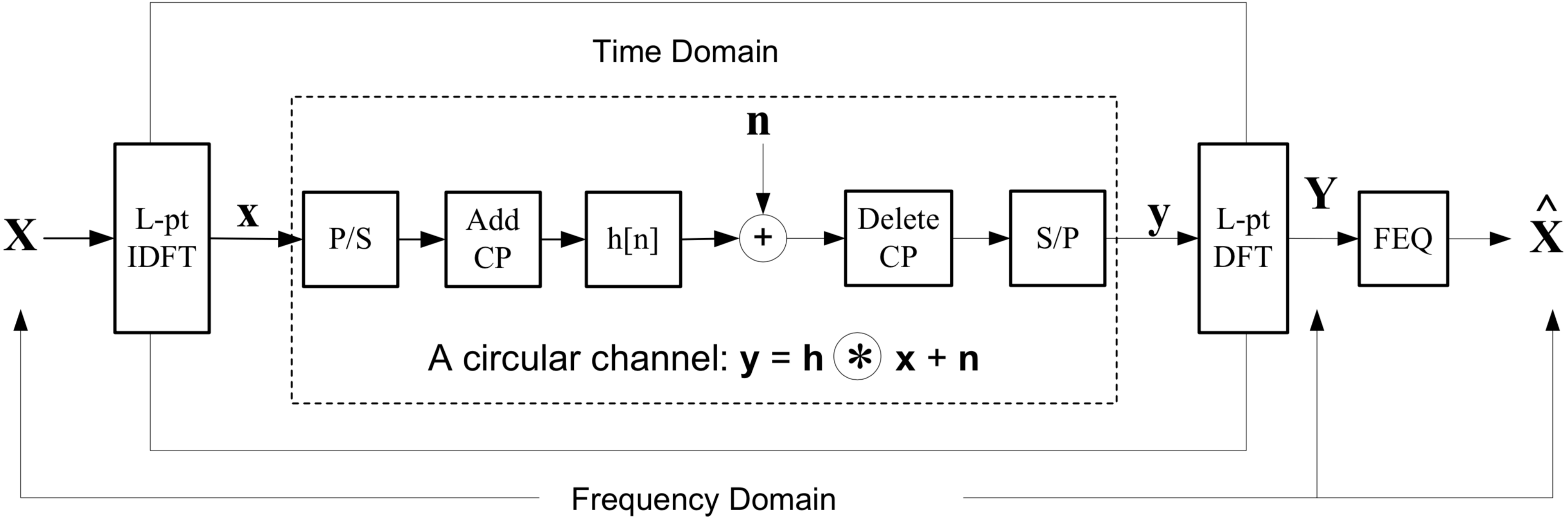
- **2003:** IEEE 802.11 committee releases 802.11g standard for operation in the 2.4GHz band.
- **2003:** The “multiband OFDM” standard for ultrawideband is developed, showing OFDM’s usefulness in low-SNR systems.
- **2005:** 802.16e standard is ratified, supporting mobile OFDMA for WiMAX.
- **2006:** First commercial LTE demonstrations by Siemens (now Nokia Siemens Networks).
- **2008:** Qualcomm, the primary backer of Ultramobile Broadband (UMB), the main future competition to LTE and WiMAX and also OFDM/OFDMA-based, announces it will end UMB development and transition to LTE, solidifying LTE as the leading beyond 3G cellular standard.

Brief History of OFDM

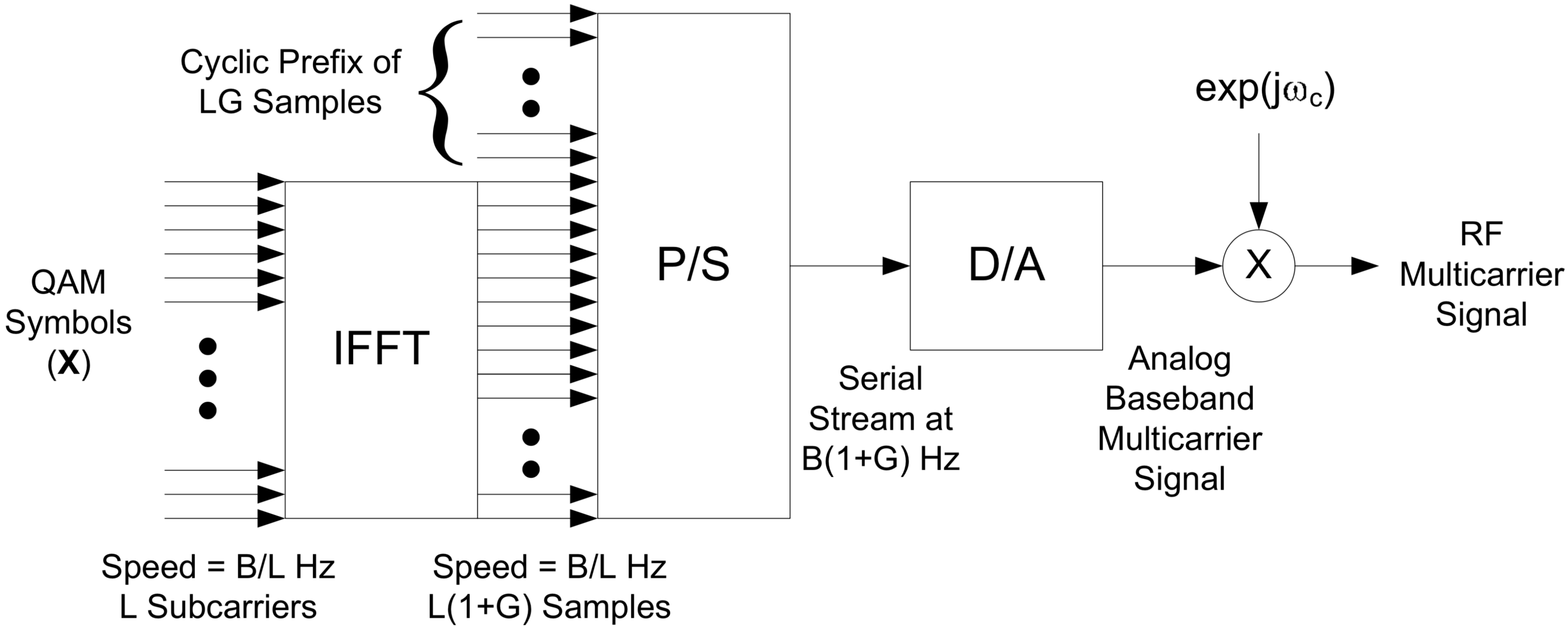
- **2009:** 3GPP Release 8 LTE/SAE specifications completed and released.
- **2009:** 802.11n standard is ratified, which performs MIMO-OFDM for wireless LANs for peak data rates of 600 Mbps.

S. B. Weinstein, "The history of orthogonal frequency-division multiplexing [History of Communications]," in IEEE Communications Magazine, vol. 47, no. 11, November 2009.

An OFDM System



An OFDM Transmitter



OFDM Parameters in LTE for 10MHz

Symbol	Description	Relation	Example LTE value
B	Nominal bandwidth	$B = 1/2f_s$	7.68MHz
B_{chan}	Transmission bandwidth	Channel spacing	10MHz
L	No. of subcarriers	Size of IFFT/FFT	1024
G	Guard fraction	% of L for CP	0.07
L_d	Data subcarriers	$L-$ pilot/null subcarriers	600
Δf	Subcarrier spacing	Independent of L	15KHz
T_s	Sample time	$T_s = 1/\max(B) = 1/\Delta f \cdot 2048$	$1/15\text{KHz} \cdot 2048$ $= 32.55$ nsec
N_g	Guard symbols	$N_g = GL$	72
T_g	Guard time	$T_g = 144T_s$ or $160T_s$	4.7 or 5.2 μsec
T	OFDM symbol time	$T = (L + N_g)/B$	142.7 μsec

OFDM Parameters in WLAN

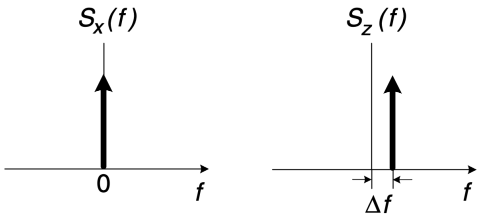
Parameter	Value (20 MHz channel spacing)	Value (10 MHz channel spacing)	Value (5 MHz channel spacing)
N_{SD}: Number of data subcarriers	48	48	48
N_{SP}: Number of pilot subcarriers	4	4	4
N_{ST}: Number of subcarriers, total	52 ($N_{SD} + N_{SP}$)	52 ($N_{SD} + N_{SP}$)	52 ($N_{SD} + N_{SP}$)
Δ_F: Subcarrier frequency spacing	0.3125 MHz (=20 MHz/64)	0.156 25 MHz (= 10 MHz/64)	0.078 125 MHz (= 5 MHz/64)
T_{FFT}: Inverse Fast Fourier Transform (IFFT) / Fast Fourier Transform (FFT) period	3.2 μs ($1/\Delta_F$)	6.4 μs ($1/\Delta_F$)	12.8 μs ($1/\Delta_F$)
$T_{PREAMBLE}$: PHY preamble duration	16 μs ($T_{SHORT} + T_{LONG}$)	32 μs ($T_{SHORT} + T_{LONG}$)	64 μs ($T_{SHORT} + T_{LONG}$)

Front-End Electronics Effects

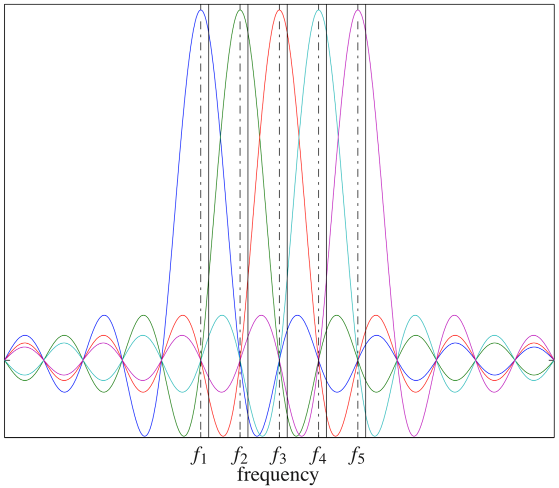
- Carrier Frequency Offset
- Sampling Clock Offset
- Phase Noise
- IQ Balance and DC Offset
- Power Amplifier Nonlinearity

Carrier Frequency Offset

- CFO occurs when the local oscillator signal for down conversion in the receiver does not synchronize with the carrier signal contained in the received signal
  - frequency mismatch in the transmitter and the receiver oscillators
  - Doppler effect as the transmitter and/or the receiver is moving
- Results in received signal shifted in frequency
- For OFDM
  - orthogonality is maintained only if receiver uses a local oscillation signal that is synchronous with the carrier signal contained in the received signal
  - Results in ICI
  - Adds additional term $e^{j2\pi\Delta F_c t}$ in baseband

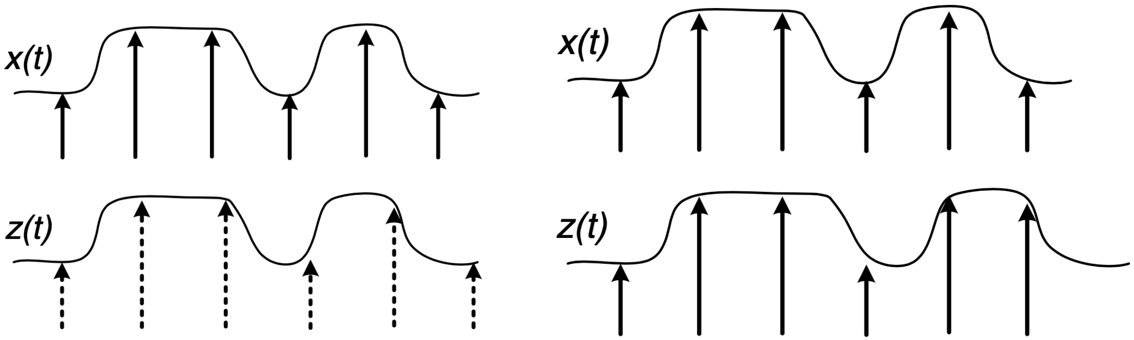


Fourier transforms of the carriers



Sampling Clock Offset (SCO)

- Occurs when oscillators with mismatched frequencies are used to drive the sampling clocks of the DAC and the ADC
- Doppler causes expansion or contraction of signal at receiver



Sampled at different clock rates

Received signal expanded due to Doppler

Phase Noise

- Related to jitter in oscillation signal
- Occurs because oscillators cannot generate pure sinusoidal waves

$$\theta_n(t_n) = \theta_n(t_{n-1}) + \psi(t_n)$$

Phase noise — Phase increment

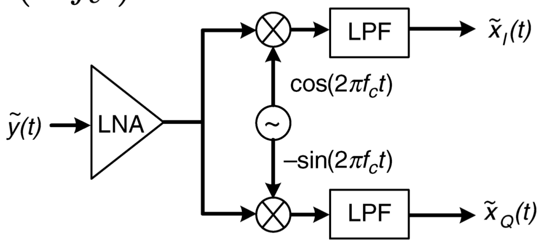
- Results in ICI

IQ Imbalance

- Direct conversion translates passband to baseband in one stage of mixing
- Occurs when there are mismatches
  - between the gain and phase of the two sinusoidal signals
  - along two branches of down-conversion mixers, gain amplifier and low-pass filters

Passband signal $y(t) = \mathrm{Re}\{x(t)e^{j2\pi f_c t}\} = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$

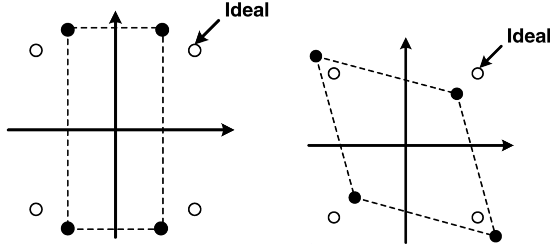
Gain Error $20\log\frac{1+\alpha}{1-\alpha}$ dB   Phase Error ϕ degrees



Baseband signal

$$\tilde{x}_I(t) = (1+\alpha)\left[x_I(t)\cos\left(\frac{\phi}{2}\right) - x_Q(t)\sin\left(\frac{\phi}{2}\right)\right]$$

$$\tilde{x}_Q(t) = (1-\alpha)\left[x_Q(t)\cos\left(\frac{\phi}{2}\right) - x_I(t)\sin\left(\frac{\phi}{2}\right)\right]$$



Gain Error   Phase Error

IQ Imbalance

$$\begin{aligned}\tilde{x}(t) &= \tilde{x}_I(t) + j\tilde{x}_Q(t) \\ &= \left[\cos\left(\frac{\phi}{2}\right) + j\alpha\sin\left(\frac{\phi}{2}\right)\right]x(t) + \left[\alpha\cos\left(\frac{\phi}{2}\right) - j\sin\left(\frac{\phi}{2}\right)\right]x^*(t) \\ &= Ax(t) + Bx^*(t),\end{aligned}$$

- In frequency domain, for multiple subcarriers in OFDM

$$\left((X_{k,I} + jX_{k,Q})e^{j2\pi kf_St}\right)^* = (X_{k,I} - jX_{k,Q})e^{-j2\pi kf_St} = X_k^* e^{j2\pi(-k)f_St}$$

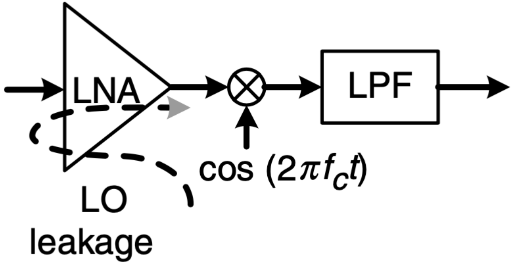
- complex conjugating the baseband signal of the k-th subcarrier carrying data X_k is identical to carrying $X_k{}^*$on the (-k)-th subcarrier
- Received baseband signal $\tilde{X}_k = AX_k + BX^*_{-k}$

a complex gain imposed on current subcarrier data

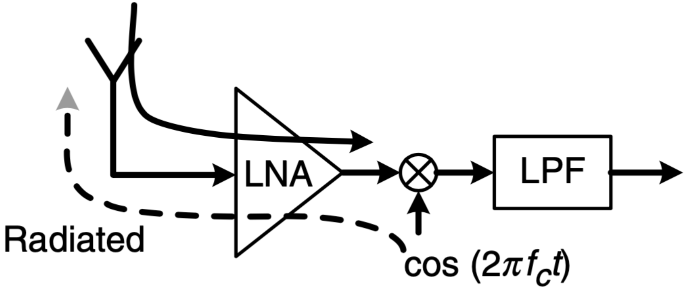
ICI introduces in the mirror subcarrier

DC Offset

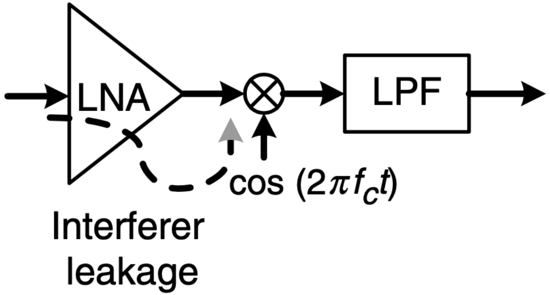
- Self mixing and nonlinearity in the front-end



LO leakage
(static DC)



LO re-radiation
(dynamic DC)

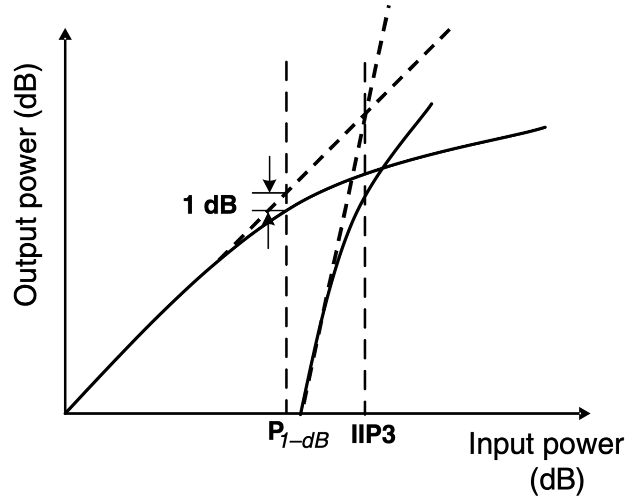


interference leakage
(strong in-band interferers)

- DC offset will saturate all the following stages and make the receiver fail

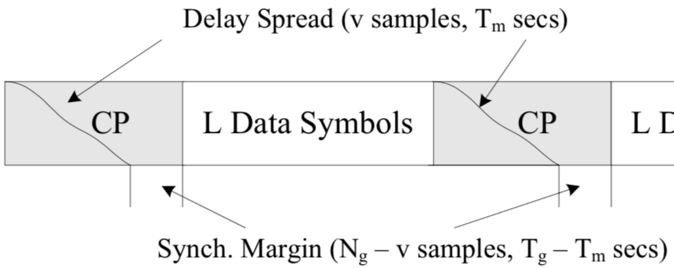
Power Amplifier Non-linearity

- Nonlinear amplification distorts Tx signal
- OFDM is particularly sensitive to non-linearity
  - Constellation Distortion
  - Intermodulation Distortion (shows up in harmonics, can be filtered)

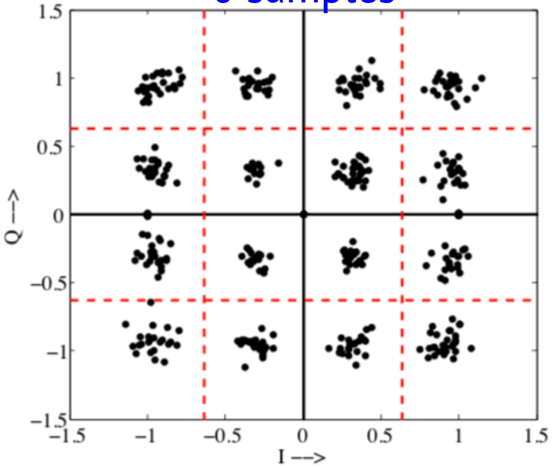


Timing Offset

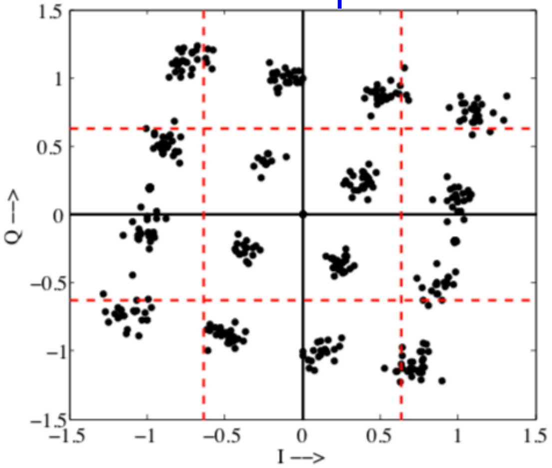
- Cyclic Prefix provides some toleration in error in timing synchronization



0 samples



-8 samples



Synchronization

- Carrier frequency offset (CFO) Δf
  - causes the received complex baseband signal to rotate at a frequency
- Carrier phase error $\phi(t)$
  - introduces an additional phase rotation term in the received complex baseband signal
- Sampling clock offset (SCO) δ
  - results in sampling the received continuous-time waveform at interval of $(1+\delta)T_s$ instead of the ideal T_s
- Symbol timing offset T_d
  - error in the symbol boundary at the receiver from the actual boundary in the received waveform