
Cyber-Physical Systems



UNIVERSITY
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Deadline based Scheduling

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Real-Time Systems

- The operating system, and in particular the scheduler, is perhaps the most important component

Examples:

- Control of laboratory experiments
- Process control in industrial plants
- Robotics
- Air traffic control
- Telecommunications
- Military command and control systems

- Correctness of the system depends not only on the logical result of the computation but also on the time at which the results are produced
- Tasks attempt to react to events that take place in the outside world
- These events occur in “real time” and tasks must be able to keep up with them

Hard and Soft Real-Time Tasks

➤ Hard

- One that must meet its deadline
- Otherwise it will cause unacceptable damage or a fatal error to the system

➤ Soft

- Has an associated deadline that is desirable but not mandatory
- It still makes sense to schedule and complete the task even if it has passed its deadline

Periodic and Aperiodic Tasks

➤ Periodic tasks

- Requirement may be stated as:
 - Once per period T
 - Exactly T units apart

➤ Aperiodic tasks

- Has a deadline by which it must finish or start
- May have a constraint on both start and finish time

Characteristics of Real Time Systems

Real-time operating systems have requirements in five general areas:

Determinism

Responsiveness

User control

Reliability

Fail-soft operation

Determinism

- Concerned with how long an operating system delays before acknowledging an interrupt
- Operations are performed at fixed, predetermined times or within predetermined time intervals
 - When multiple processes are competing for resources and processor time, no system will be fully deterministic

The extent to which an operating system can deterministically satisfy requests depends on:

The speed with which it can respond to interrupts

Whether the system has sufficient capacity to handle all requests within the required time

Responsiveness

- Together with determinism make up the response time to external events
 - Critical for real-time systems that must meet timing requirements imposed by individuals, devices, and data flows external to the system
- Concerned with how long, after acknowledgment, it takes an operating system to service the interrupt

Responsiveness includes:

- Amount of time required to initially handle the interrupt and begin execution of the interrupt service routine
- Amount of time required to perform the ISR
- Effect of interrupt nesting

User Control

- Generally much broader in a real-time operating system than in ordinary operating systems
- It is essential to allow the user fine-grained control over task priority
- User should be able to distinguish between hard and soft tasks and to specify relative priorities within each class
- May allow user to specify such characteristics as:

Paging or
process
swapping

What processes
must always be
resident in main
memory

What disk
transfer
algorithms are
to be used

What rights the
processes in
various priority
bands have

Reliability

- More important for real-time systems than non-real time systems
- Real-time systems respond to and control events in real time so loss or degradation of performance may have catastrophic consequences such as:
 - Financial loss
 - Major equipment damage
 - Loss of life

Fail-Soft Operation

- A characteristic that refers to the ability of a system to fail in such a way as to preserve as much capability and data as possible
- Important aspect is stability
 - A real-time system is stable if the system will meet the deadlines of its most critical, highest-priority tasks even if some less critical task deadlines are not always met

Features common to Most RTOSs

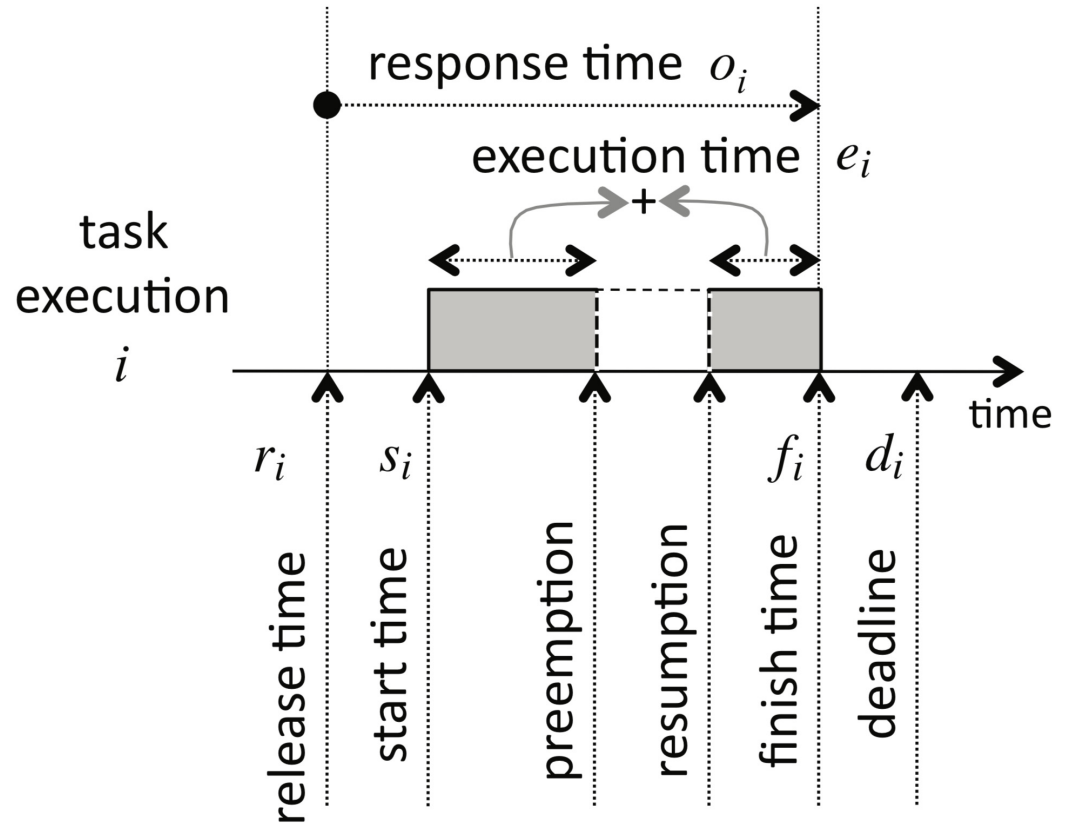
- A **stricter use of priorities** than in an ordinary OS, with preemptive scheduling that is designed to meet real-time requirements
- **Interrupt latency is bounded** and relatively short
- More **precise and predictable timing** characteristics than general purpose OSs

Task Model

$$s_i \geq r_i$$

$$f_i \geq s_i$$

$$o_i = f_i - r_i$$



Scheduling Strategies

- **Goal:** all task executions meet their deadlines

$$f_i \leq d_i$$

- A schedule that accomplishes this is called a **feasible schedule**.
- A scheduler that yields a feasible schedule for any task set is said to be **optimal** with respect to feasibility.

Criteria or Metrics

➤ Processor Utilization μ

➤ Maximum Lateness

$$L_{\max} = \max_{i \in T} (f_i - d_i)$$

➤ Total Completion Time or Makespan

$$M = \max_{i \in T} f_i - \min_{i \in T} r_i$$

➤ Average Response Time

$$\bar{t}_r = \frac{1}{n} \sum_{i=1}^n (f_i - a_i)$$

Rate Monotonic Scheduling

- Simple process model: n tasks invoked periodically with:
 - periods T_1, \dots, T_n , which equal the deadlines
 - known worst-case execution times (WCET) C_1, \dots, C_n
 - no mutexes, semaphores, or blocking I/O
 - independent tasks, no precedence constraints
 - fixed priorities
 - preemptive scheduling
- Rate Monotonic Scheduling (RMS): **priorities ordered by period** (smallest period has the highest priority)

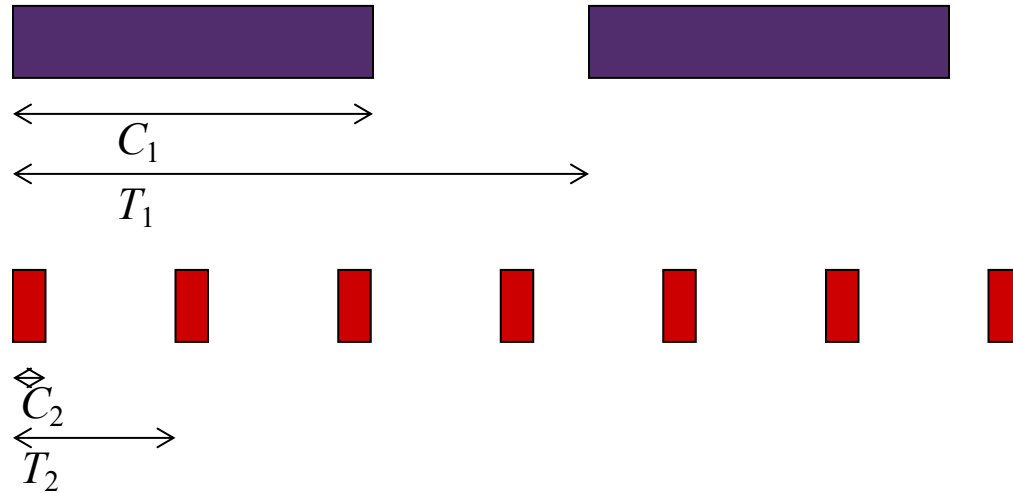
Feasibility for RMS

- Feasibility is defined for RMS to mean that every task executes to completion once within its designated period.
- Theorem: Under the simple process model, if any priority assignment yields a feasible schedule, then RMS also yields a feasible schedule.
- RMS is optimal in the sense of feasibility.

Liu and Layland, "Scheduling algorithms for multiprogramming in a hard-real-time environment," J. ACM, 1973.

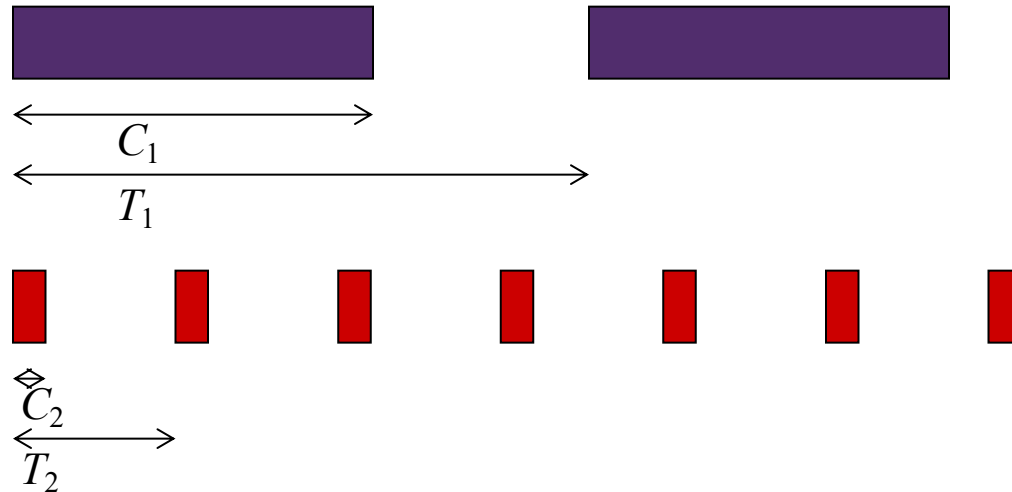
Showing Optimality of RMS:

- Consider two tasks with different periods.
- Is a non-preemptive schedule feasible?



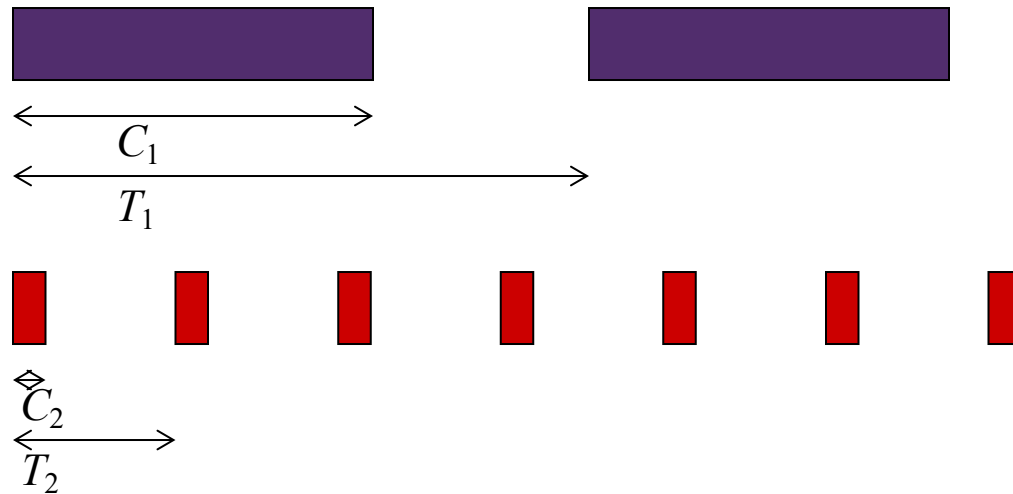
Showing Optimality of RMS:

- Non-preemptive schedule is not feasible. Some instance of the Red Task (2) will not finish within its period if we do non-preemptive scheduling.



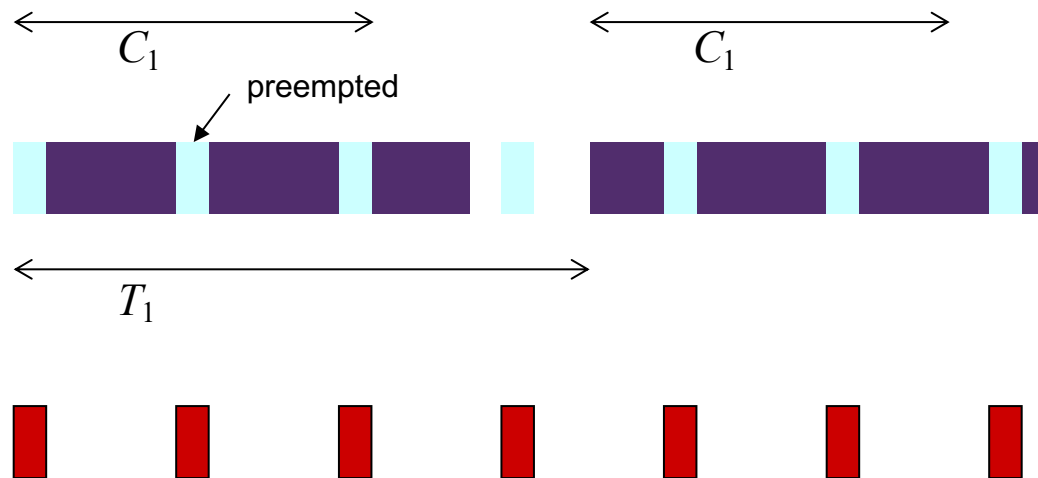
Showing Optimality of RMS:

- What if we had a preemptive scheduling with higher priority for red task?



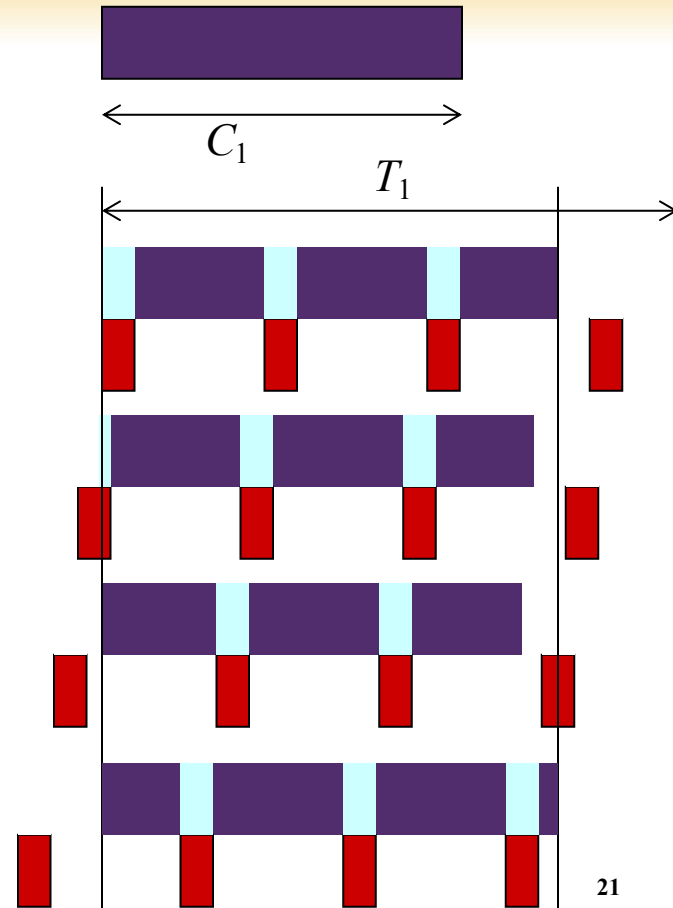
Showing Optimality of RMS:

- Preemptive schedule with the red task having higher priority is feasible. Note that preemption of the purple task extends its completion time.



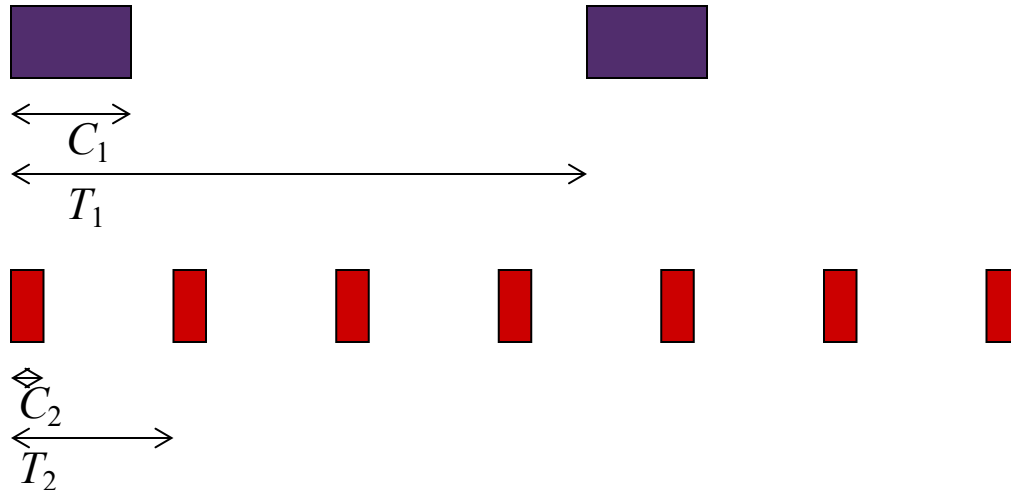
Alignment of tasks

- Completion time of the lower priority task is worst when its *starting phase* matches that of higher priority tasks.
- Thus, when checking schedule feasibility, it is sufficient to consider only the worst case: All tasks start their cycles at the same time.



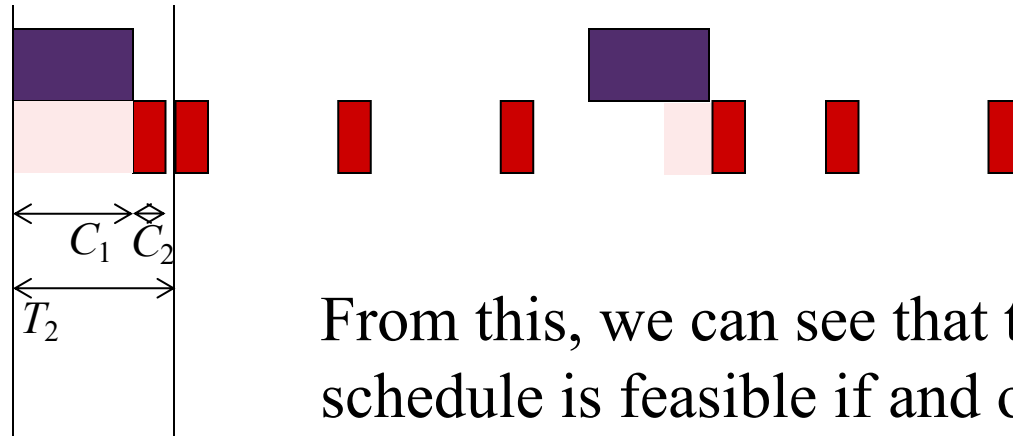
Showing Optimality of RMS: (two tasks)

- It is sufficient to show that if a non-RMS schedule is feasible, then the RMS schedule is feasible.
- Consider two tasks as follows:



Showing Optimality of RMS: (two tasks)

The non-RMS, fixed priority schedule looks like this:



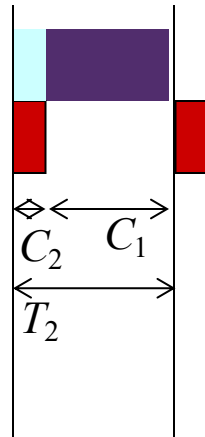
From this, we can see that the non-RMS schedule is feasible if and only if

$$C_1 + C_2 \leq T_2$$

We can then show that this condition implies that the RMS schedule is feasible.

Showing Optimality of RMS: (two tasks)

The RMS schedule looks like this: (task with smaller period moves earlier)



The condition for the non-RMS schedule feasibility:

$$C_1 + C_2 \leq T_2$$

is clearly sufficient (though not necessary) for feasibility of the RMS schedule.

Comments

- This proof can be extended to an arbitrary number of tasks (though it gets much more tedious).
- This proof gives optimality only w.r.t. feasibility.
- Practical implementation:
 - Timer interrupt at greatest common divisor of the periods.
 - Multiple timers

RM Scheduler: Processor Utilization

$$\mu = \sum_{i=1}^n \frac{e_i}{p_i}$$

- If $\mu > 1$ for any task set, then that task set has no feasible schedule
- Utilization Bound: RMS is feasible $\mu \leq n(2^{1/n} - 1)$
- As n gets large, $\lim_{n \rightarrow \infty} n(2^{1/n} - 1) = \ln(2) \approx 0.693$.
- If a task set with any number of tasks does not attempt to use more than 69.3% of the available processor time, then the RM schedule will meet all deadlines.

Liu and Layland, "Scheduling algorithms for multiprogramming in a hard-real-time environment," J. ACM, 1973.

Jackson's Algorithm: EDD (1955)

- Given n independent one-time tasks with deadlines d_1, \dots, d_n , schedule them to minimize the maximum lateness, defined as

$$L_{\max} = \max_{1 \leq i \leq n} \{f_i - d_i\}$$

- where f_i is the finishing time of task i . Note that this is negative iff all deadlines are met.
- **Earliest Due Date (EDD)** algorithm: Execute them in order of non-decreasing deadlines.
- Note that this does not require preemption.

EDD is Optimal

- Optimal in the Sense of Minimizing Maximum Lateness
 - To prove, use an *interchange argument*.
 - Given a schedule S that is not EDD, there must be tasks a and b where a immediately precedes b in the schedule but $d_a > d_b$. Why?
 - We can prove that this schedule can be improved by interchanging a and b . Thus, no non-EDD schedule achieves smaller max lateness than EDD
 - So the EDD schedule must be optimal.

Maximum Lateness

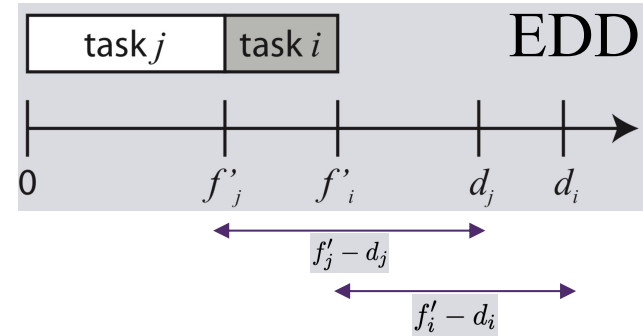
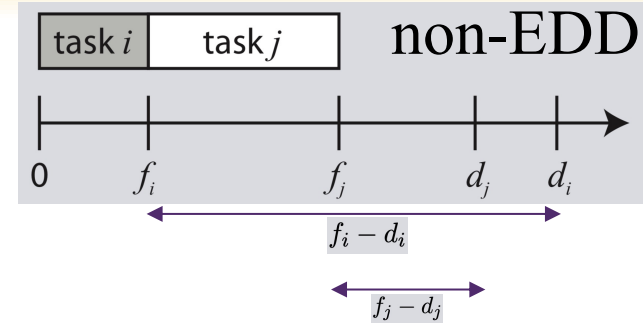
➤ First Schedule (non-EDD)

$$L_{\max} = \max(f_i - d_i, f_j - d_j) = f_j - d_j$$

- where $f_i \leq f_j$ and $d_j < d_i$

➤ Second Schedule (EDD)

$$L'_{\max} = \max(f'_i - d_i, f'_j - d_j)$$



Consider Cases

Case 1: $L'_{\max} = f'_i - d_i$

Since $f'_i = f_j$ $d_j < d_i$

$$L'_{\max} = f_j - d_i \leq f_j - d_j$$

Hence, $L'_{\max} \leq L_{\max}$

Case 2: $L'_{\max} = f'_j - d_j$

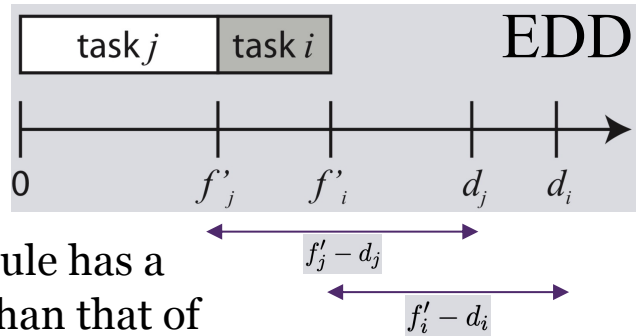
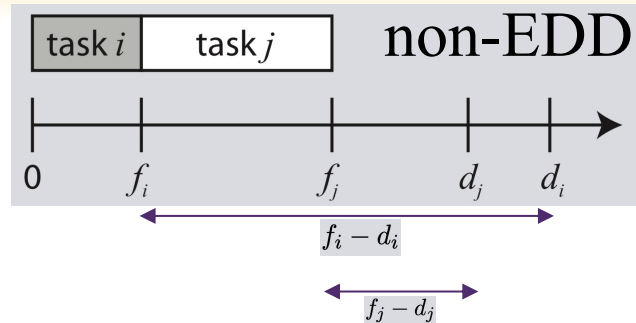
Since $f'_j \leq f_j$

$$L'_{\max} \leq f_j - d_j$$

Hence, $L'_{\max} \leq L_{\max}$

In both cases, the second schedule has a maximum lateness no greater than that of the first schedule.

EDD minimizes maximum lateness.



Horn's algorithm: EDF (1974)

- Extend EDD by allowing tasks to “arrive” (become ready) at any time.
- **Earliest deadline first (EDF)**: Given a set of n independent tasks with *arbitrary arrival times*, any algorithm that at any instant executes the task with the earliest absolute deadline among all arrived tasks is optimal w.r.t. minimizing the maximum lateness.
- Proof uses a similar interchange argument.

Using EDF for Periodic Tasks

- The EDF algorithm can be applied to periodic tasks as well as aperiodic tasks.
 - Simplest use: Deadline is the end of the period.
 - Alternative use: Separately specify deadline (relative to the period start time) and period.

RMS vs. EDF? Which one is better?

- What are the pros and cons of each?

Comparison of EDF and RMS

➤ Favoring RMS

- Scheduling decisions are simpler (fixed priorities vs. the dynamic priorities required by EDF. EDF scheduler must maintain a list of ready tasks that is sorted by priority.)

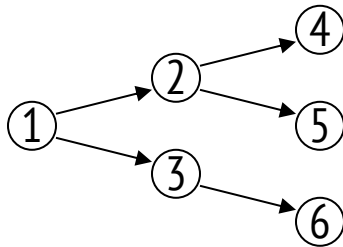
Comparison of EDF and RMS

➤ Favoring EDF

- Since EDF is optimal w.r.t. maximum lateness, it is also optimal w.r.t. feasibility. RMS is only optimal w.r.t. feasibility.
- For infeasible schedules, RMS completely blocks lower priority tasks, resulting in unbounded maximum lateness.
- EDF can achieve full utilization where RMS fails to do that.
- EDF results in fewer preemptions in practice, and hence less overhead for context switching.
- Deadlines can be different from the period.

Precedence Constraints

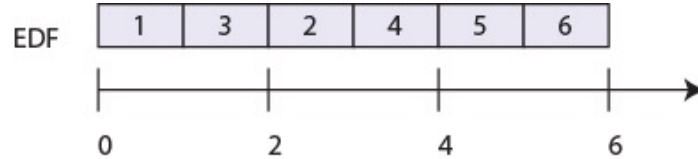
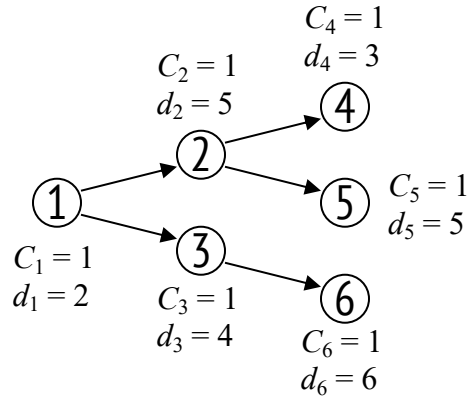
- A directed acyclic graph (DAG) shows precedences, which indicate which tasks must complete before other tasks start.



DAG, showing that task 1 must complete before tasks 2 and 3 can be started, etc.

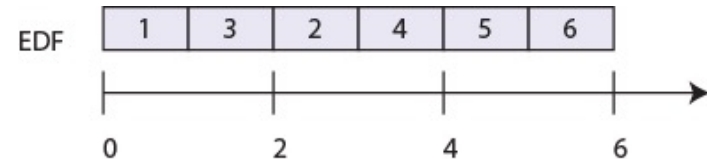
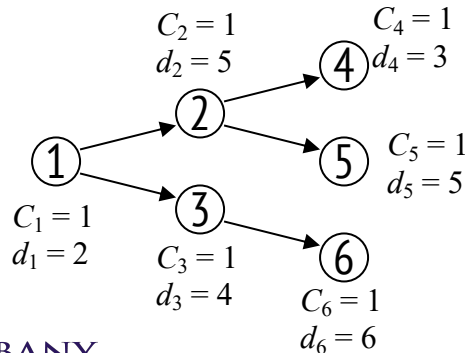
Example: EDF Schedule

➤ Is this feasible?

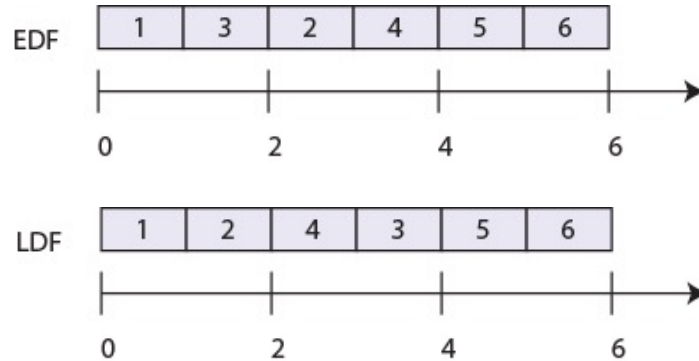
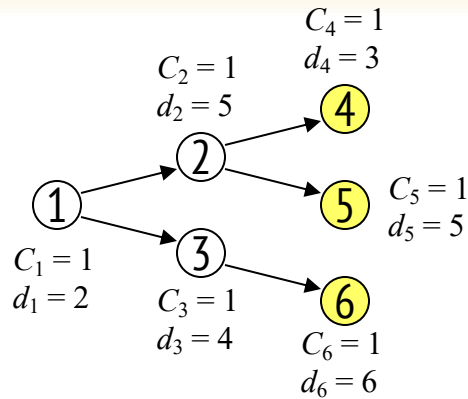


EDF is not optimal under precedence constraints

- The EDF schedule chooses task 3 at time 1 because it has an earlier deadline. This choice results in task 4 missing its deadline.

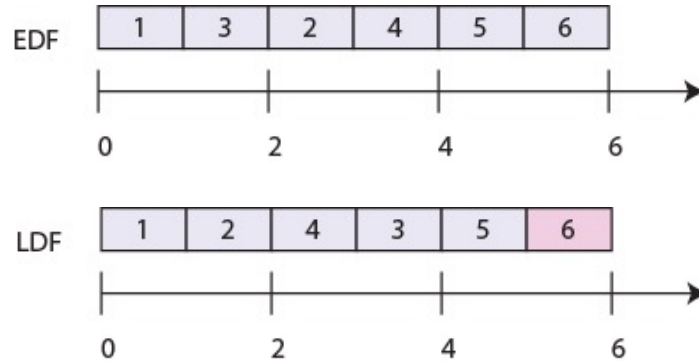
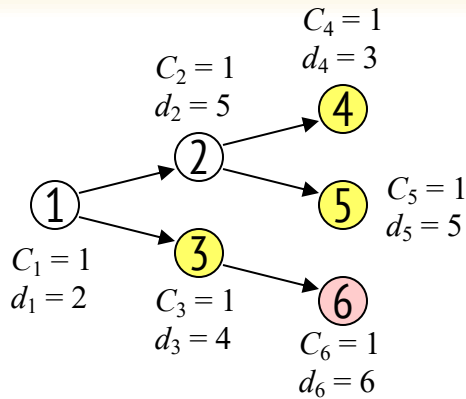


Latest Deadline First (LDF) Lawler 1973



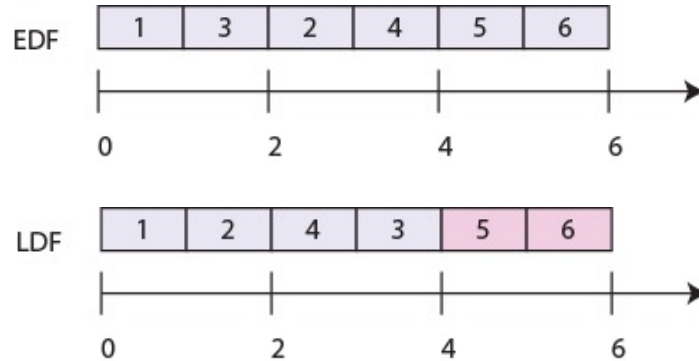
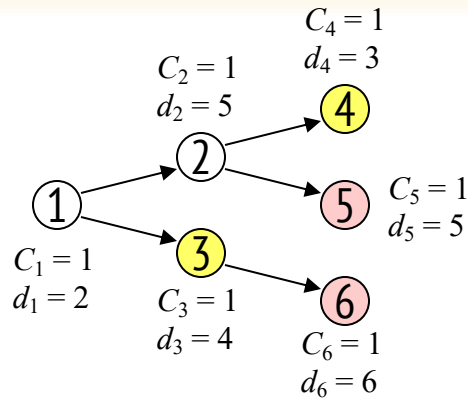
- The LDF scheduling strategy **builds a schedule backwards**. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

Latest Deadline First (LDF)



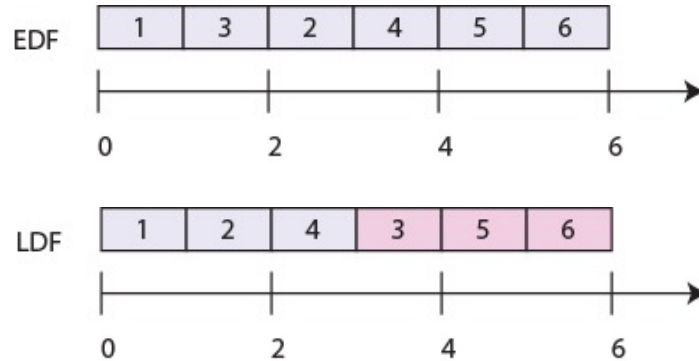
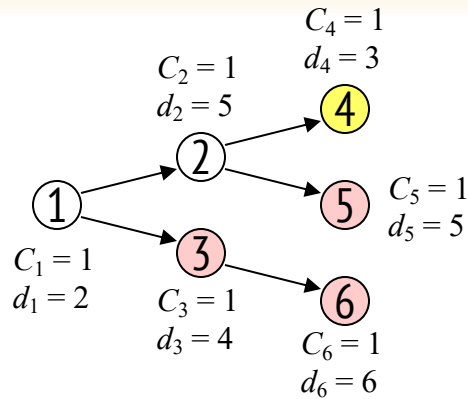
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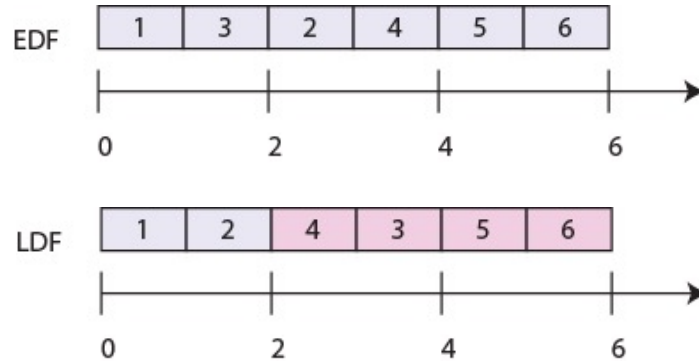
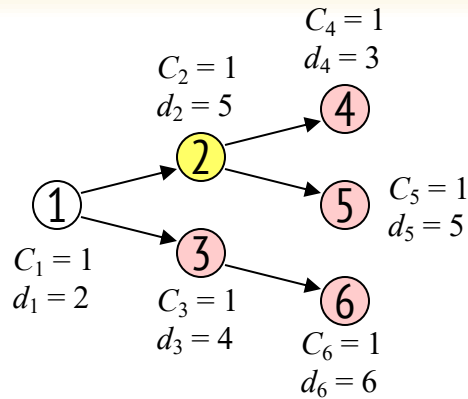
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Latest Deadline First (LDF)



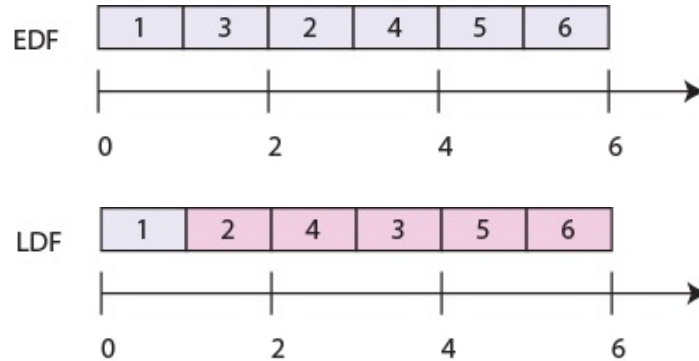
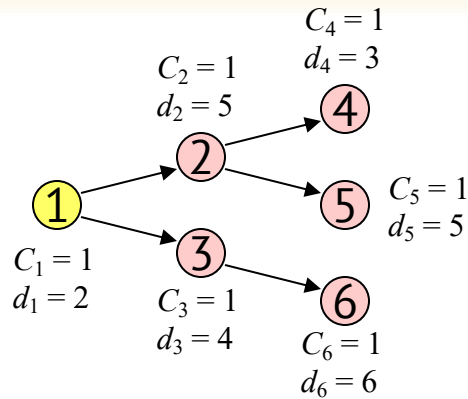
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Latest Deadline First (LDF)



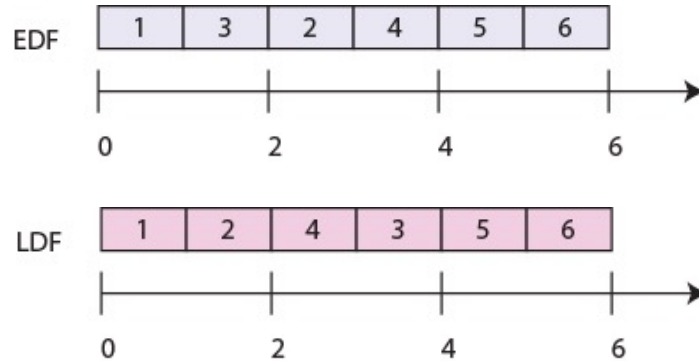
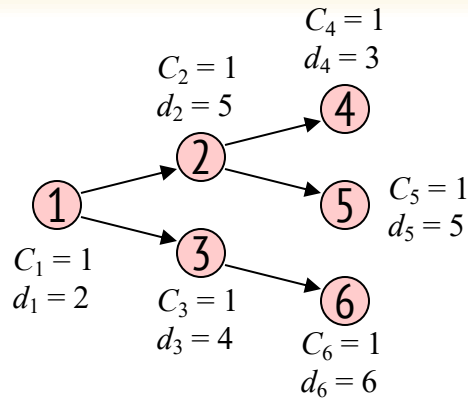
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Latest Deadline First (LDF)



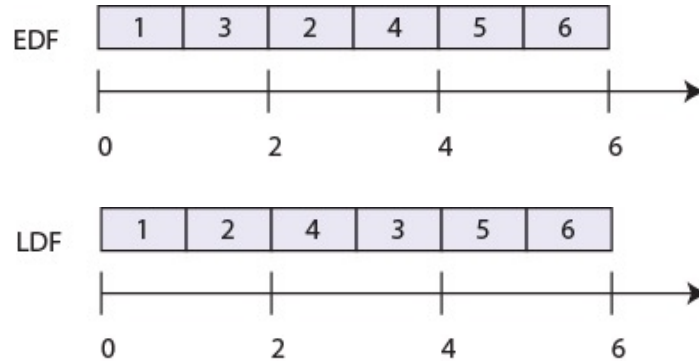
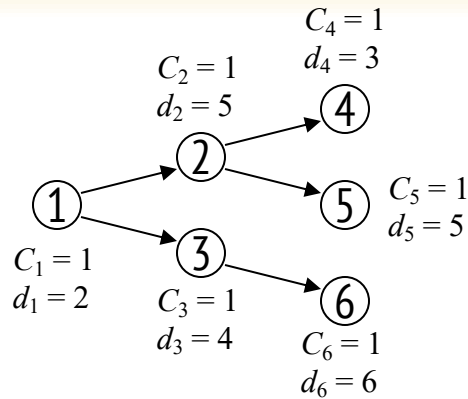
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Latest Deadline First (LDF)



- The LDF scheduling strategy builds a schedule backwards. Given a DAG, choose the leaf node with the latest deadline to be scheduled last, and work backwards.

LDF is optimal for precedence constraints



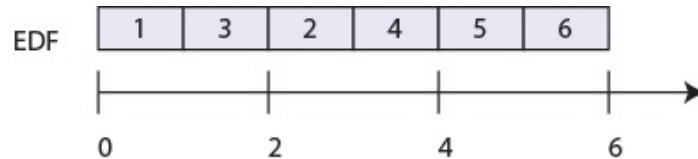
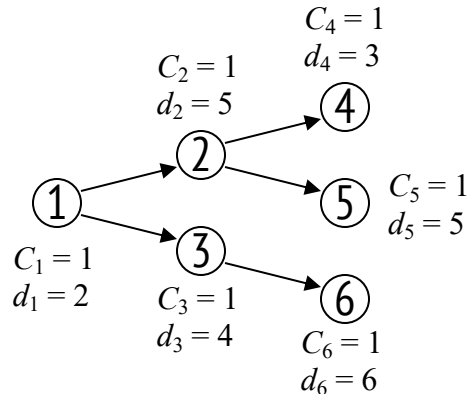
- The LDF schedule shown at the bottom respects all precedences and meets all deadlines.
- Also minimizes maximum lateness

Latest Deadline First (LDF)

- LDF is optimal in the sense that it minimizes the maximum lateness.
- It does not require preemption. (EDF can be made to work with preemption.)
- However, it requires that all tasks be available and their precedences known before any task is executed.

EDF with Precedences or EDF*

- With a **preemptive** scheduler, EDF can be modified to account for precedences and to allow tasks to arrive at arbitrary times.
- Adjust the deadlines and arrival times according to the precedences.



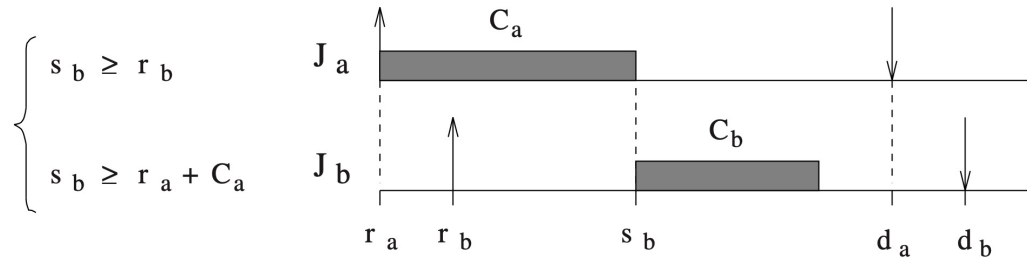
Recall that for the tasks at the left, EDF yields the schedule above, where task 4 misses its deadline.

Modification of Release Times

➤ Observations:

$s_b \geq r_b$ (that is, J_b must start the execution not earlier than its release time);

$s_b \geq r_a + C_a$ (that is, J_b must start the execution not earlier than the minimum finishing time of J_a).

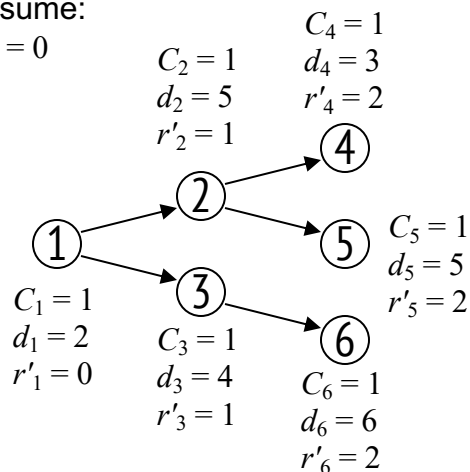


➤ Modification: $r_b^* = \max(r_b, r_a + C_a)$

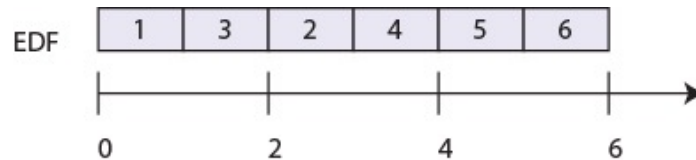
EDF with Precedences: Modifying Release Times

- Given n tasks with precedences and release times r_i , if task i immediately precedes task j , then modify the release times as follows:

assume:
 $r_i = 0$



$$r'_j = \max(r_j, r_i + C_i)$$

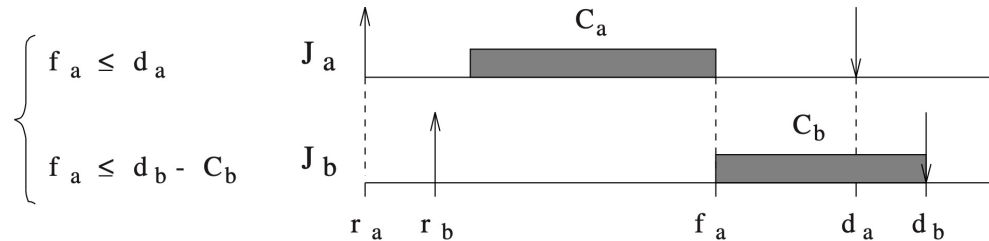


Modification of Deadlines

➤ Observations:

$f_a \leq d_a$ (that is, J_a must finish the execution within its deadline);

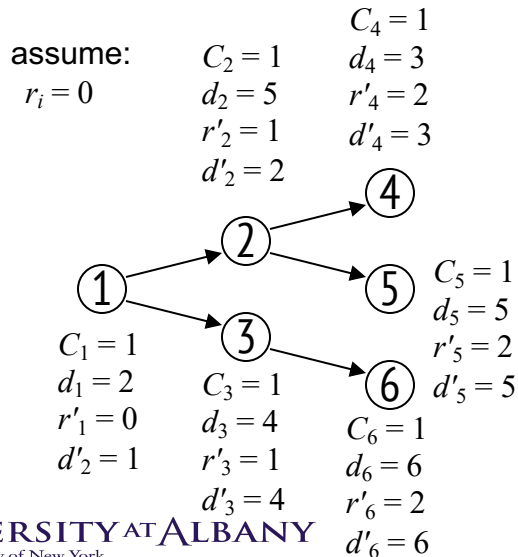
$f_a \leq d_b - C_b$ (that is, J_a must finish the execution not later than the maximum start time of J_b).



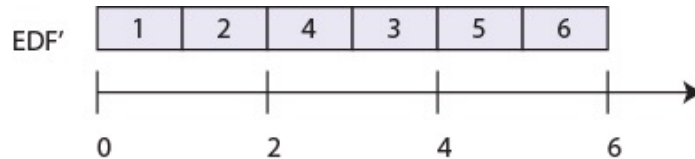
➤ Modification: $d_a^* = \min(d_a, d_b - C_b)$

EDF with Precedences: Modifying Deadlines

- Given n tasks with precedences and deadlines d_i , if task i immediately precedes task j , then modify the deadlines as follows:



$$d'_i = \min(d_i, d'_j - C_j)$$



Using the revised release times and deadlines, the above EDF schedule is optimal and meets all deadlines.

Optimality

- Generalized modified deadline

$$d'_i = \min(d_i, \min_{j \in D(i)} (d'_j - e_j))$$

- EDF with precedences is **optimal** in the sense of minimizing the maximum lateness.