Cyber-Physical Systems

Sensors and Actuators

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What is a sensor? An actuator?

- A sensor is a device that measures a physical quantity
- → Input / "Read from physical world"

- An actuator is a device that modifies a physical quantity
- → Output / "Write to physical world"

The Bridge between Cyber and Physical

- Sensors:
  - Cameras
  - Accelerometers
  - Gyroscopes
  - Strain gauges
  - Microphones
  - Magnetometers
  - Radar/Lidar
  - Chemical sensors
  - Pressure sensors
  - Switches
- Actuators:
  - Motor controllers
  - Solenoids
  - LEDs, lasers
  - LCD and plasma displays
  - Loudspeakers
  - Switches
  - Valves
- Modeling Issues:
  - Physical dynamics, Noise, Bias, Sampling, Interactions, Faults

Sensor-Rich Cars

- Source: Analog Devices

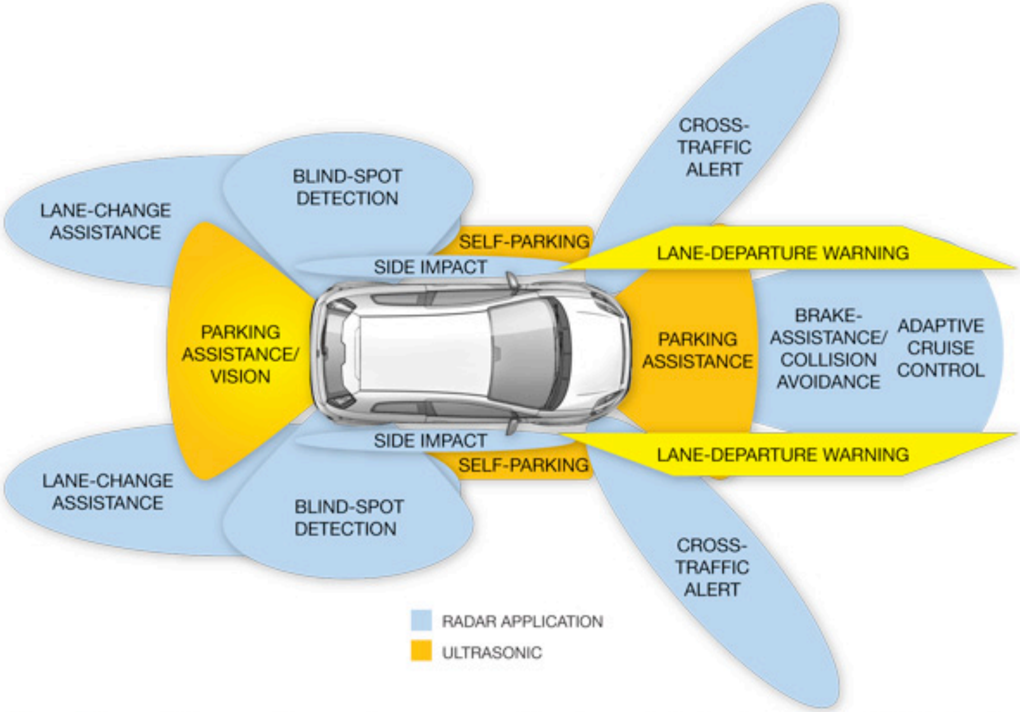
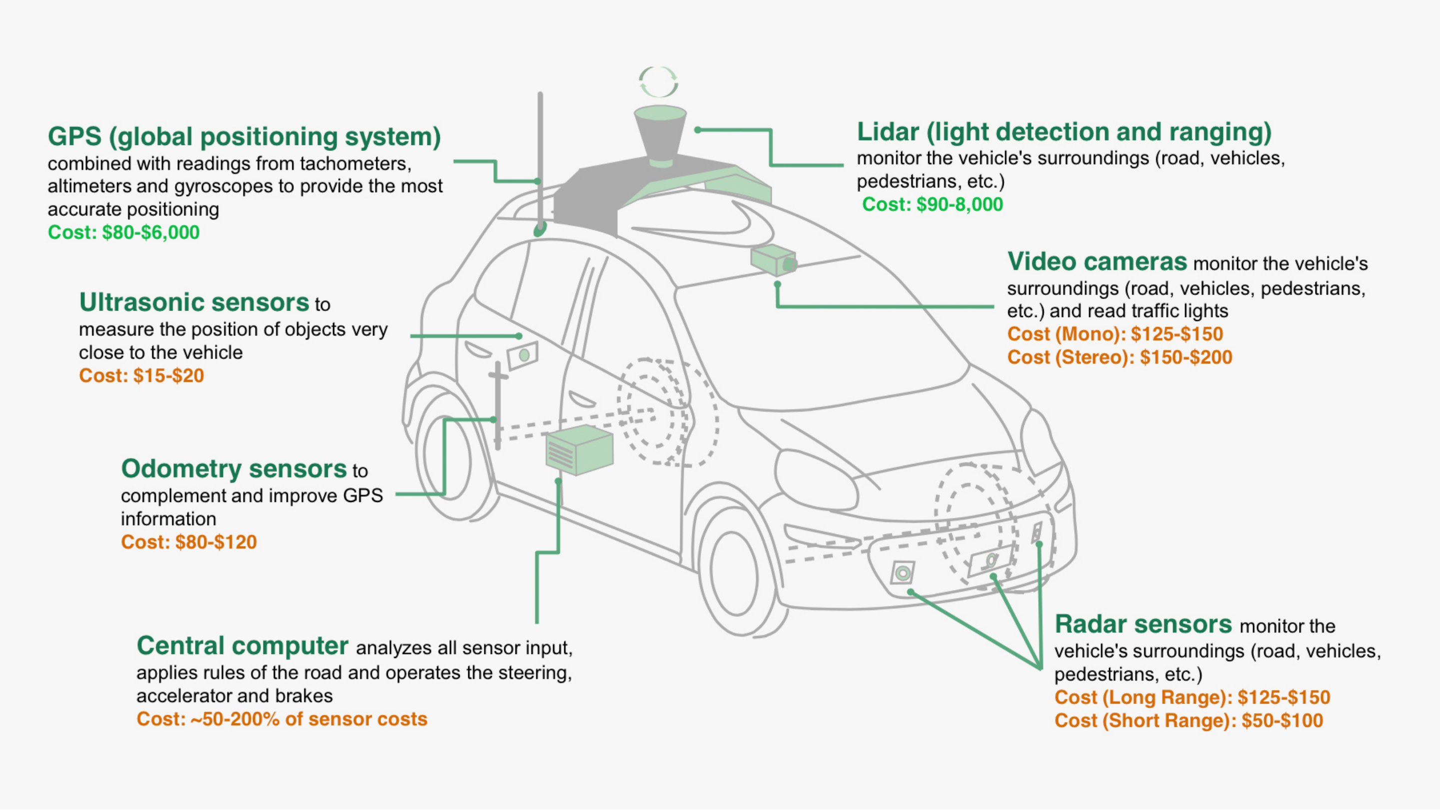


Figure 2 Several driver-assistance systems are currently using radar technology to provide blind-spot detection, parking assistance, collision avoidance, and other driver aids (courtesy Analog Devices).

Sensor-Rich Cars

- Source: Wired Magazine



Self-Driving Cars

Berkeley PATH Project Demo, 1999, San Diego.

Google self-driving car 2.0

Kingvale Blower

- Berkeley PATH Project, March 2005

Sensor Model

- Linear and Affine Functions

$$f\big(x(t)\big) = ax(t)$$

$$f\big(x(t)\big) = ax(t) + b$$

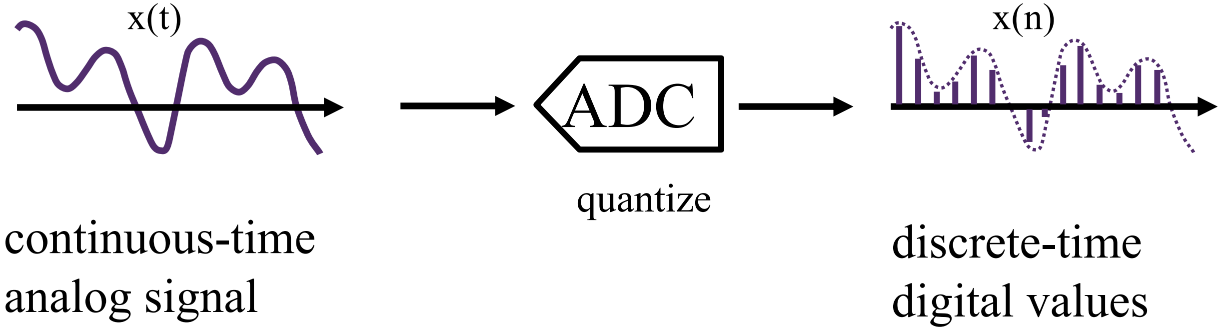
- Affine Sensor Model

$$f\big(x(t)\big) = ax(t) + b + n$$

- Sensitivity (a), Bias (b) and Noise (n)
  - Sensitivity specifies the degree to which the measurement changes when the physical quantity changes

Analog-to-Digital Converter (ADC)

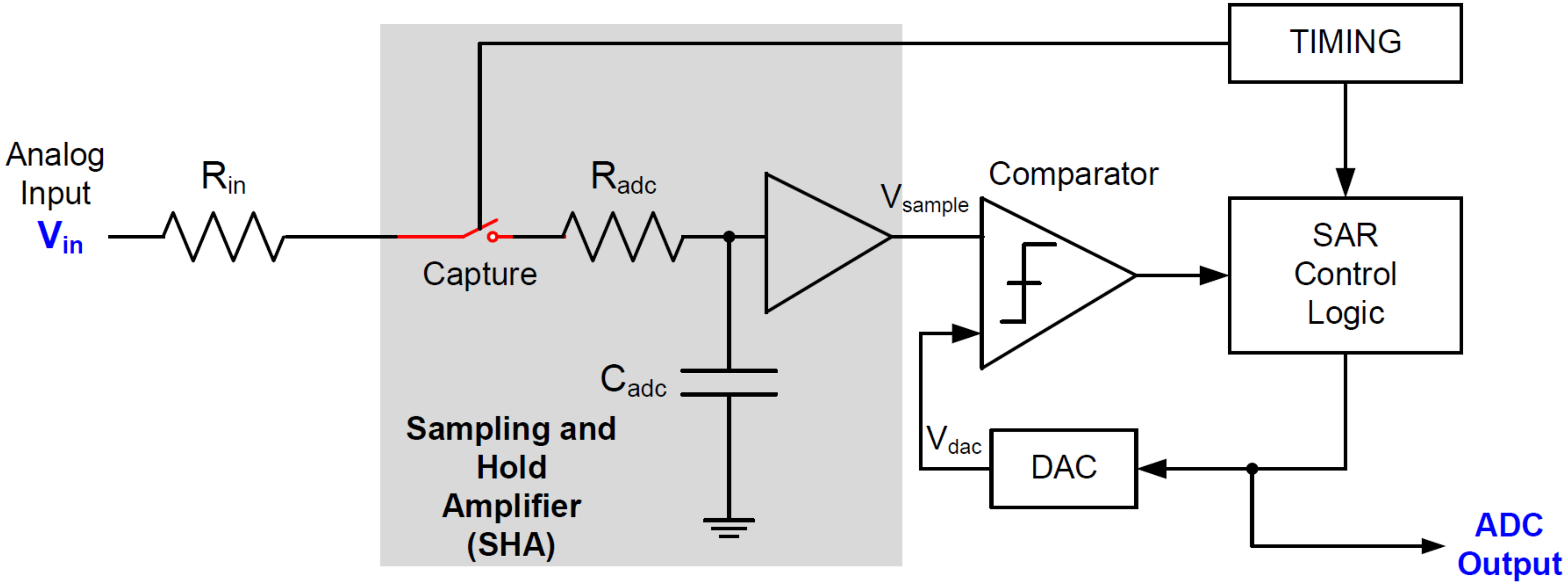
- ADC is important almost to all application fields
- Converts a continuous-time voltage signal within a given range to discrete-time digital values to quantify the voltage's amplitudes



Analog-to-Digital Converter (ADC)

- Three performance parameters:
  - sampling rate – number of conversions per unit time
  - Resolution – number of bits an ADC output
  - power dissipation – power efficiency
- Many ADC implementations:
  - sigma-delta (low sampling rate, high resolution)
  - successive-approximation (low power data acquisition)
  - Pipeline (high speed applications)

Successive-approximation (SAR) ADC



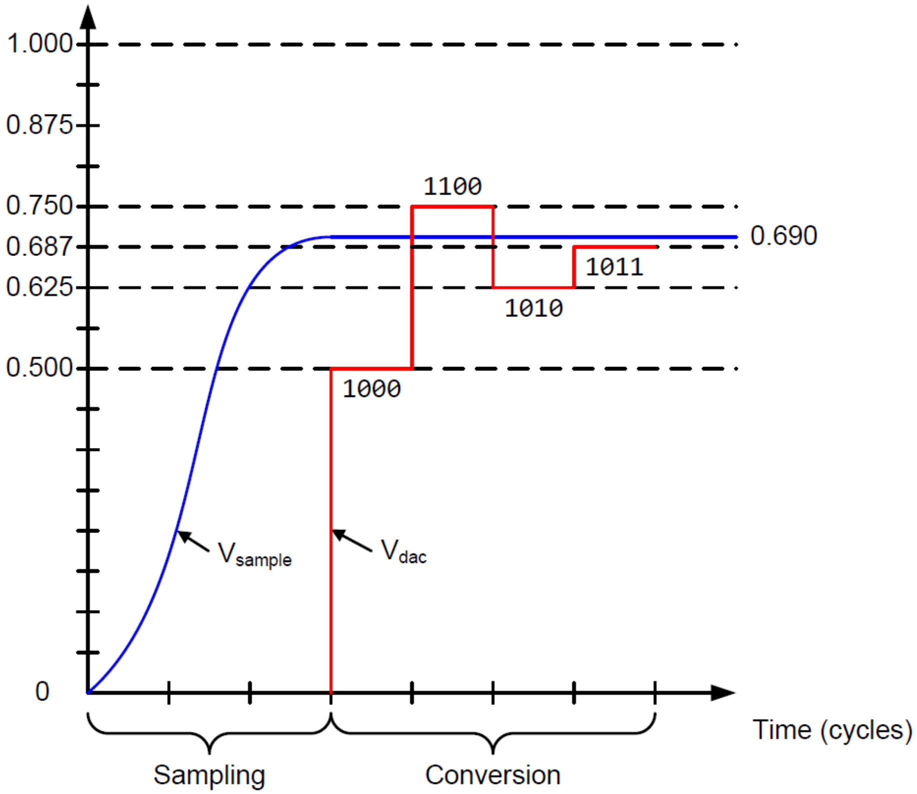
Successive-approximation (SAR) ADC

- A sample and hold circuit to acquire input voltage(V_{in})
- An analog voltage comparator
  - compares V_{in} to the output of the internal DAC and outputs the result of the comparison to the successive approximation register (SAR)
- A successive approximation register subcircuit
  - Supplies an approximate digital code of V_{in} to the internal DAC
- An internal reference DAC
  - for comparison with V_{REF}, supplies the comparator with an analog voltage equal to the digital code output of the SAR_{in}.

Digital Quantization

- SAR Control Logic performs Binary Search algorithm
  - DAC output is set to $1/2V_{REF}$
  - If $V_{IN} > V_{REF}$, SAR Control Logic sets the MSB of ADC, else MSB is cleared
  - V_{DAC} is set to ¾ V_{REF} or ¼ V_{REF} depending on output of previous step
  - Repeat until ADC output has been determined
- How long does it take to converge?

Successive-approximation (SAR) ADC



- Binary search algorithm to gradually approaches the input voltage
- Settle into ±½ LSB bound within the time allowed

$T_{ADC} = T_{sampling} + T_{Conversion}$

$T_{Conversion} = N \times T_{ADC\_Clock}$

$T_{sampling}$ is software configurable

ADC Conversion Time

$$T_{ADC} = T_{sampling} + T_{Conversion}$$

- Suppose ADC_{CLK} = 16 MHz and Sampling time = 4 cycles

For 12-bit ADC

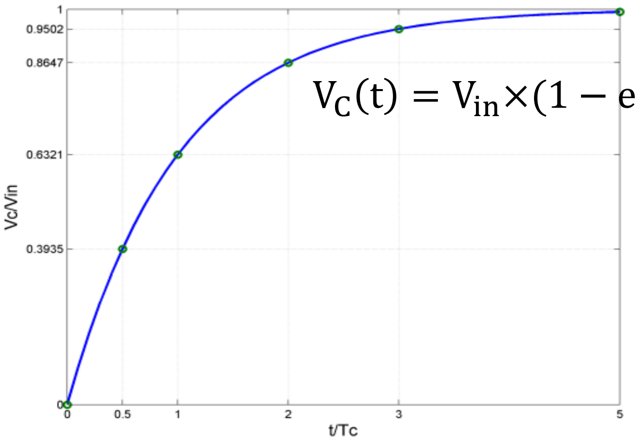
$$T_{ADC} = 4 + 12 = 16 \text{ cycles} = 1\mu s$$

For 6-bit ADC

$$T_{ADC} = 4 + 6 = 10 \text{ cycles} = 625ns$$

Determining Minimum Sampling Time

- When the switch is closed, the voltage across the capacitor increases exponentially.



$$V_C(t) = V_{in} \times (1 - e^{-\frac{t}{T_c}})$$

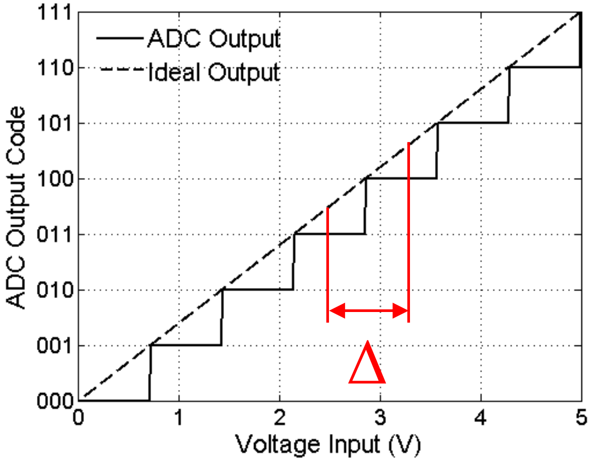
t= time required for the sample capacitor voltage to settle to within *one-fourth* of an LSB of the input voltage

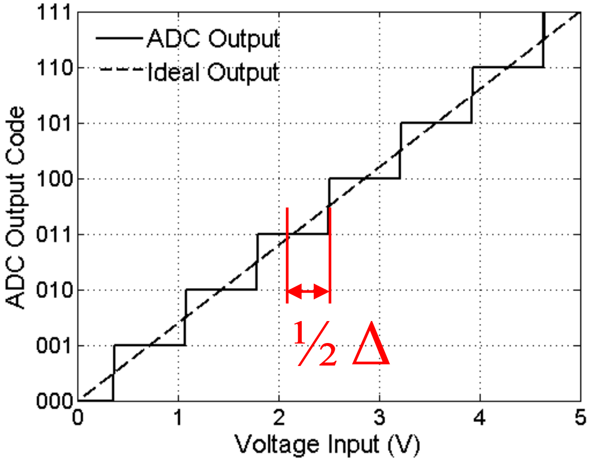
Sampling time is often software programmable!

Larger sampling time → Smaller sampling error / Slower ADC speed — Tradeoff

Resolution

- Resolution is determined by number of bits (in binary) to represent an analog input.
- Example of two quantization methods (N = 3)





$$\text{Digital Result} = \text{floor}\left(2^3 \times \frac{V}{V_{REF}}\right)$$

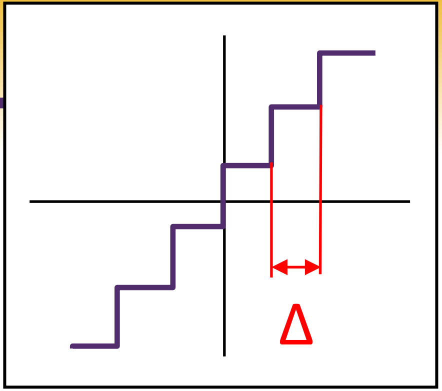
Max quantization error = $\Delta = V_{REF}/2^3$

$$\text{Digital Result} = \text{round}\left(2^3 \times \frac{V}{V_{REF}}\right)$$

Max quantization error = $\pm \frac{1}{2} \Delta = \pm V_{REF}/2^4$

$$\text{round}(x) = \text{floor}(x + 0.5)$$

Quantization Error



- For N-bit ADC, it is limited to $\pm\frac{1}{2}\Delta$
- Δ = is the step size of the converter.

- Example: for 12-bit ADC and input voltage range [0, 3V]

$$Max\ Quantization\ Error = \frac{1}{2}\Delta = \frac{3V}{2\times 2^{12}} = 0.367mV$$

- How to reduce error?

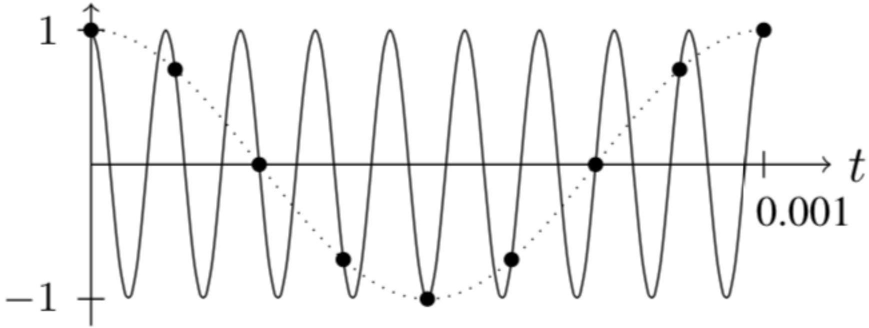
Aliasing

- Example 1:
  - Consider a sinusoidal sound signal at 1 kHz : $x(t) = \cos(2000\pi t)$
  - Sampling interval T = 1/8000
  - Samples $s(n) = f\big(x(nT)\big) = \cos(\pi n/4)$
- Example 2:
  - Consider a sinusoidal sound signal at 9 kHz : $x'(t) = \cos(18000\pi t)$
  - Sampling interval T = 1/8000
  - Samples $s'^{(n)} = f\big(x(nT)\big) = \cos\left(\frac{9\pi n}{4}\right) = \cos\left(\frac{\pi n}{4} + 2\pi n\right) = \cos\left(\frac{\pi n}{4}\right) = s(n)$
- There are many distinct functions x that when sampled will yield the same signal s.

Minimum Sampling Rate

- In order to be able to reconstruct the analog input signal, the sampling rate should be at least twice the maximum frequency component contained in the input signal
- Example of two sine waves have the same sampling values. This is called aliasing.

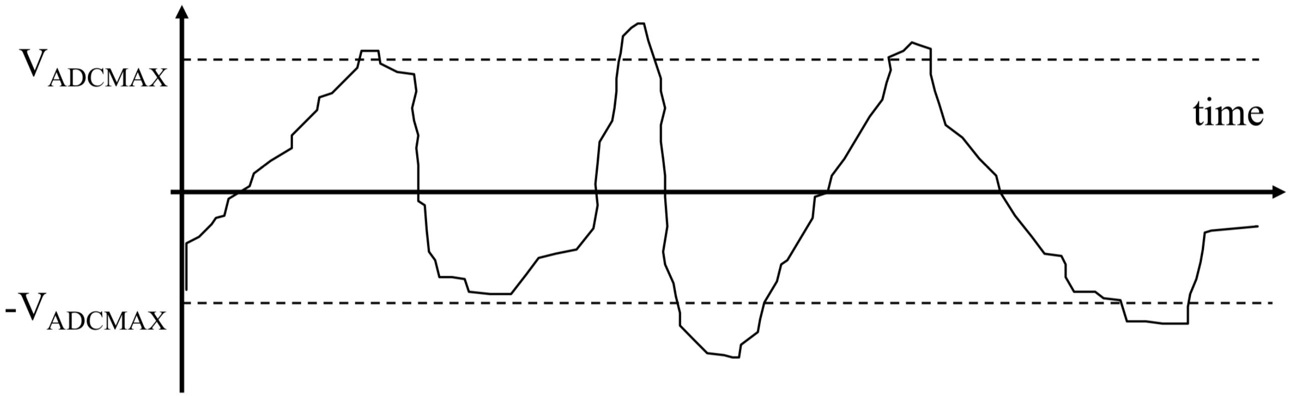
Nyquist–Shannon Sampling Theorem



- Antialiasing
  - Pre-filtering: use analog hardware to filtering out high-frequency components and only sampling the low-frequency components. The high-frequency components are ignored.
  - Post-filtering: Oversample continuous signal, then use software to filter out high-frequency components

ADC Conversion

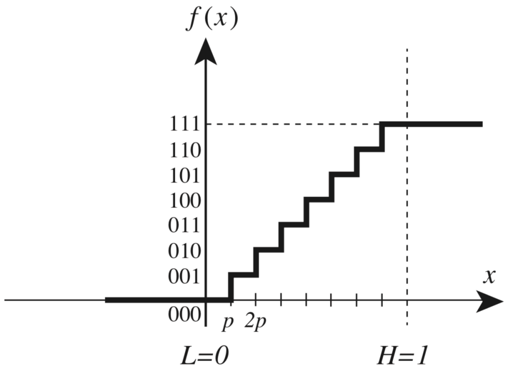
- Input Range
  - Unipolar (0, V_{ADCMAX})
  - Bipolar ($-V_{ADCMAX}$, $+V_{ADCMAX}$)
  - Clipping:
    - If $|V_{IN}| > |V_{ADCMAX}|$, then $|V_{OUT}| = |V_{ADCMAX}|$



Automatic Gain Control (AGC)

- Closed loop Feedback regulating circuit in an amplifier
- Maintains a suitable signal amplitude at its output, despite variation of the signal amplitude at the input
- The *average or peak output signal* level is used to dynamically adjust the gain of the amplifiers
- Example Use: Radio Receivers, Audio Recorders, Microphone

Range and Dynamic Range



- Range

$$f(x(t)) = \begin{cases} ax(t)+b & \text{if } L \le x(t) \le H \\ aH+b & \text{if } x(t) > H \\ aL+b & \text{if } x(t) < L, \end{cases}$$

- Dynamic Range

$$D = \frac{H-L}{p}, \qquad D_{dB} = 20\log_{10}\left(\frac{H-L}{p}\right)$$

- Precision (p) $\quad p = (H-L)/2^n$

Power and RMS of Signal

- Average Power of a signal $$P_x = \frac{1}{N}\sum_{n=0}^{N-1}|x_n|^2$$
- Crest Factor $$C = \frac{|x_{PEAK}|}{x_{RMS}}$$
- Square root of the arithmetic mean of the squares of the values
$$x_{RMS} = \sqrt{\frac{1}{n}({x_1}^2 + {x_2}^2 + \cdots + {x_n}^2)}$$
- Crest Factor
  - Sine Wave ~ 3.01dB, OFDM ~12dB

PAPR

- Crest Factor in dB

$$C_{dB} = 20log_{10}\frac{|x_{PEAK}|}{x_{RMS}}$$

- Peak to Average Power Ratio (PAPR)

$$PAPR = \frac{|x_{PEAK}|^2}{{x_{RMS}}^2}$$

$$PAPR_{dB} = 10log_{10}\frac{|x_{PEAK}|^2}{{x_{RMS}}^2} = C_{dB}$$

Noise

- Measured signal – Actual signal

$$n(t) = x'(t) - x(t)$$

$$x'(t) = x(t) + n(t)$$

- Sensor Distortion Function: Sensor imperfections and errors due to quantization can be modeled as noise $$f(x(t)) = x(t) + n(t)$$

Noise measured

- The root mean square (RMS) of the noise is equal to the square root of the average value of $n(t)^2$
- Noise Power

$$N = \lim_{T \to \infty} \sqrt{\frac{1}{2T} \int_{-T}^{T} (n(\tau))^2 d\tau}.$$

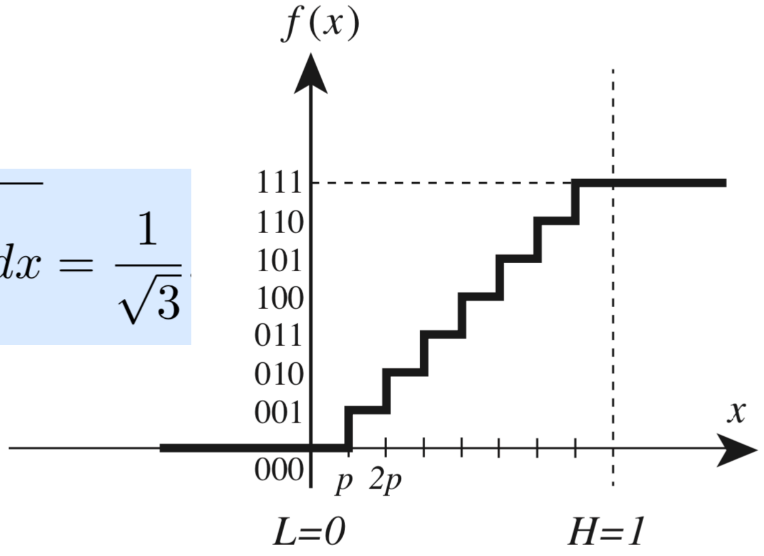
- Signal to Noise Ratio (SNR)

$$SNR_{dB} = 20 \log_{10} \left(\frac{X}{N} \right)$$

Noise modeled as statistical property

- x(t) is a random variable with uniform distribution ranging from 0 to 1

$$X = \sqrt{\int_0^1 x^2 dx} = \frac{1}{\sqrt{3}}$$



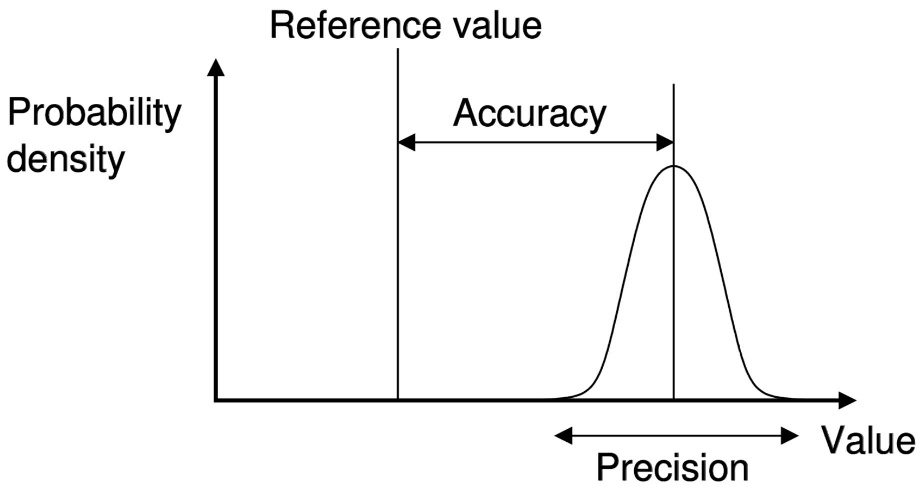
- n(t) = f(x(t)) – x(t)
  - ranges from −1/8 to 0

$$N = \sqrt{\int_{-1/8}^{0} 8n^2 dn} = \sqrt{\frac{1}{3 \cdot 64}} = \frac{1}{8\sqrt{3}}$$

$$SNR_{dB} = 20\log_{10}\left(\frac{X}{N}\right) = 20\log_{10}(8) \approx 18dB$$

Precision and Accuracy

- Precision: how close the two measured values can be
- Accuracy: how close is the measured value to the true value



Noise & Signal Conditioning

Parsevals theorem relates the energy or the power in a signal in the time and frequency domains. For a finite energy signal x, the energy is

$$\int_{-\infty}^{\infty} (x(t))^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

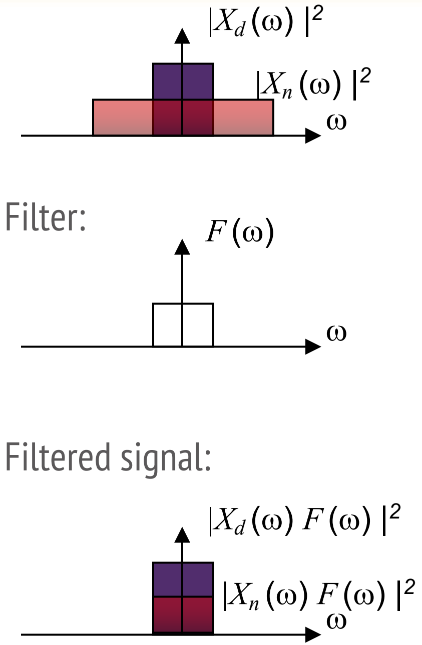
where X is the Fourier transform. If there is a desired part x_d and an undesired part (noise) x_n,

$$x(t) = x_d(t) + x_n(t)$$

then

$$X(\omega) = X_d(\omega) + X_n(\omega)$$

Suppose that x_d is a narrowband signal and x_n is a broadband signal. Then the *signal to noise ratio* (SNR) can be greatly improved with filtering.



Example Gain Control

- AD8338

FUNCTIONAL BLOCK DIAGRAM

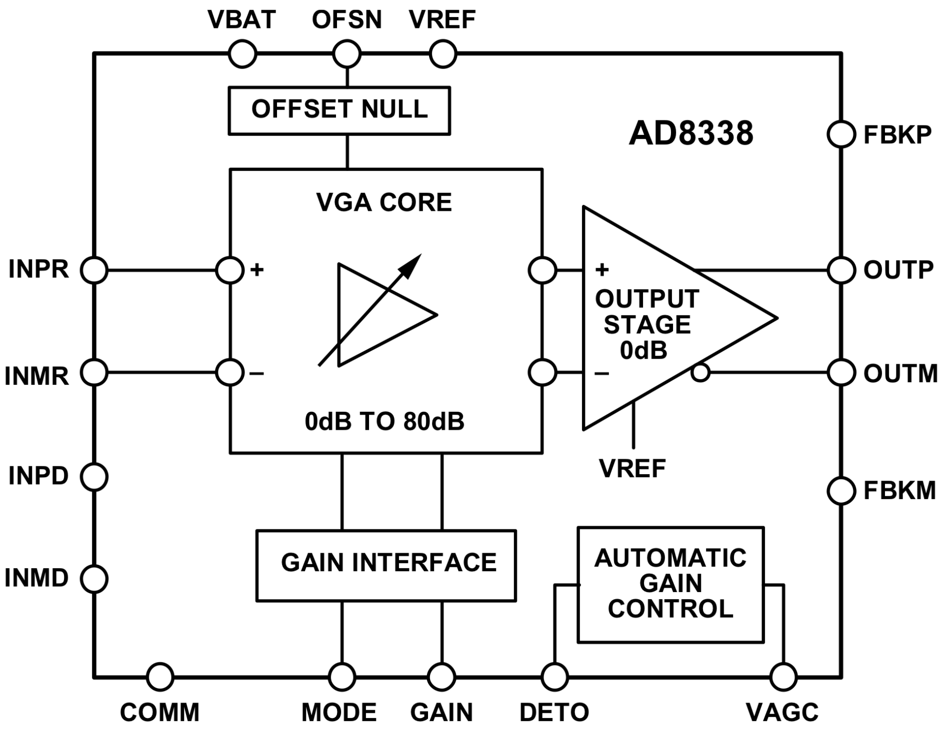


Figure 1.

Digital-to-analog converter (DAC)

- Converts digital data into a voltage signal by a N-bit DAC

$$DAC_{output} = V_{ref} \times \frac{Digital\ Value}{2^N}$$

- For 12-bit DAC

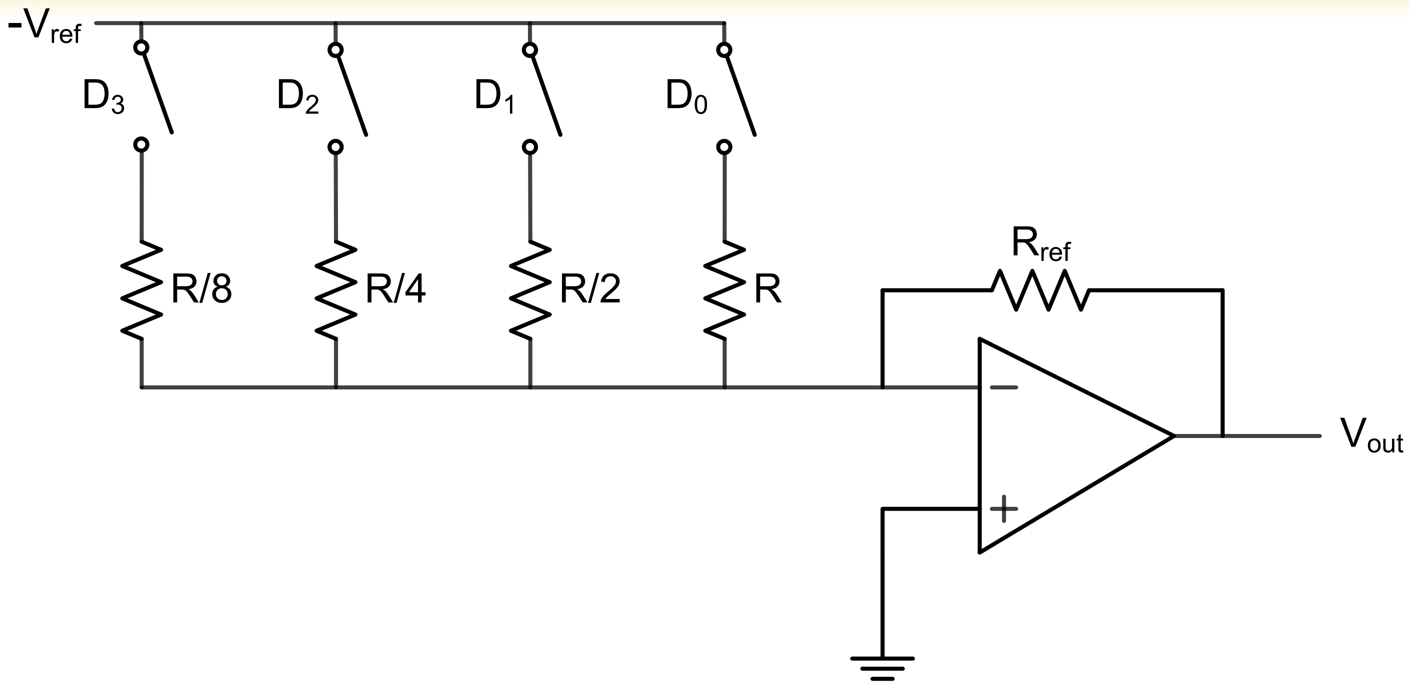
$$DAC_{output} = V_{ref} \times \frac{Digital\ Value}{4096}$$

- Many applications:
  - digital audio
  - waveform generation
- Performance parameters
  - speed
  - resolution
  - power dissipation
  - glitches

DAC Implementations

- Pulse-width modulator (PWM)
- Binary-weighted resistor (We will use this one as an example)
- R-2R ladder (A special case of binary-weighted resistor)

Binary-weighted Resistor DAC



$$V_{out} = V_{ref} \times \frac{R_{ref}}{R} \times (D_3 \times 2^3 + D_2 \times 2^2 + D_1 \times 2 + D_0)$$

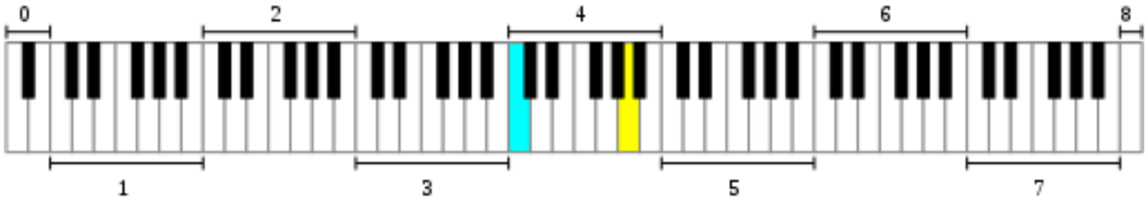
Digital Music

	0	1	2	3	4	5	6	7	8
C	16.352	32.703	65.406	130.813	261.626	523.251	1046.502	2093.005	4186.009
C#	17.324	34.648	69.296	138.591	277.183	554.365	1108.731	2217.461	4434.922
D	18.354	36.708	73.416	146.832	293.665	587.330	1174.659	2349.318	4698.636
D#	19.445	38.891	77.782	155.563	311.127	622.254	1244.508	2489.016	4978.032
E	20.602	41.203	82.407	164.814	329.628	659.255	1318.510	2637.020	5274.041
F	21.827	43.654	87.307	174.614	349.228	698.456	1396.913	2793.826	5587.652
F#	23.125	46.249	92.499	184.997	369.994	739.989	1479.978	2959.955	5919.911
G	24.500	48.999	97.999	195.998	391.995	783.991	1567.982	3135.963	6271.927
G#	25.957	51.913	103.826	207.652	415.305	830.609	1661.219	3322.438	6644.875
A	27.500	55.000	110.000	220.000	440.000	880.000	1760.000	3520.000	7040.000
A#	29.135	58.270	116.541	233.082	466.164	932.328	1864.655	3729.310	7458.620
B	30.868	61.735	123.471	246.942	493.883	987.767	1975.533	3951.066	7902.133

Musical Instrument Digital Interface (MIDI) standard assigns the note A as pitch 69.

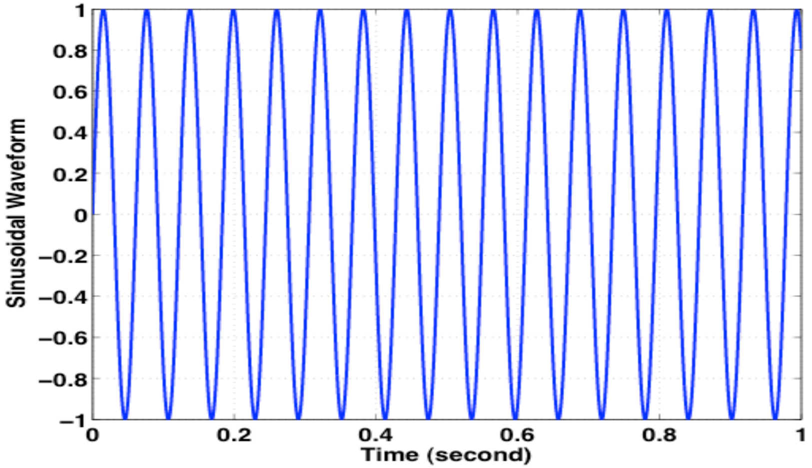
$$f = 440 \times 2^{(p-69)/12} = 440$$

$$p = 69 + 12 \times \log_2\left(\frac{f}{440}\right)$$



Digital Music

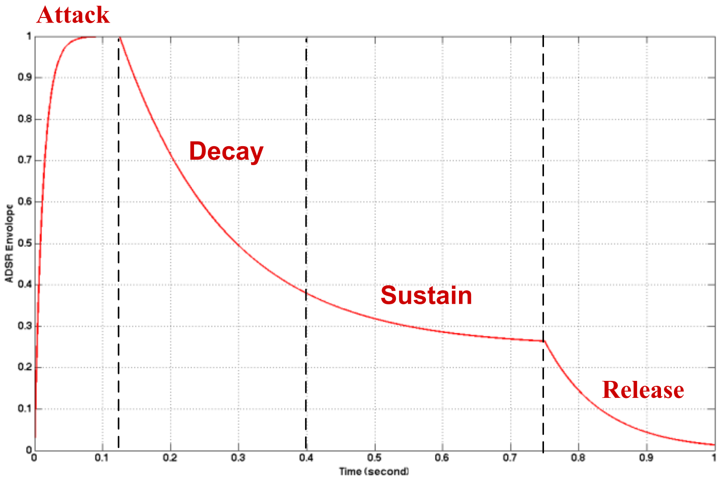
Generate Sine Wave



- No (Floating Point Unit) FPU available on the processor to compute sine functions
- Software FP to compute sine is slow
- Solution: Table Lookup
  - Compute sine values and store in table as fix-point format
  - Look up the table for result
  - Linear interpolation if necessary

Digital Music: Attack, Decay, Sustain, Release

- ADSR: Amplitude Modulation of Tones (modulate music amplitude)

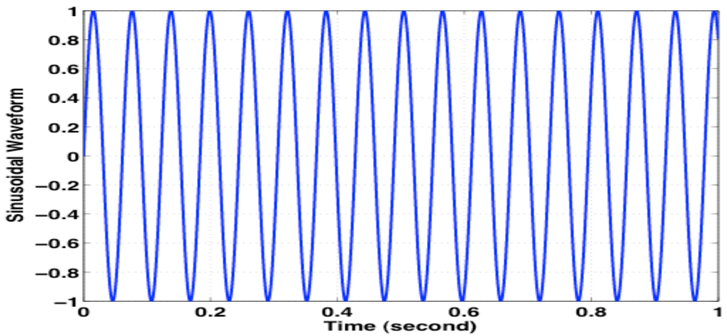


Implemented by a simple digital filter:

$$ADSR(n) = g \times \overrightarrow{ADSR} + (1 - g) \times ADSR(n-1)$$

where $\overrightarrow{ADSR}$ is the target modulated amplitude value, g is the gain parameter.

Digital Music: ADSR Amplitude Modulation



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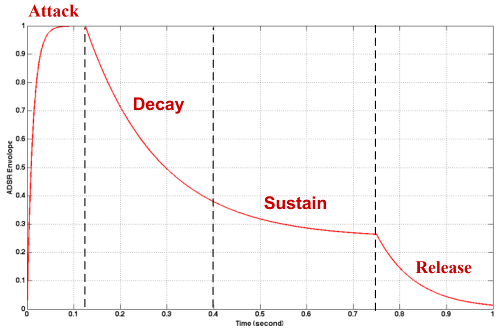
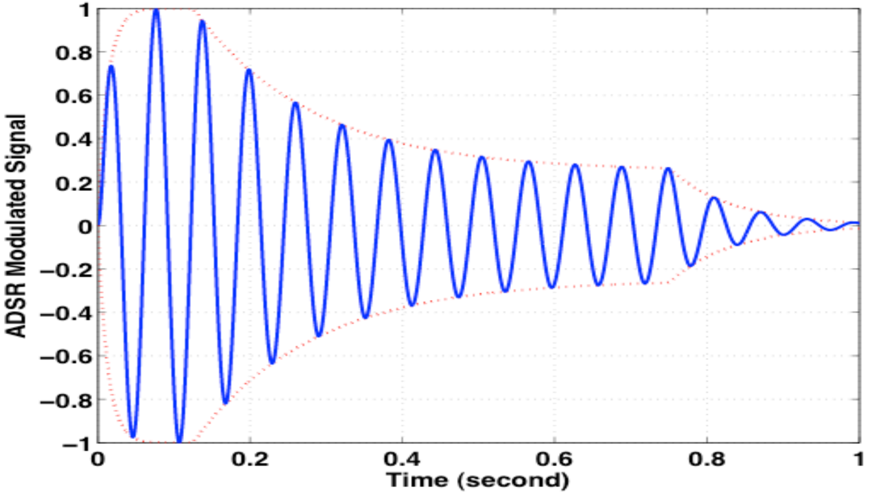
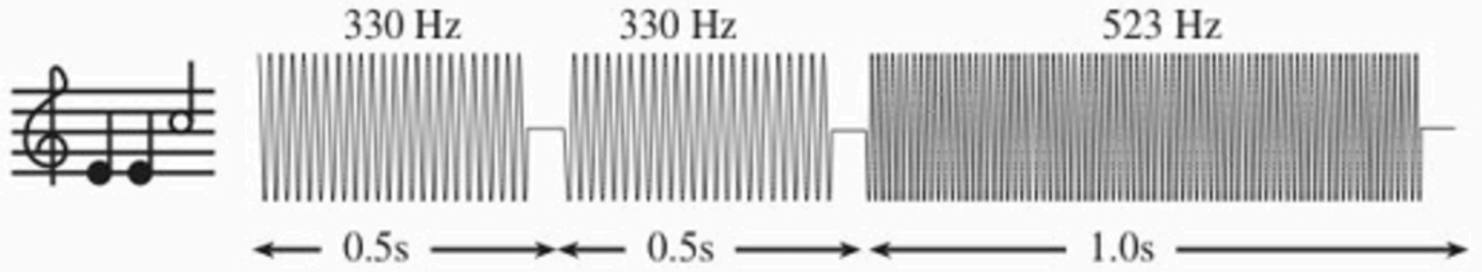
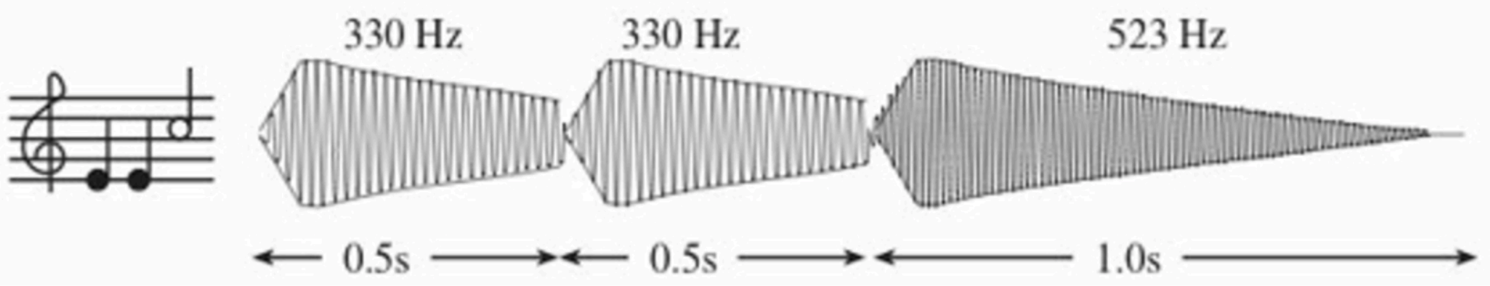




Music with ADSR





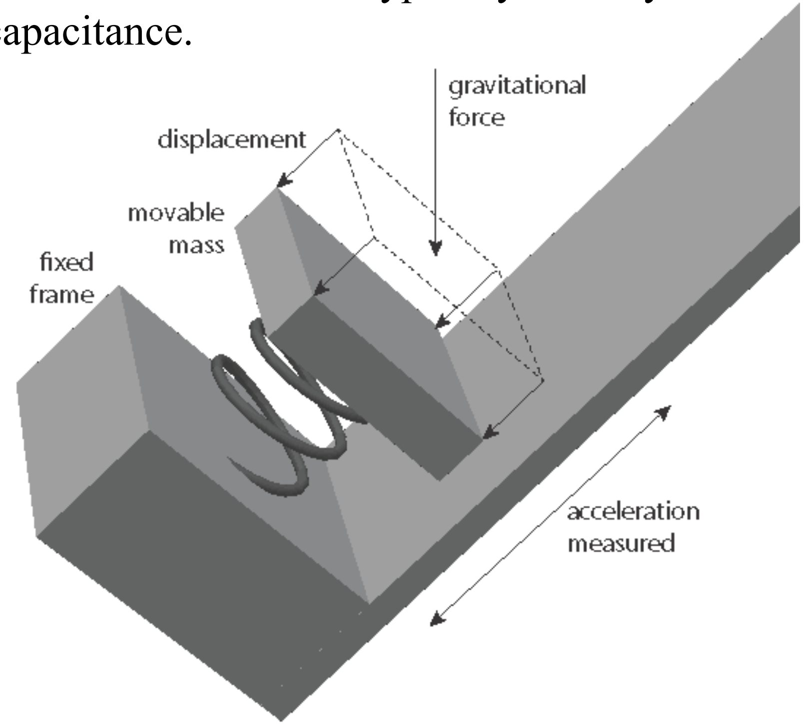
Degrees of Freedom (DoF)

- Movement of a rigid body in space
- 3 DoF
  - Translational Movement (x, y, z)
  - Rotational Movement (roll, yaw, pitch)
- 6 DoF
  - Combine 3 Translational Movement and 3 Rotational Movement
- 9DoF
  - Sensor Fusion with Magnetometer

Accelerometers

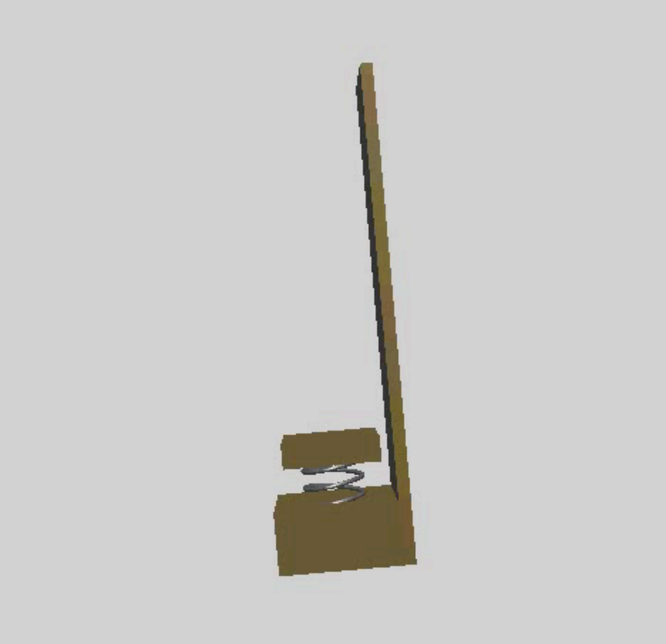
- Uses:
  - Navigation
  - Orientation
  - Drop detection
  - Image stabilization
  - Airbag systems
  - VR/AR systems

The most common design measures the distance between a plate fixed to the platform and one attached by a spring and damper. The measurement is typically done by measuring capacitance.



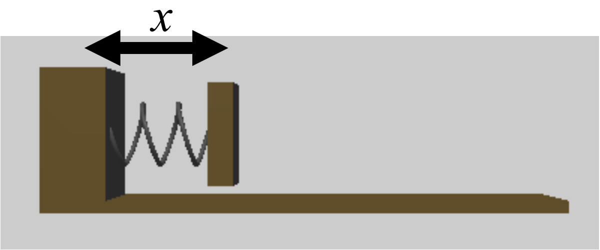
Spring-Mass-Damper Accelerometer

- By Newton's second law, F=ma.
- For example, F could be the Earth's gravitational force.
- The force is balanced by the restoring force of the spring.



Spring-Mass-Damper System

- mass: M
- spring constant: k
- spring rest position: p
- position of mass: x
- viscous damping constant: c



Force due to spring extension:

$$F_1(t) = k(p - x(t))$$

Force due to viscous damping:

$$F_2(t) = -c\dot{x}(t)$$

Newton's second law:

$$F_1(t) + F_2(t) = M\ddot{x}(t)$$

or

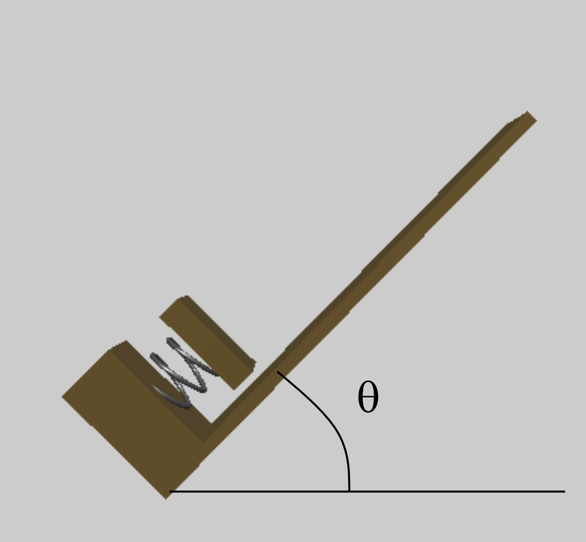
$$M\ddot{x}(t) + c\dot{x}(t) + kx(t) = kp.$$

Measuring tilt

Component of gravitational force in the direction of the accelerometer axis must equal the spring force:

$$Mg\sin(\theta) = k(p - x(t))$$

Given a measurement of x, you can solve for θ, up to an ambiguity of π.



- Digital Accelerometer produces measurement f(x)

$$f : (L, H) \to \{0, \ldots, 2^b - 1\}$$

Difficulties Using Accelerometers

- Separating tilt from acceleration
- Vibration
- Nonlinearities in the spring or damper
- Integrating twice to get position: Drift

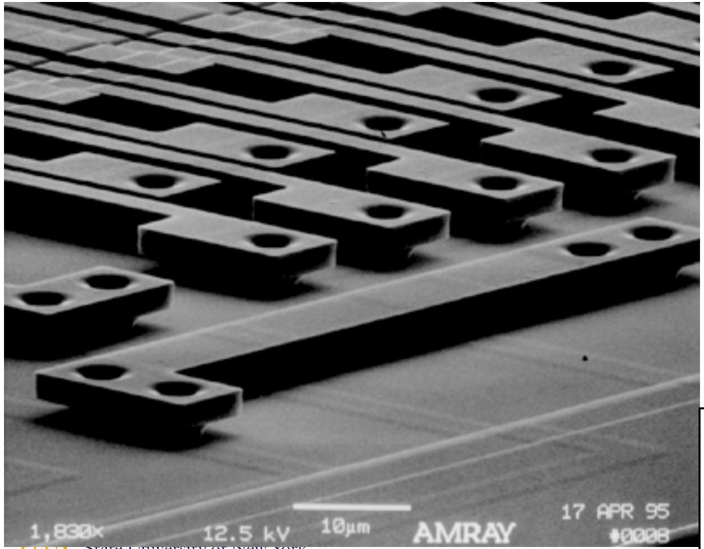
$$p(t) = p(0) + \int_0^t v(\tau)d\tau,$$

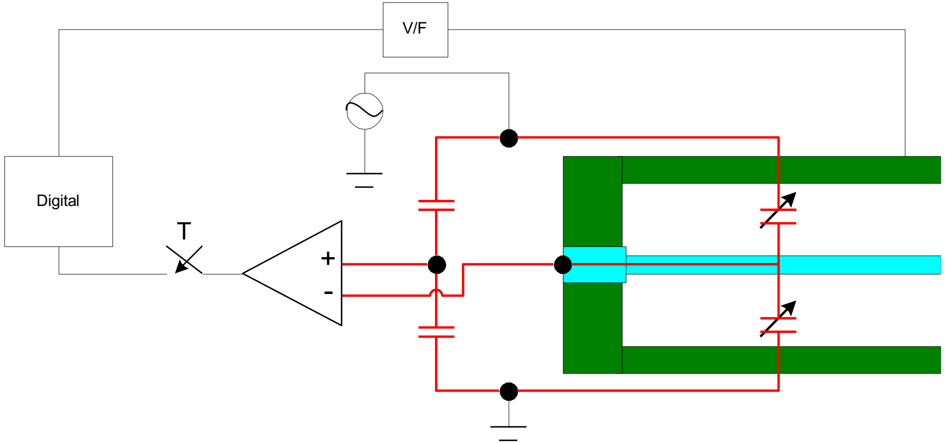
$$v(t) = v(0) + \int_0^t x(\tau)d\tau.$$

Position is the integral of velocity, which is the integral of acceleration. Bias in the measurement of acceleration causes position estimate error to increase quadraticly.

Feedback

- Improves Accuracy & Dynamic Range
- The Berkeley Sensor and Actuator Center (BSAC) created the first silicon microaccelerometers, MEMS devices now used in airbag systems, computer games, disk drives (drop sensors), etc.

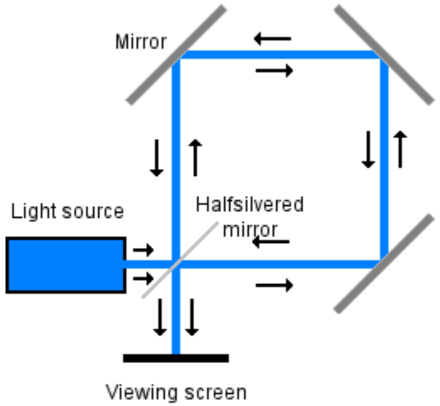




M. A. Lemkin, "Micro Accelerometer Design with Digital Feedback Control", Ph.D. dissertation, EECS, University of California, Berkeley, Fall 1997

Changes in Orientation: Gyroscopes

- Gyroscopes measure angular velocity
  - how fast something is spinning about an axis.
- MEMS Gyros: microelectromechanical systems using small resonating structures
- Optical Gyros:
  - Sagnac effect, where a laser light is sent around a loop in opposite directions and the interference is measured.
  - When the loop is rotating, the distance the light travels in one direction is smaller than the distance in the other.
  - This shows up as a change in the interference.



Magnetometers

- Hall Effect magnetometer

- Charge particles electrons (1) flow through a conductor (2) serving as a Hall sensor. Magnets (3) induce a magnetic field (4) that causes the charged particles to accumulate on one side of the Hall sensor, inducing a measurable voltage difference from top to bottom.

- The four drawings at the right illustrate electron paths under different current and magnetic field polarities.

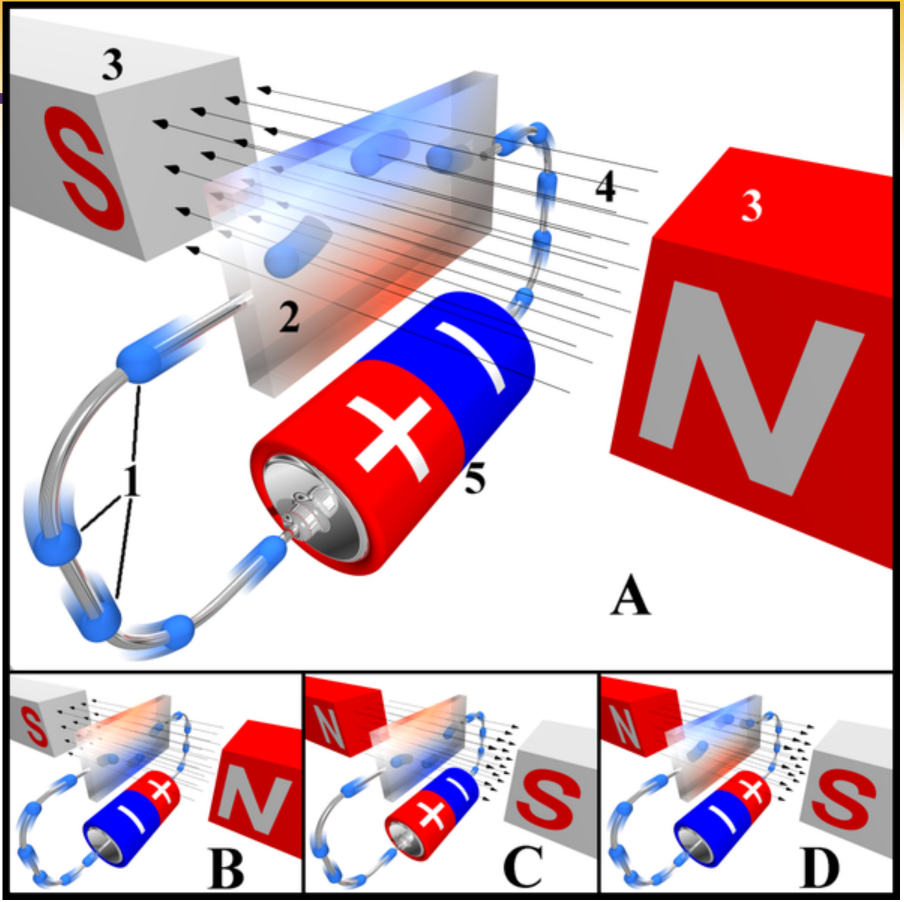
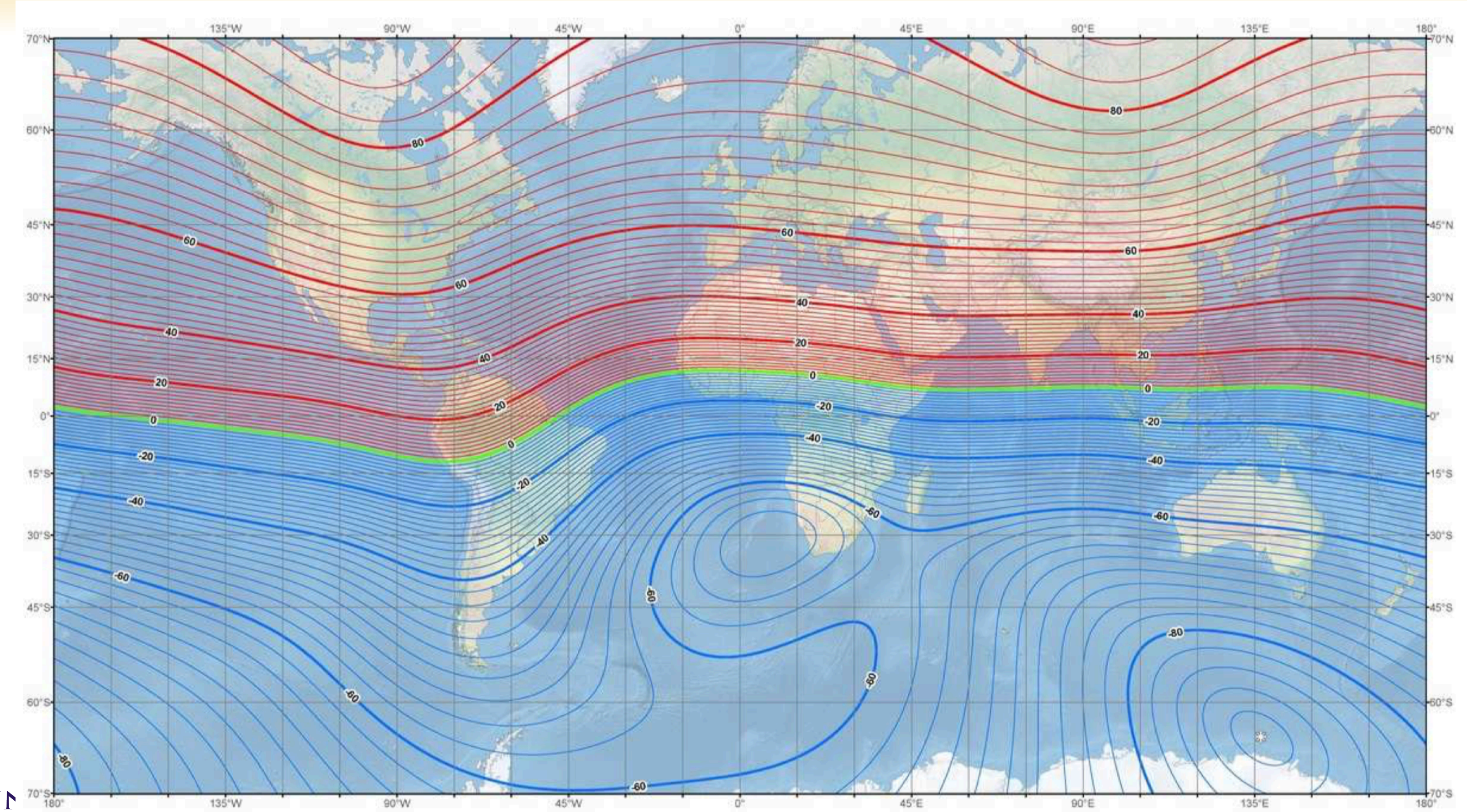


Image source: Wikipedia Commons

Edwin Hall discovered this effect in 1879.

Magnetometers



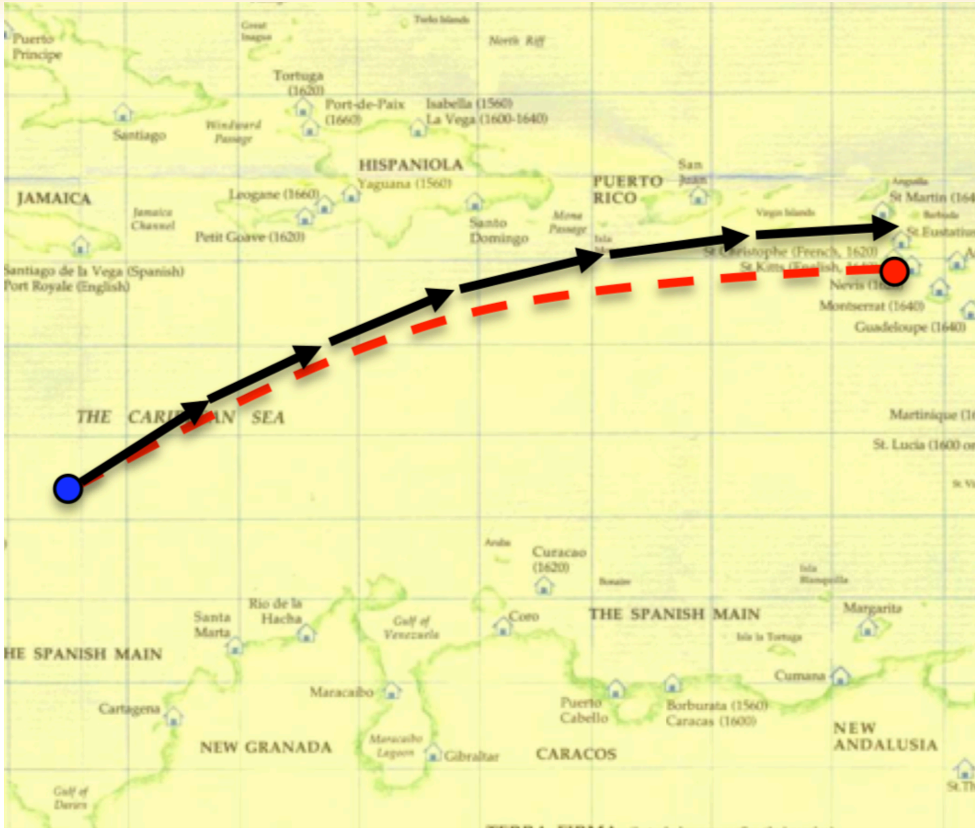
Magnetometers: Issues

- Dependent on location
- Magnetic field near a sensor changes the result
- Indoor: a building generates its own field due to ferromagnetic metals
- Moving elevator (for example) changes magnetic field

Inertial Navigation Systems

- Combinations of:
  - GPS (for initialization and periodic correction).
  - Three axis gyroscope measures orientation.
  - Three axis accelerometer, double integrated for position after correction for orientation.
- Typical drift for systems used in aircraft have to be:
  - 0.6 nautical miles per hour
  - tenths of a degree per hour
- Good enough? It depends on the application!

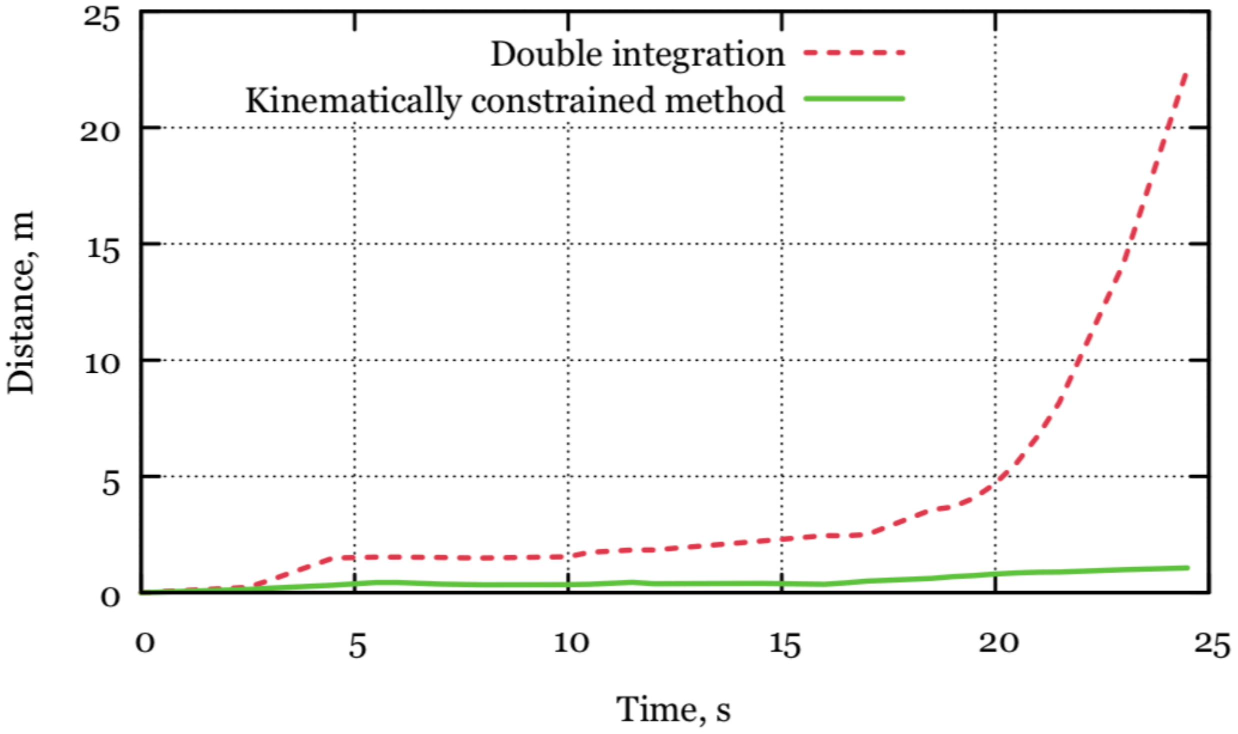
How often to calibrate?



Design Issues with Sensors

- Calibration
  - Relating measurements to the physical phenomenon
  - Can dramatically increase manufacturing costs
- Nonlinearity
  - Measurements may not be proportional to physical phenomenon
  - Correction may be required
  - Feedback can be used to keep operating point in the linear region
- Sampling
  - Aliasing
  - Missed events
- Noise
  - Analog signal conditioning
  - Digital filtering
  - Introduces latency
- Failures
  - Redundancy (sensor fusion problem)
  - Attacks (e.g. Stuxnet attack)

Minimizing Error



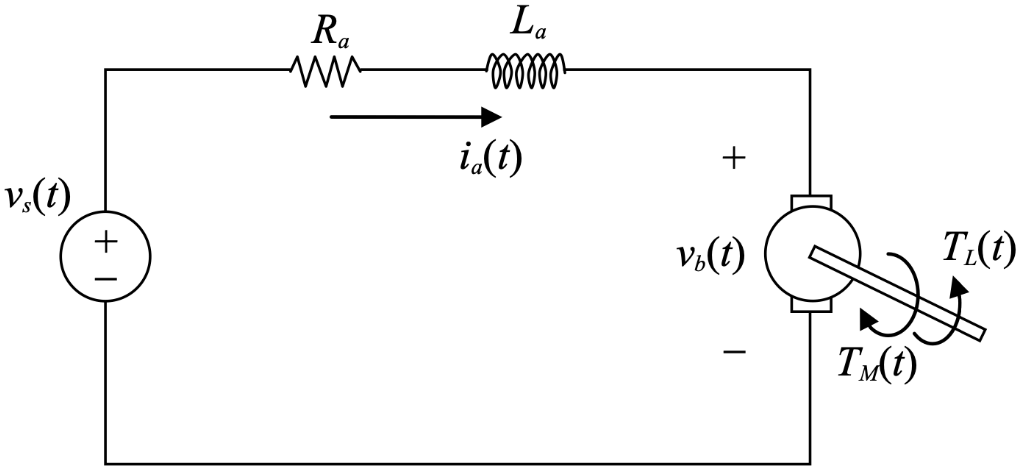
Head Tracking for the Oculus Rift, 2014

Light Emitting Diodes

- Read from book – 7.3.1

Motor

- DC motor consists of an electromagnet
- When current flows through the wires, the core spins



Model of a Motor

- Electrical Model:

Back electromagnetic force constant

Angular velocity

$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \boxed{k_b\omega(t)}$$

Back EMF Voltage

R is the resistance and L the inductance of the coils in the motor

- Mechanical Model (angular version of Newton's second law):

$$I\frac{d\omega(t)}{dt} = k_T i(t) - \eta\omega(t) - \tau(t)$$

Moment of inertia

Torque constant

Friction

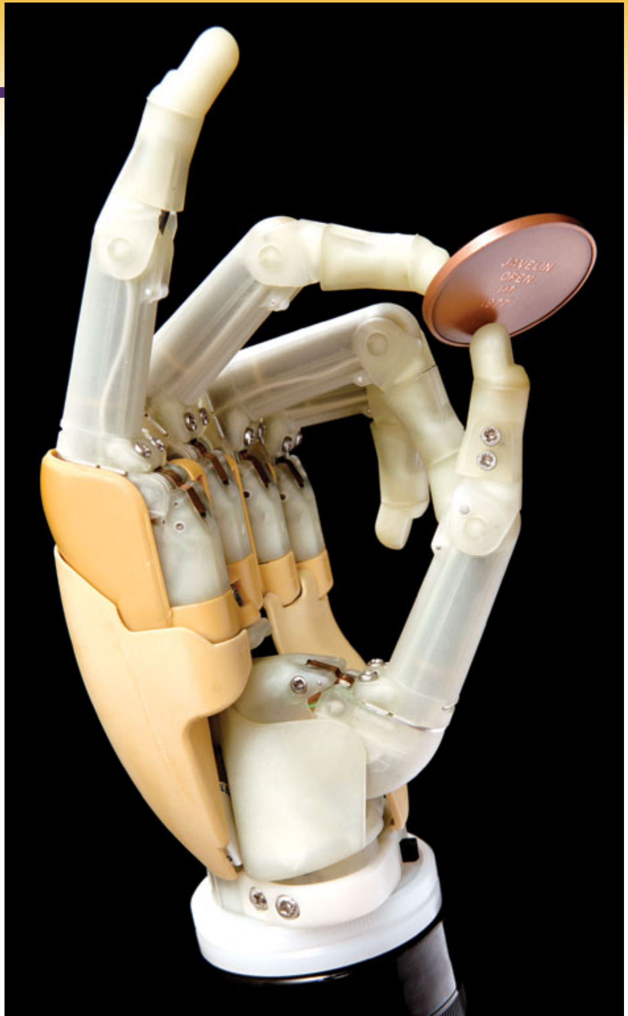
Load torque

Torque is proportional to the current flowing through the motor, adjusted by friction and any torque that might be applied by the mechanical load

Motor Controllers

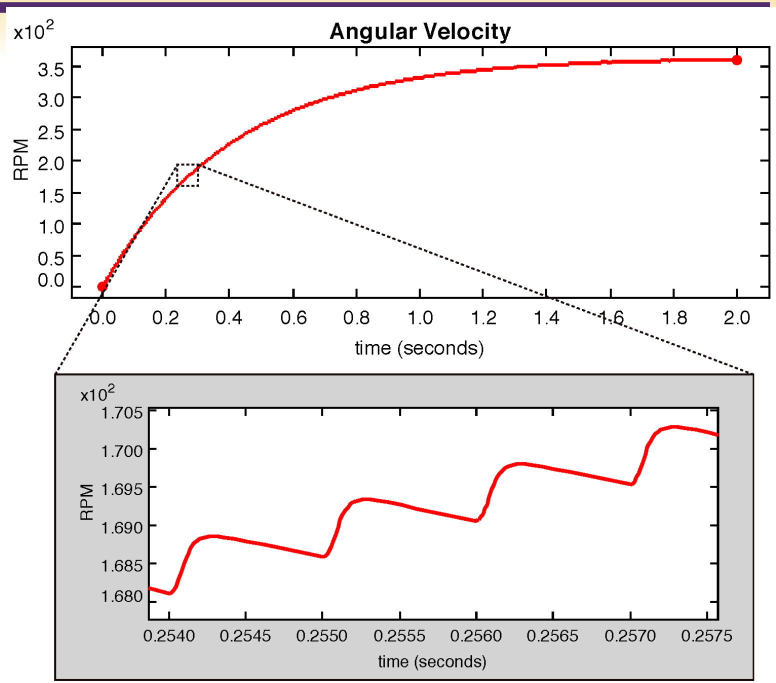
- Bionic hand from Touch Bionics costs $18,500, has and five DC motors, can grab a paper cup without crushing it, and turn a key in a lock. It is controlled by nerve impulses of the user’s arm, combined with autonomous control to adapt to the shape of whatever it is grasping.

Source: IEEE Spectrum, Oct. 2007.

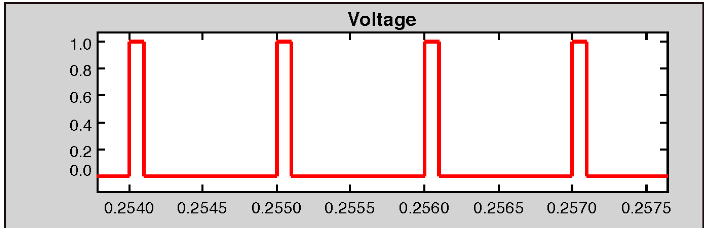


Pulse-Width Modulation (PWM)

- Delivering power to actuators can be challenging. If the device tolerates rapid on-off controls (“bang-bang” control), then delivering power becomes much easier.

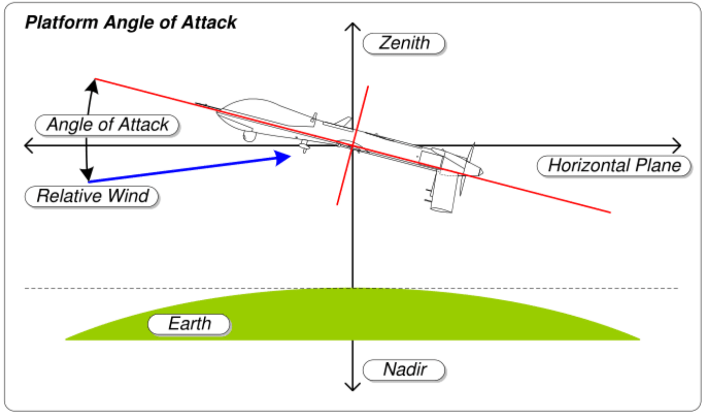


Duty cycle around 10%



Violent Pitching of Qantas Flight 72

- An Airbus A330 en-route from Singapore to Perth on 7 October 2008
- Started pitching violently, unrestrained passengers hit the ceiling, 12 serious injuries, so counts it as an accident.
- Three Angle Of Attack (AOA) sensors, one on left (#1), two on right (#2, #3) of nose.
- Have to deal with inaccuracies, different positions, gusts/spikes, failures.



Faults in Sensors

- Sensors are physical devices
- Like all physical devices, they suffer wear and tear, and can have manufacturing defects
- Cannot assume that *all* sensors on a system will work correctly at *all* times
- Solution: Use redundancy
- → However, must be careful *how* you use it!

How to deal with Sensor Errors

- Difficult Problem, still research to be done

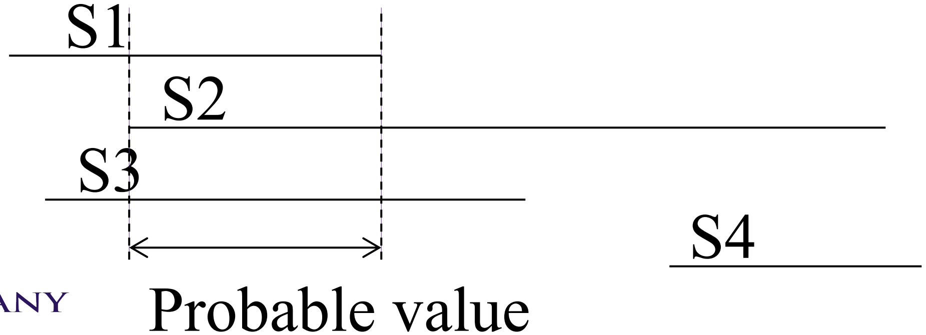
- Possible approach: Intelligent sensor communicates an interval, not a point value
  - Width of interval indicates confidence, health of sensor

Sensor Fusion: Marzullo's Algorithm

- Axiom: if sensor is non-faulty, its interval contains the true value
- Observation: true value must be in overlap of non-faulty intervals
- Consensus (fused) Interval to tolerate f faults in n: Choose interval that contains all overlaps of n – f; i.e., from least value contained in n – f intervals to largest value contained in n – f

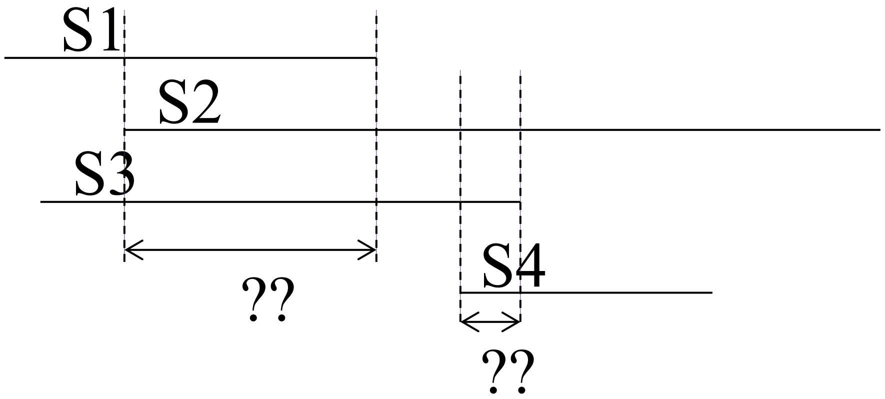
Example: 4 sensors, at most one faulty

- Interval reports range of possible values.
- Of S1 and S4, one must be faulty.
- Of S3 and S4, one must be faulty.
- Therefore, S4 is faulty.
- Sound estimate is the overlap of the remaining three.



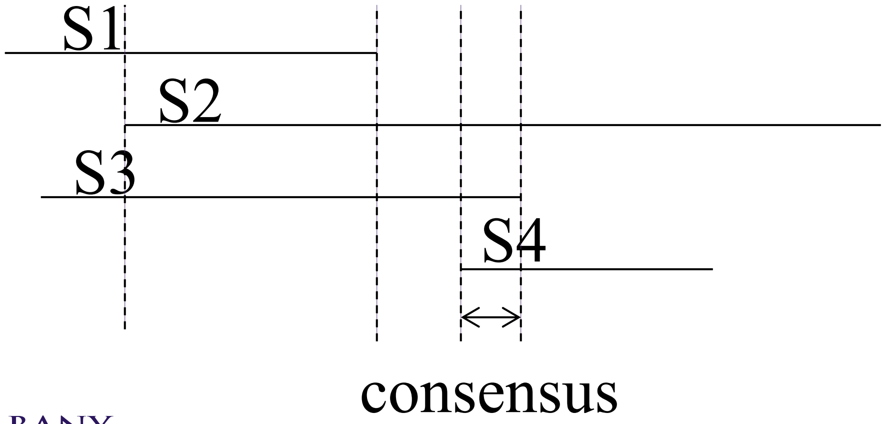
Example: 4 sensors, at most one faulty

- Suppose S4's reading moves to the left
- Which interval should we pick?



Example: 4 sensors, at most one faulty

- Marzullo's algorithm picks the smallest interval that is sure to contain the true value, under the assumption that at most one sensor failed.
- But this yields big discontinuities. Jumps!

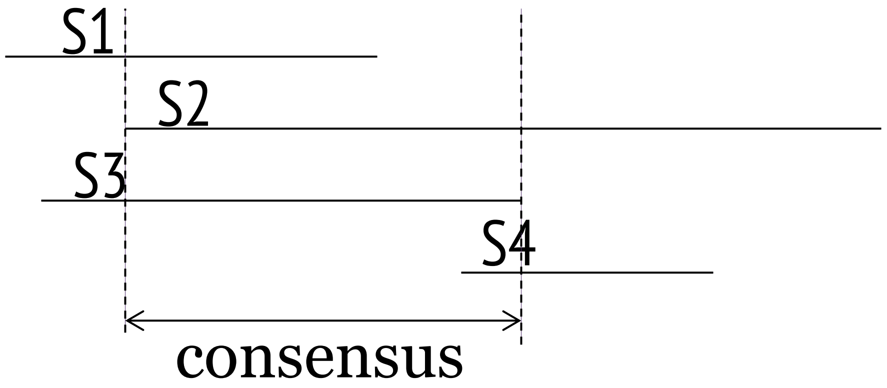


Schmid & Schossmaier's Fusion Method

- Recall: n sensors, at most f faulty
- Choose interval from f+1st largest lower bound to f+1st smallest upper bound
- Optimal among selections that satisfy continuity conditions.

Example: 4 sensors, at most one faulty

- Assuming at most one faulty, Schmid and Schossmaier’s method choose the interval between:
  - Second largest lower bound
  - Second smallest upper bound
  - This preserves continuity, but not soundness

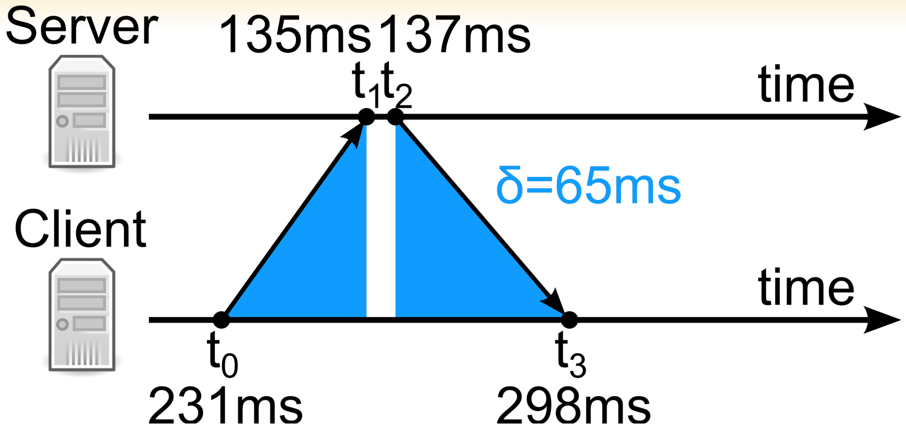


Algorithm

- sort the lower and upper bounds of all the sensor readings into ascending order → O(nlogn)
- scan the sorted list from smallest to largest, maintaining an intersection count
  - increments by one for every lower bound and decrements by one for every upper bound
- the lower bound l of the fusion interval is the first value where the count reaches n – f

Network Time Protocol (NTP)

- Intersection Algorithm: (Modified Marzullo's Algorithm)
- NTP client regularly polls one or more NTP servers
- Client computes its time offset and round-trip delay



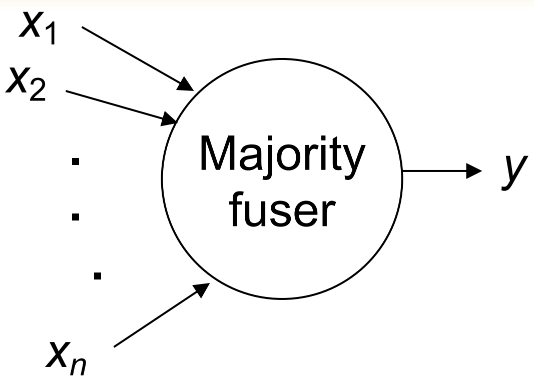
Source: https://en.wikipedia.org/wiki/Network_Time_Protocol

$$\theta = \frac{(t_1 - t_0) + (t_2 - t_3)}{2}$$

$$\delta = (t_3 - t_0) - (t_2 - t_1)$$

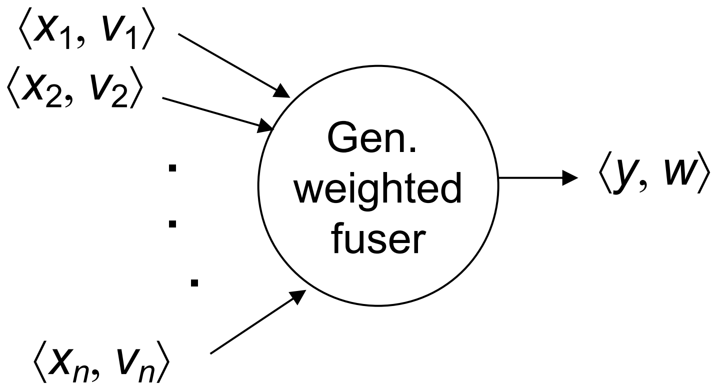
Voting and Data Fusion

- Majority voting: Select the value that appears on at least $\lfloor n/2 \rfloor + 1$ of the n inputs
- Majority fusers can be realized by means of comparators and multiplexers



Weighted Voting

Given n input data objects $x_1, x_2, \ldots,$ x_n and associated nonnegative real weights $v_1, v_2, \ldots, v_n$, with S$v_i = V$, compute output y and its weight w such that y is “supported by” a set of input objects with weights totaling w, where w satisfies a condition associated with the voting subscheme



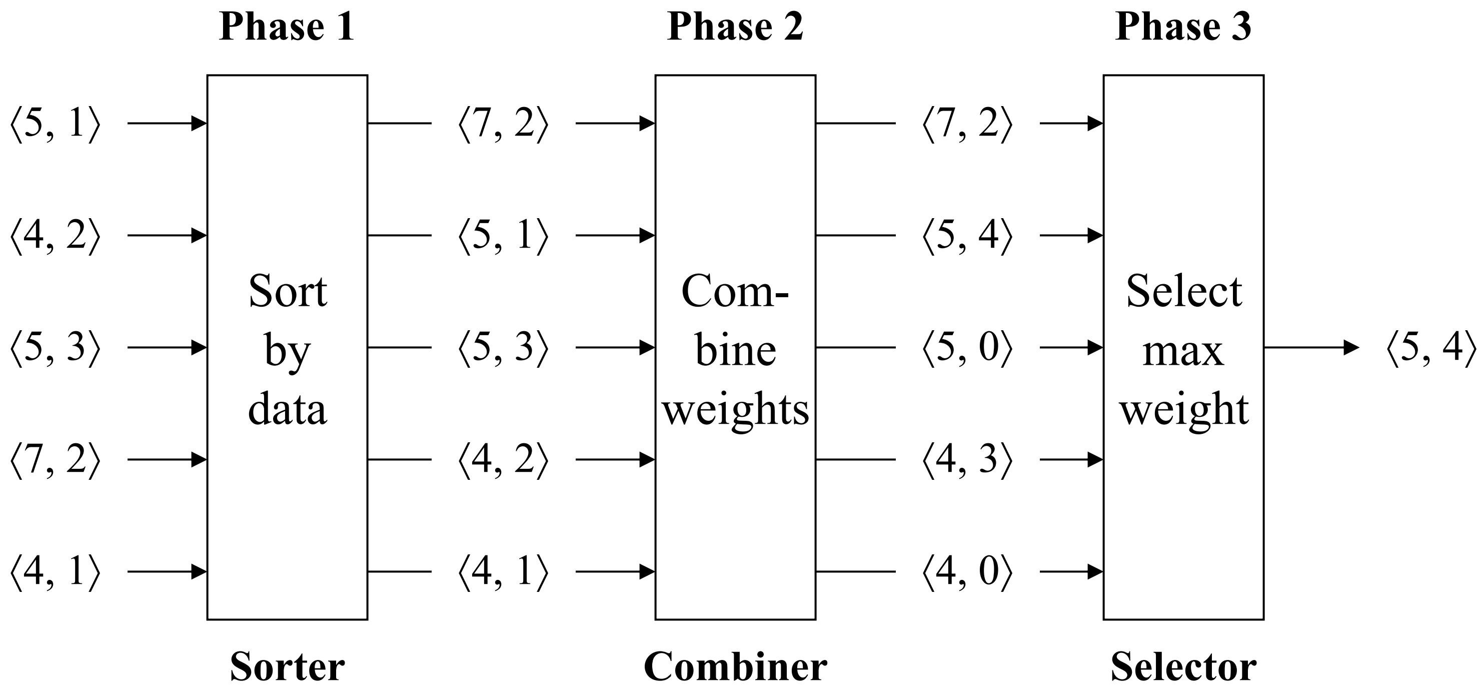
Possible voting subschemes:

Unanimity	$w = V$
Majority	$w > V/2$
Supermajority	$w \geq 2V/3$
Byzantine	$w > 2V/3$
Plurality	(w for y) $\geq$ (w for any $z \neq y$)
Threshold	$w >$ a preset lower bound

Weighted Plurality Voting Units

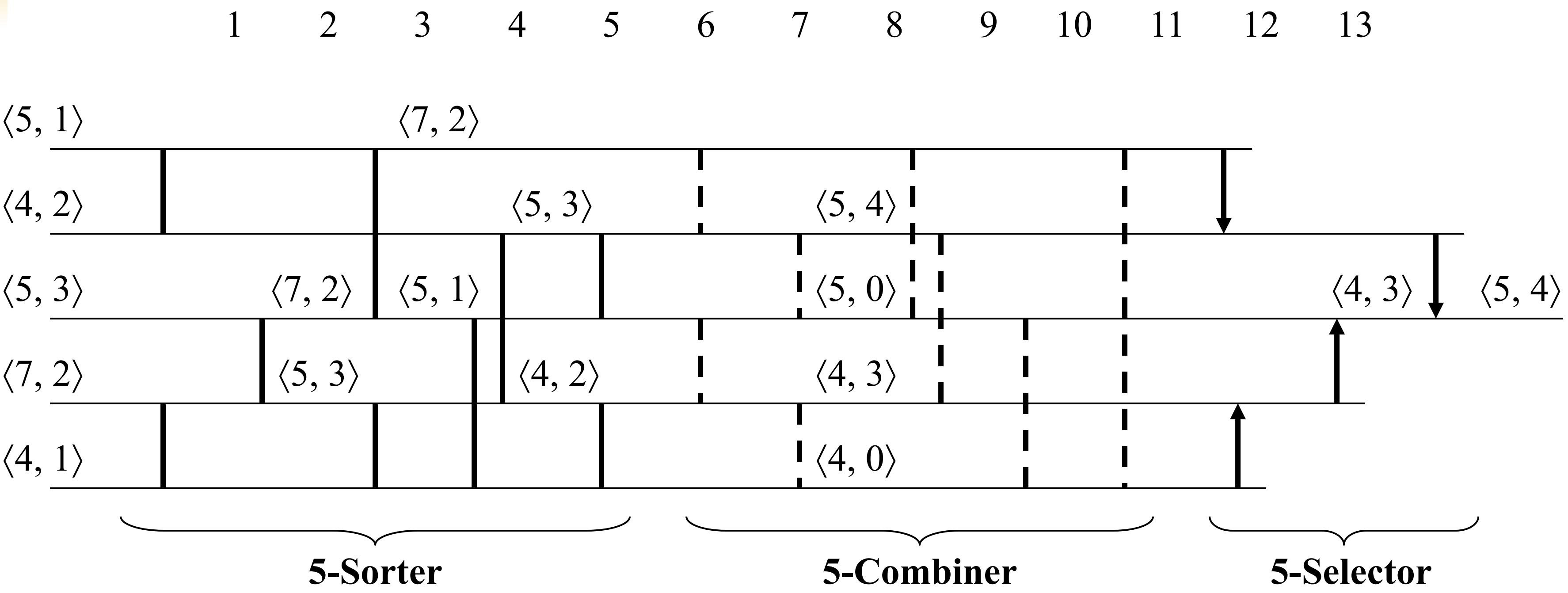
Inputs: Data-weight pairs

Output: Data with maximal support and its associated tally



Source: B. Parhami, *IEEE Trans. Reliability*, Vol. 40, No. 3, pp. 380-394, August 1991

Stages of delay



The first two phases (sorting and combining) can be merged, producing a 2-phase design – fewer, more complex cells (lead to tradeoff)