Modern Wireless Networks

Wireless Fundamentals

UNIVERSITY AT ALBANY
State University of New York

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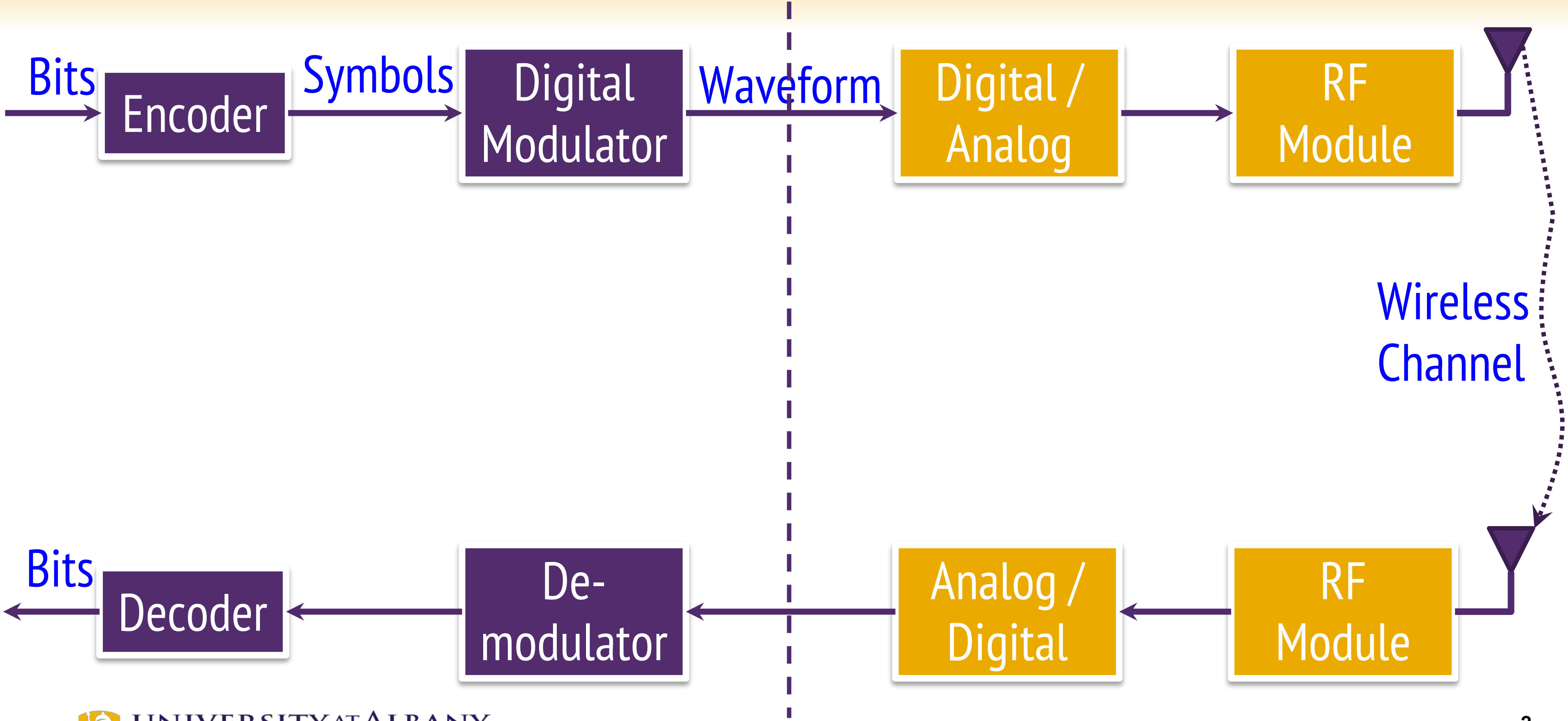
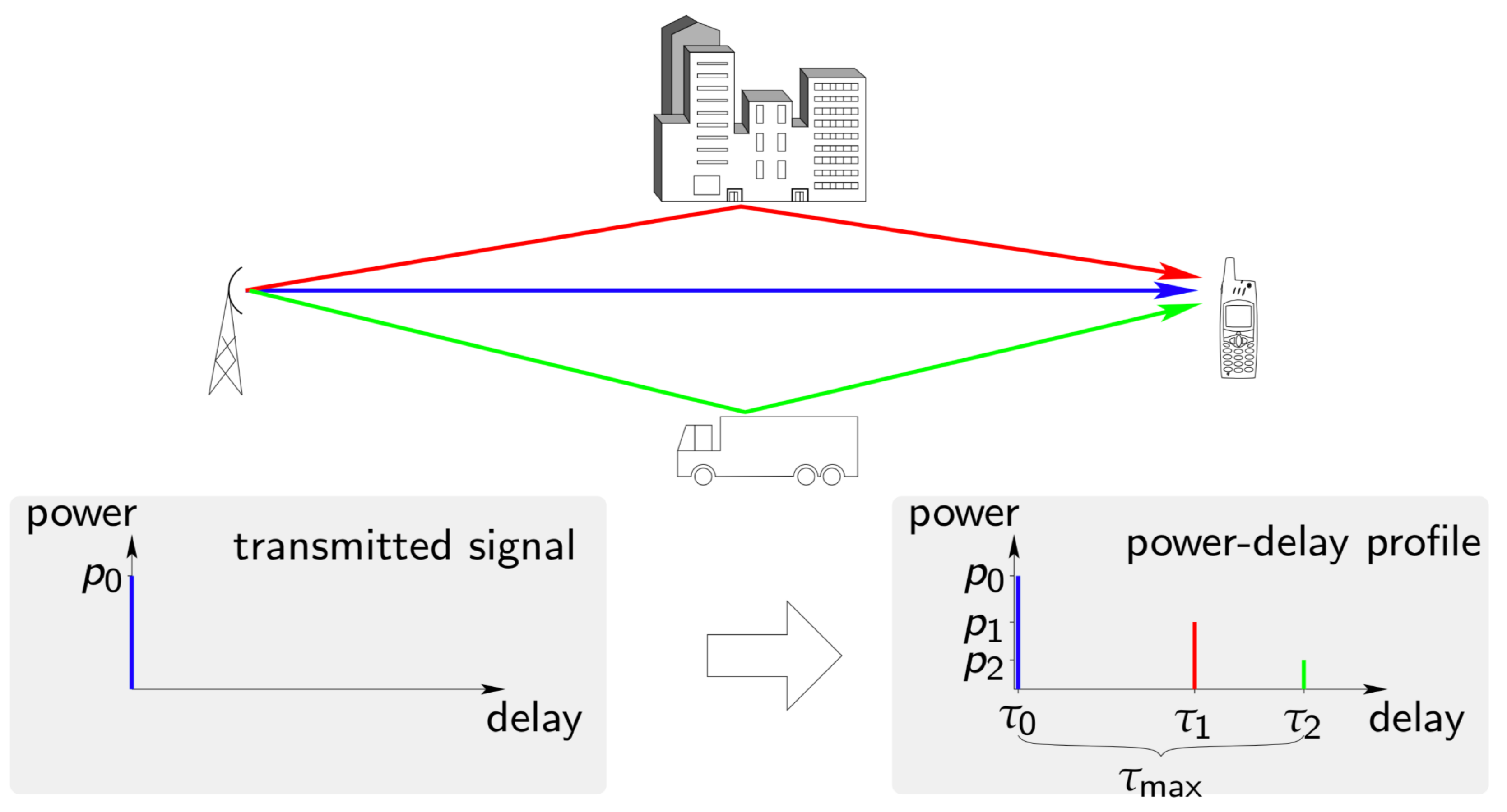
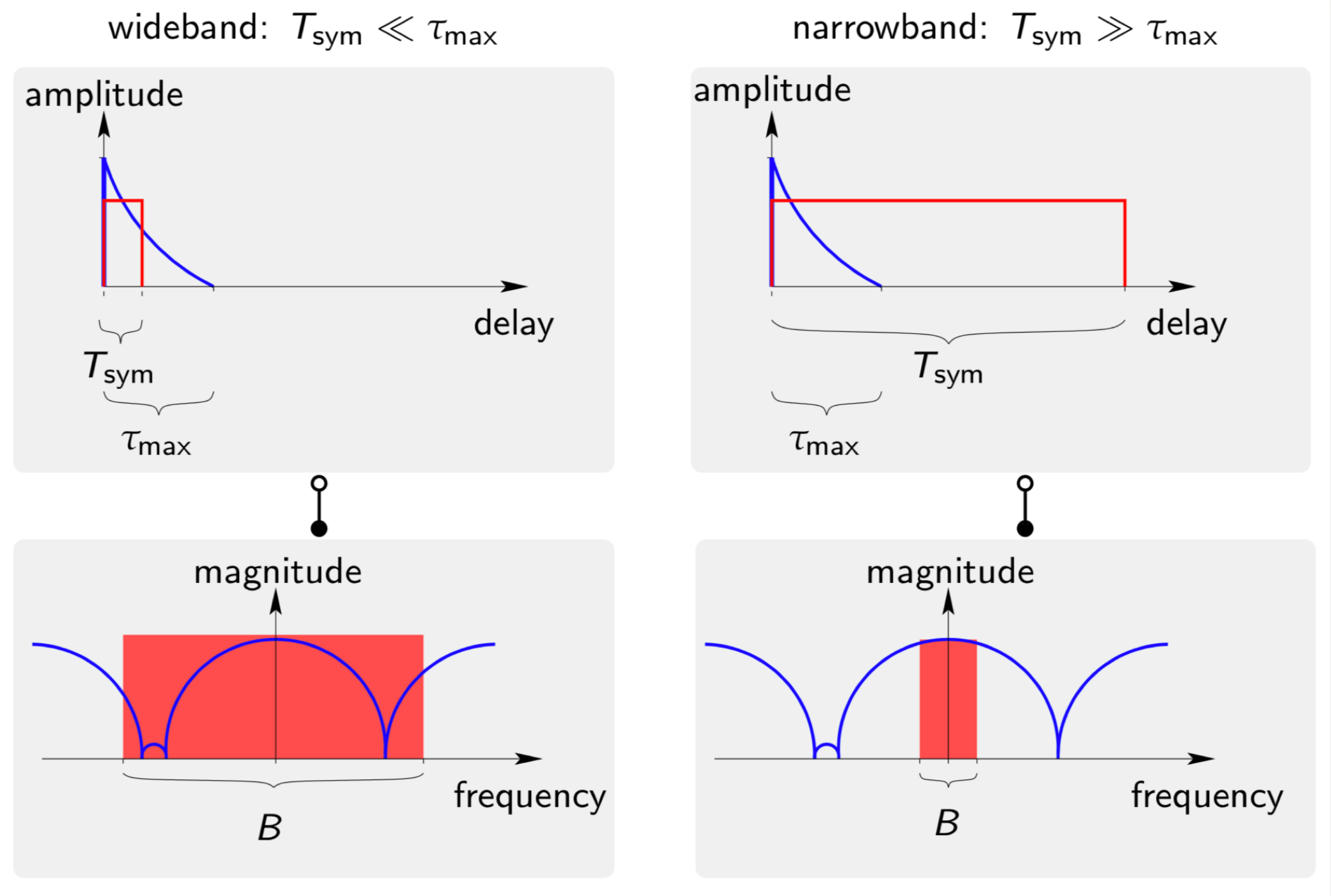
Wireless Digital Communication System



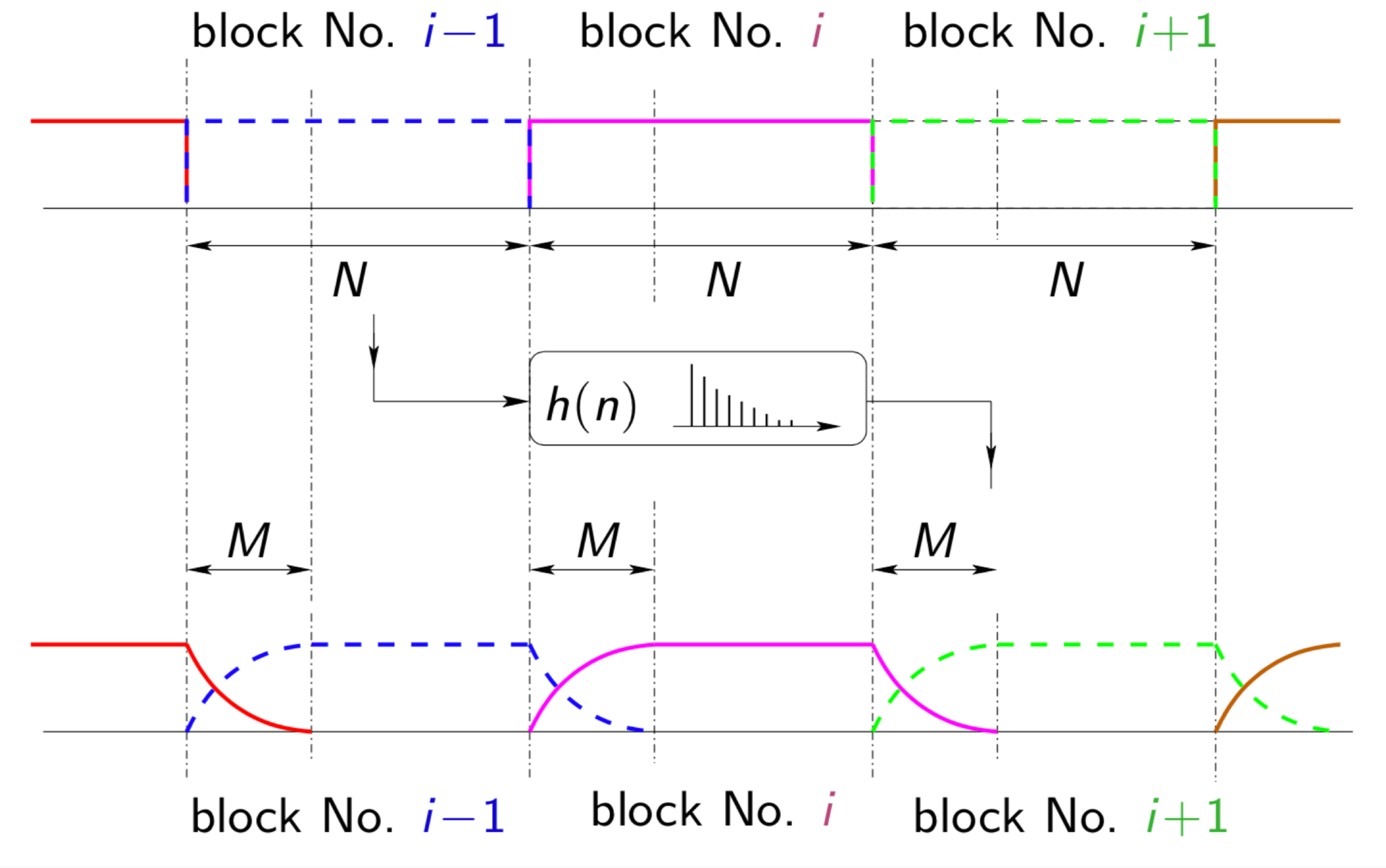
Multipath Channel Effects



Wideband vs Narrowband

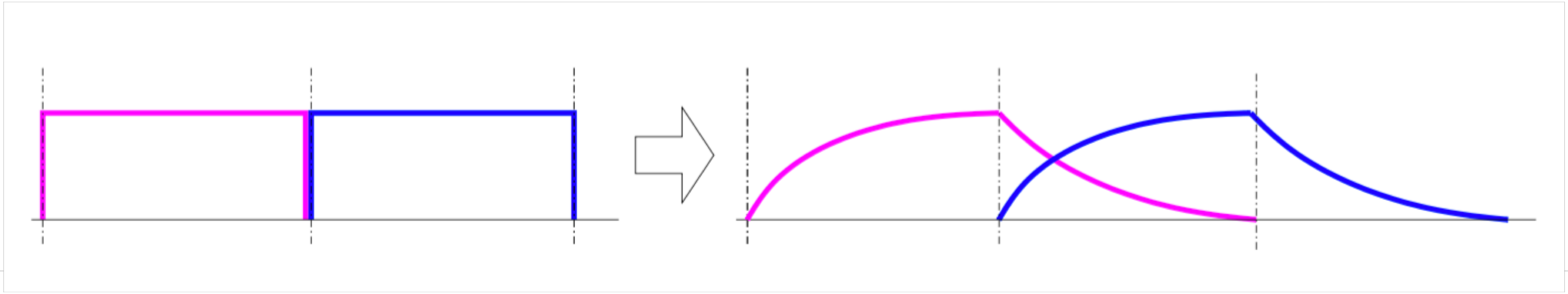


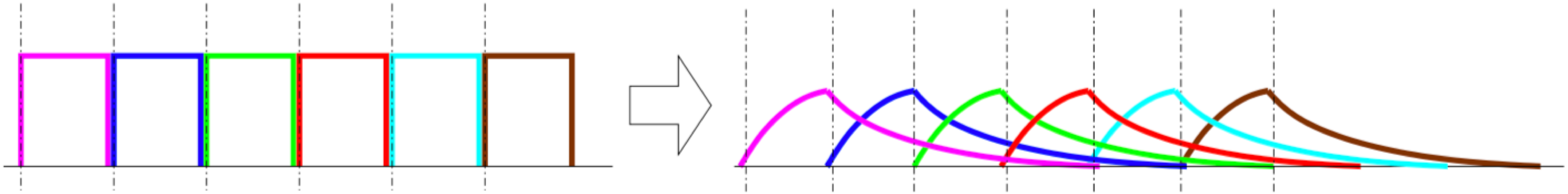
Effect of dispersion (Inter Symbol Interference)



ISI as an impediment to increase data rate

- Need for higher data rate urges to transmit at higher symbol ➔ Higher ISI



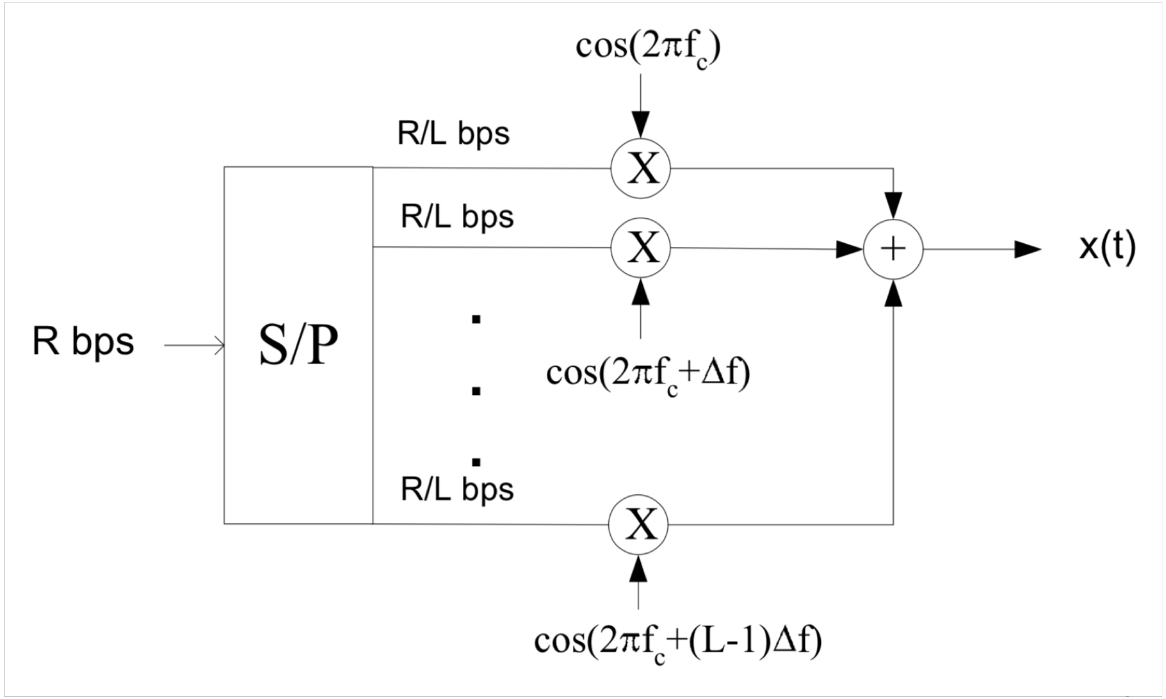


Multicarrier Modulation to combat ISI

- Divides the wideband incoming data stream into L narrowband substreams
- Each substream is then transmitted over a different orthogonal frequency subchannel
- Number of substreams L is chosen to make the symbol time on each substream much greater than the delay spread of the channel
- Make the substream bandwidth less then the channel coherence bandwidth

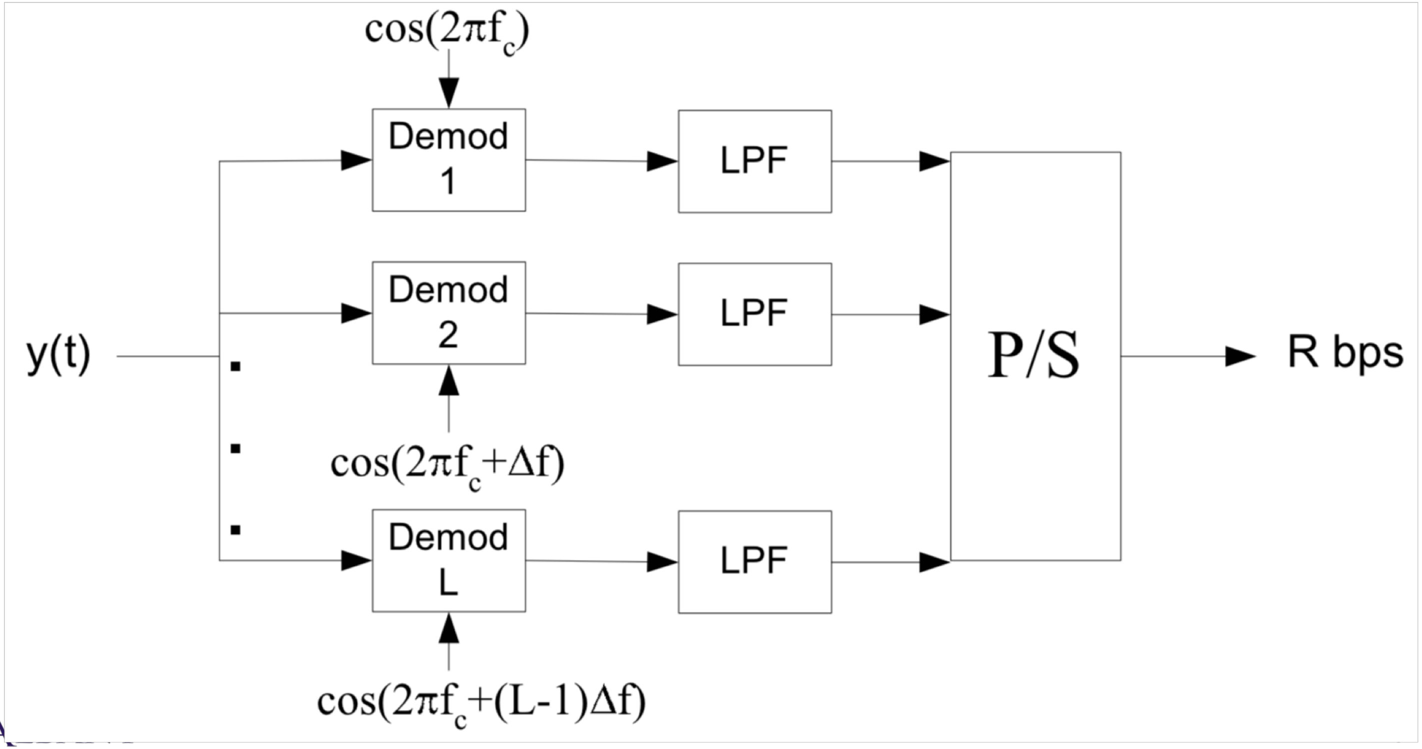
Basic Multicarrier Transmitter

- a high rate stream of R bps is broken into L parallel streams each with rate R/L and then multiplied by a different carrier frequency



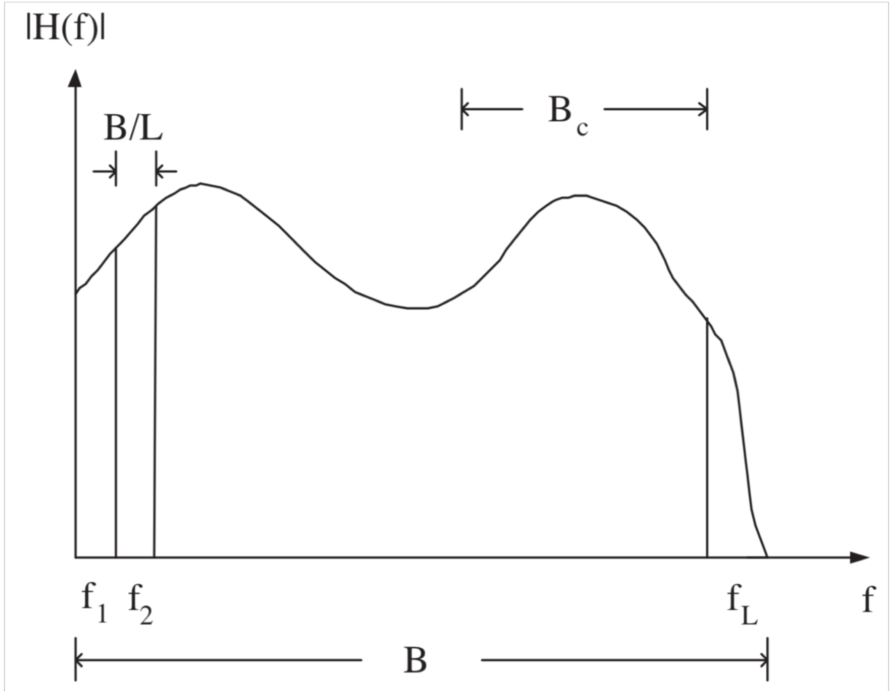
A Basic Multicarrier Receiver

- each subcarrier is decoded separately, requiring L independent receivers



Channel Effects

- flat fading on each subchannel since $B/L \ll B_c$, even though the overall channel experiences frequency selective fading, i.e. $B > B_c$.



Possible but not practical

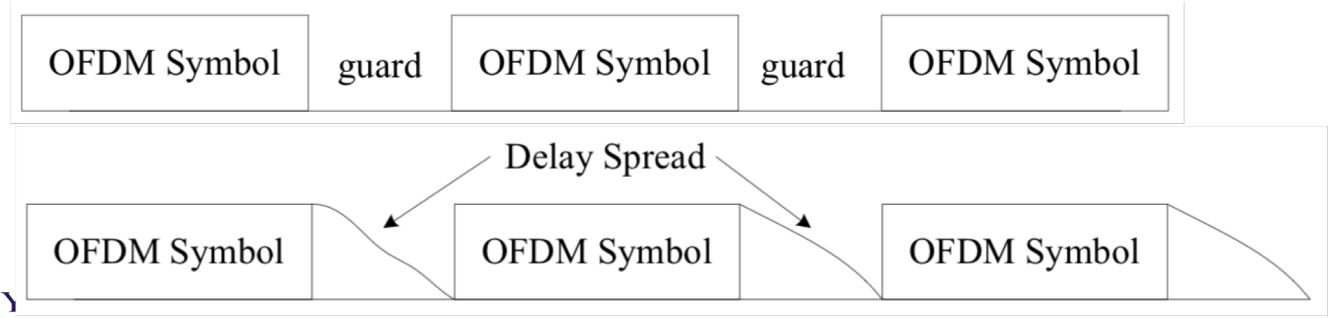
- a large bandwidth penalty will be inflicted since the subcarriers *can't have perfectly rectangular pulse shapes* and still be time-limited
- very high quality (and hence, expensive) *low pass filters* will be required to maintain the orthogonality of the the subcarriers at the receiver.
- this scheme requires *L independent RF units* and demodulation paths

OFDM Basics

- OFDM utilizes an efficient computational technique known as the Discrete Fourier Transform (DFT), more commonly known as the Fast Fourier Transform (FFT)
- No need for multiple radios

OFDM Basics

- L data symbols are grouped into a block – OFDM symbol
- T_s = symbol time for each data symbol
- $T = LT_s$ = OFDM symbol duration
- τ = Delay spread of the channel
- If guard time Tg > τ, no interference between subsequent OFDM symbols
- No ISI



Exploiting Properties of DFT

- Circular Convolution $\mathrm{DFT}\{y[n]\} = \mathrm{DFT}\{h[n] \circledast x[n]\}$
- Frequency domain output $Y[m] = H[m]X[m]$
- It is ISI-free channel in the frequency domain, where each input symbol X[m] is simply scaled by a complex-value H[m]
- Note that the *duality* between circular convolution in the time domain and simple multiplication in the frequency domain is a property unique to the DFT.

Exploiting Properties of DFT

- L point DFT

$$\text{DFT}\{x[n]\} = X[m] \triangleq \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} x[n] e^{-j\frac{2\pi nm}{L}}$$

- Inverse DFT (IDFT)

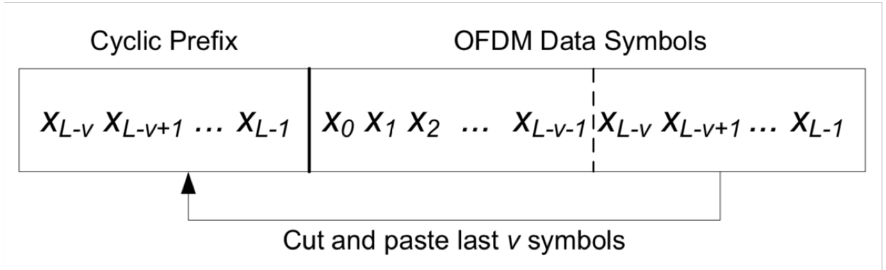
$$\text{IDFT}\{X[m]\} = x[n] \triangleq \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} X[m] e^{j\frac{2\pi nm}{L}}$$

- At receiver, if channel frequency response H[m] is known, input is derived as

$$\hat{X}[m] = \frac{Y[m]}{H[m]}$$

How to create circular convolution in channel?

- Cyclic Prefix



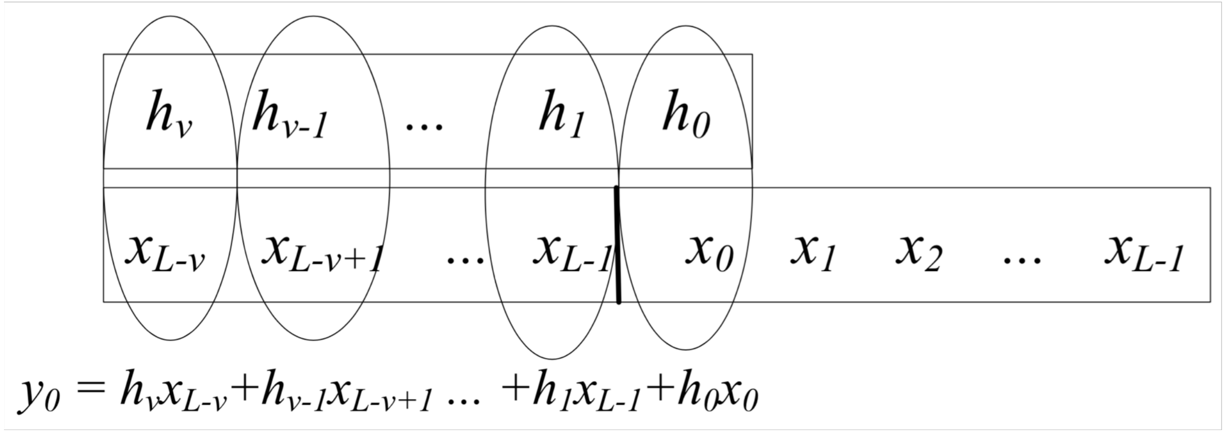
- If max channel delay spread = τ_{max} = v samples
- Then add a guard of v samples
- Time domain representation:

$$\mathbf{x} = [x_1\ x_2\ \ldots x_L]$$

$$\mathbf{x}_{cp} = [\underbrace{x_{L-v}\ x_{L-v+1}\ \ldots\ x_{L-1}}_{\text{Cyclic Prefix}}\ \underbrace{x_0\ x_1\ \ldots\ x_{L-1}}_{\text{Original data}}]$$

CP creates Circular Convolution

- Output of Channel: $y_{CP} = h * x_{CP}$



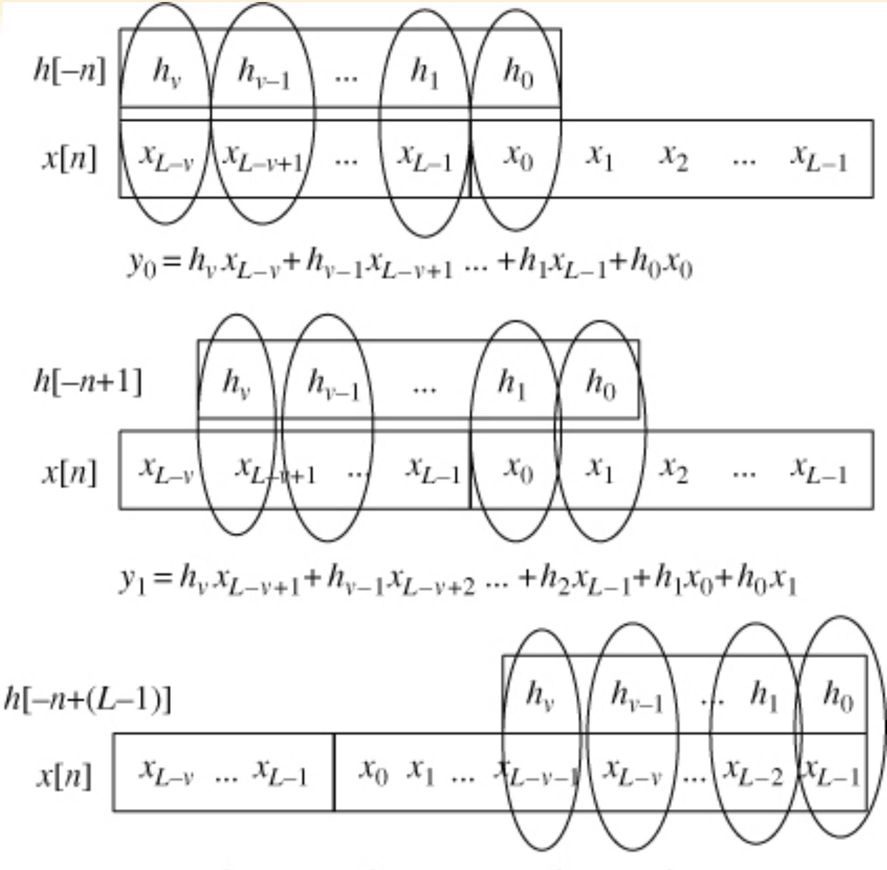
$$y_0 = h_v x_{L-v} + h_{v-1} x_{L-v+1} \dots + h_1 x_{L-1} + h_0 x_0$$

- for L ≫ v, the inefficiency due to the cyclic prefix can be made arbitrarily small by increasing the number of subcarriers

CP creates a circular convolution

- creates a circular convolution at the receiver (signal *y*) even though the actual channel causes a linear convolution.

$$\begin{aligned} y_0 &= h_0x_0 + h_1x_{L-1} + \cdots + h_v x_{L-v} \\ y_1 &= h_0x_1 + h_1x_0 + \cdots + h_v x_{L-v+1} \\ &\vdots \\ y_{L-1} &= h_0x_{L-1} + h_1x_{L-2} + \cdots + h_v x_{L-v-1} \end{aligned}$$



$$y_0 = h_v x_{L-v} + h_{v-1}x_{L-v+1} \ldots + h_1x_{L-1} + h_0x_0$$

$$y_1 = h_v x_{L-v+1} + h_{v-1}x_{L-v+2} \ldots + h_2x_{L-1} + h_1x_0 + h_0x_1$$

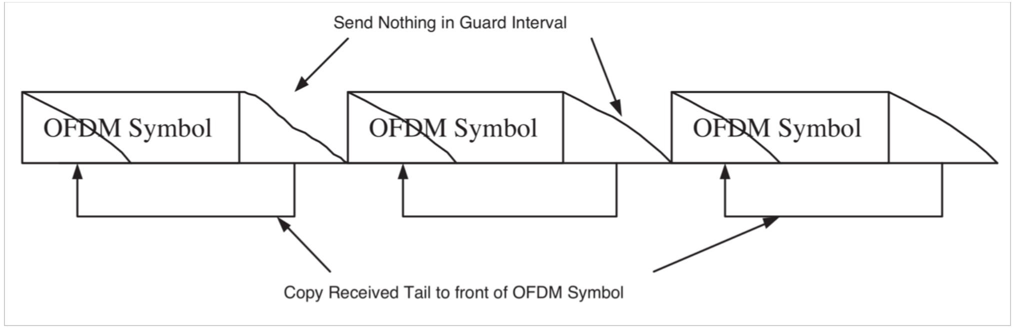
$$y_{L-1} = h_v x_{L-v-1} + h_{v-1}x_{L-v} \ldots + h_1x_{L-2} + h_0x_{L-1}$$

Penalties of CP

- v redundant symbols are sent
- Required bandwidth increases from B to $\frac{L+v}{L}B$
- Transmit power penalty $10log_{10}\frac{L+v}{L}dB$
- Rate loss = Power loss = $\frac{L}{L+v}$

Zero Prefix

- Null Guard Band
- At the receiver, the “tail” can be added back in
- Recreates the effect of a CP



- Reduces Tx power by $10log_{10}\frac{L+v}{L}dB$

Zero Prefix Issues

- Increases the receiver power by $10log_{10}\frac{L+v}{L}dB$
- With CP transmitted, the tail can be ignored
- Additional noise from the received tail symbols is added back into the signal
- Higher noise power compared to transmitted CP
- $\sigma^2 \rightarrow \frac{L+v}{L}\sigma^2$

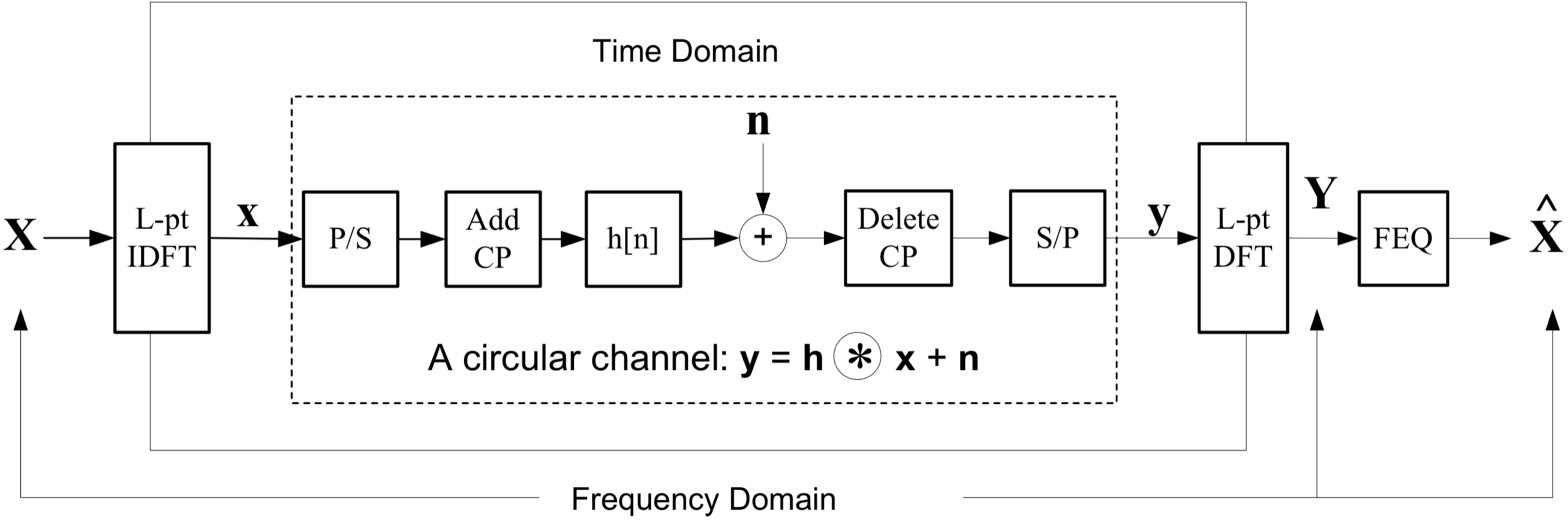
Frequency Equalization

- data symbols are estimated using a one-tap frequency domain equalizer

$$\hat{X}_l = \frac{Y_l}{H_l}$$

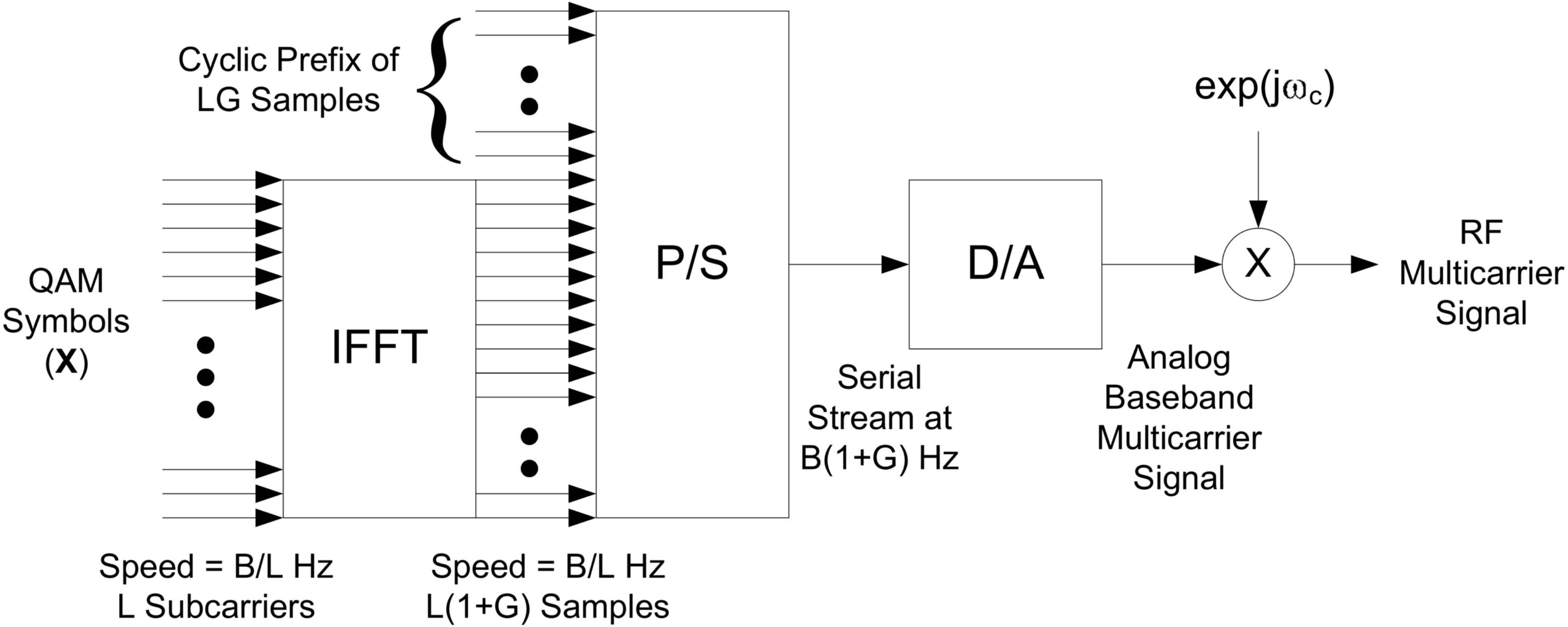
- H_l is the *complex* response of the channel at the frequency $f_c + (l - 1)\Delta f$

An OFDM System

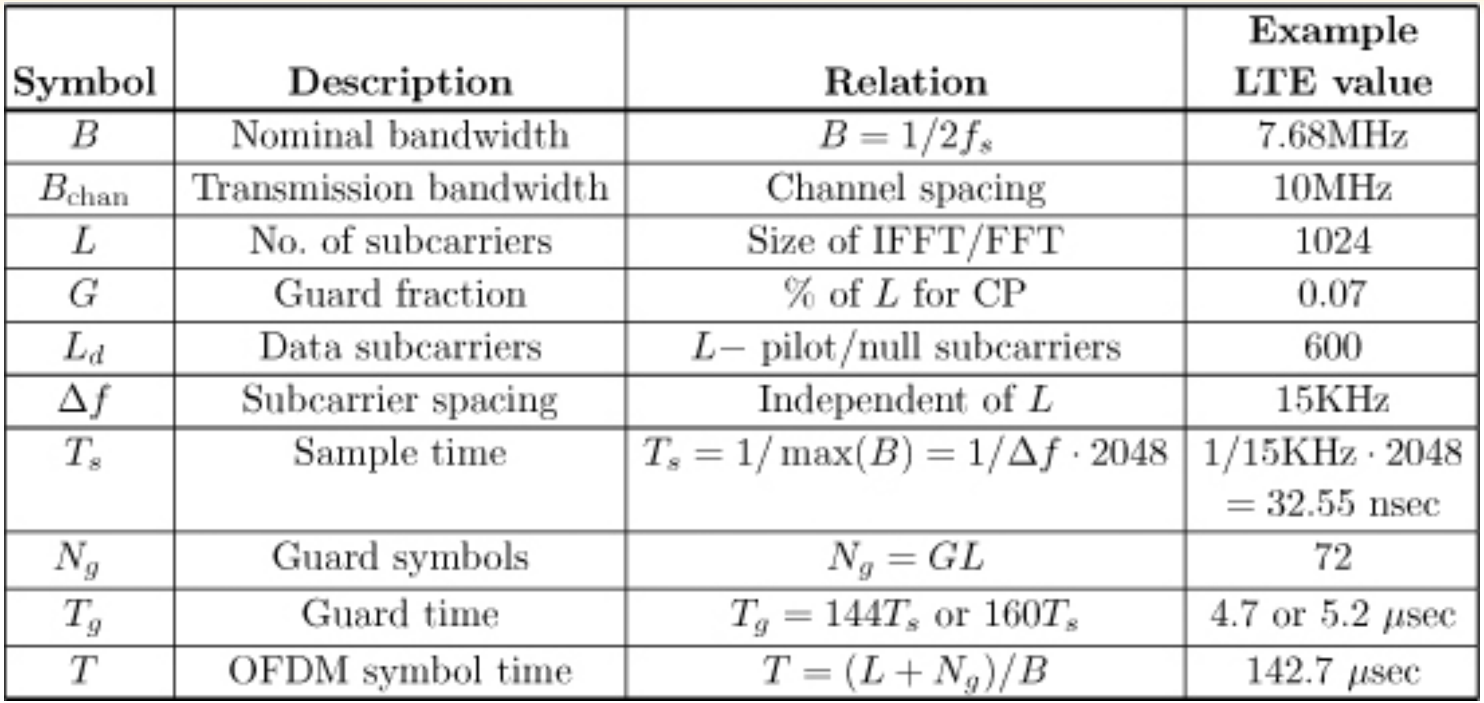


A circular channel: $\mathbf{y} = \mathbf{h} \circledast \mathbf{x} + \mathbf{n}$

An OFDM Transmitter



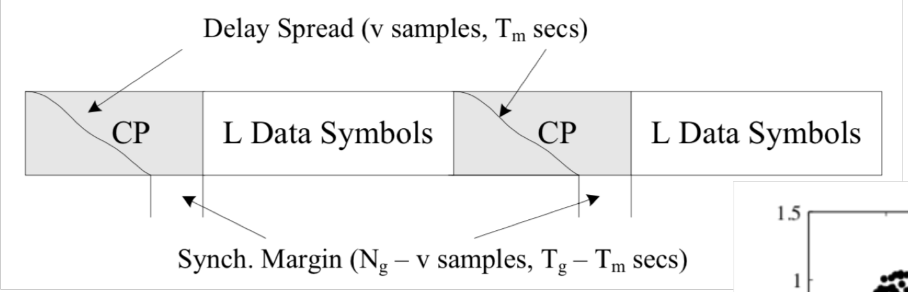
OFDM Parameters in LTE for 10MHz Channel



Symbol	Description	Relation	Example LTE value
B	Nominal bandwidth	$B = 1/2f_s$	7.68MHz
B_{chan}	Transmission bandwidth	Channel spacing	10MHz
L	No. of subcarriers	Size of IFFT/FFT	1024
G	Guard fraction	% of L for CP	0.07
L_d	Data subcarriers	$L-$ pilot/null subcarriers	600
Δf	Subcarrier spacing	Independent of L	15KHz
T_s	Sample time	$T_s = 1/\max(B) = 1/\Delta f \cdot 2048$	$1/15\text{KHz} \cdot 2048$ $= 32.55$ nsec
N_g	Guard symbols	$N_g = GL$	72
T_g	Guard time	$T_g = 144T_s$ or $160T_s$	4.7 or 5.2 μsec
T	OFDM symbol time	$T = (L + N_g)/B$	142.7 μsec

Timing Offset

- Cyclic Prefix provides some toleration in error in timing synchronization



Delay Spread (v samples, T_m secs)

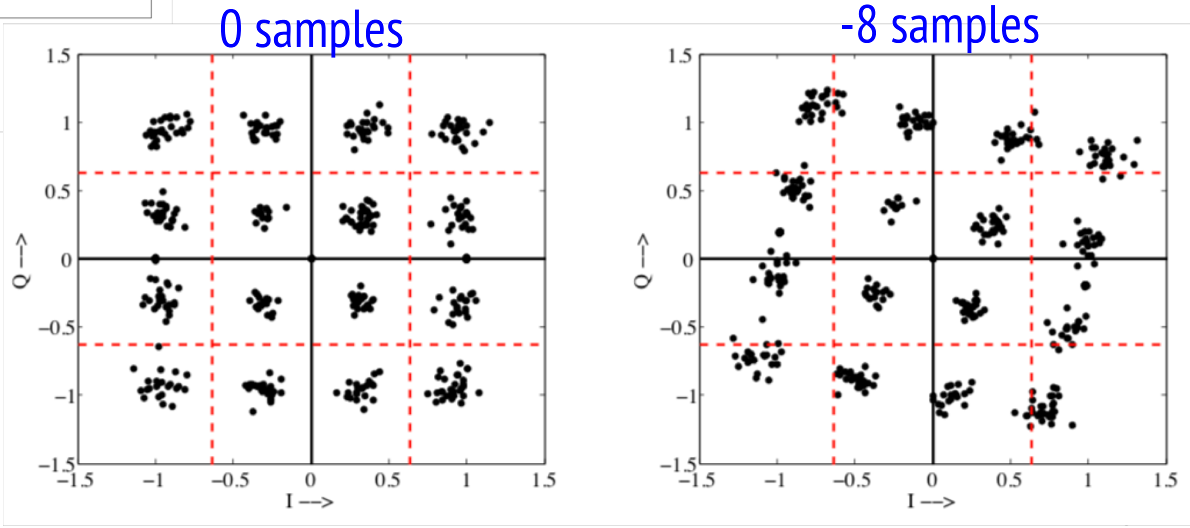
CP

L Data Symbols

CP

L Data Symbols

Synch. Margin (N_g – v samples, T_g – T_m secs)



0 samples

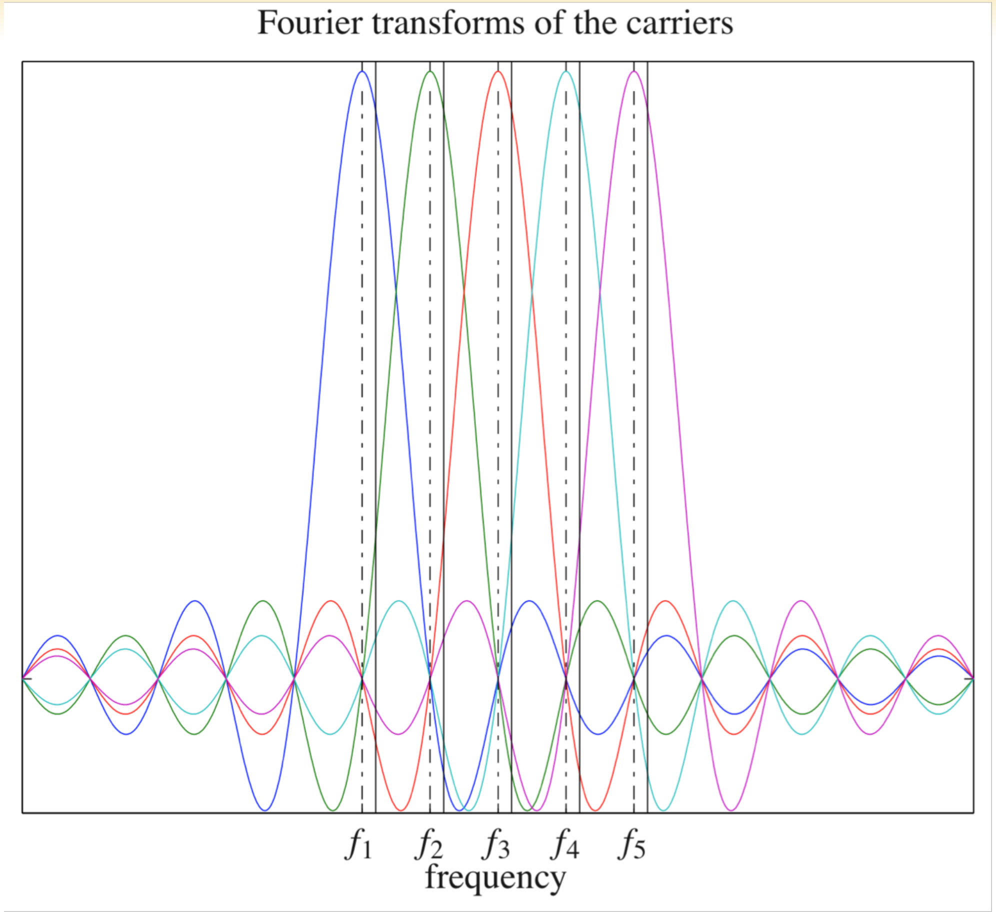
-8 samples

Frequency Offset

- carrier frequency/phase of transmitter's local oscillator (LO) and receiver's LO can be off
- resulting frequency difference ΔF_c Hz between transmitter's and receiver's carrier introduces the additional term $e^{j2\pi\Delta F_c/F_s n}$ in the baseband multiplex $\longrightarrow$ ICI (Inter-Carrier Interference)
- receiver needs to compensate this offset

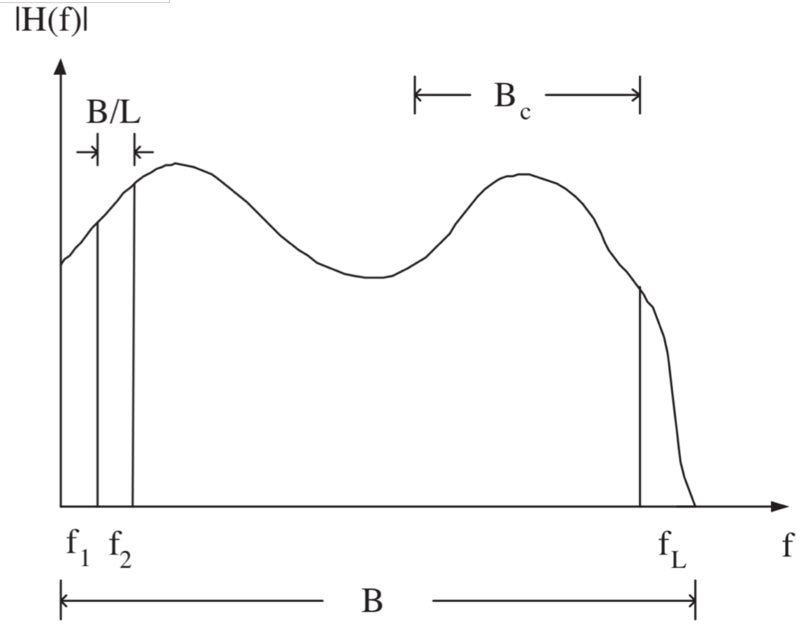
ICI due to frequency offset

- Coarse correction
  - Short preamble based
- Fine correction
  - Long preamble based, pilot tracking



Channel Fading and Recovery

- Recall: at receiver, if channel frequency response H[m] is known, input is derived as $\hat{X}[m] = \dfrac{Y[m]}{H[m]}$
- OFDM is wideband
- Each SC is narrowband
- Flat fading on each SC
- But overall channel experiences frequency selective fading



Fading across subcarriers

- transmit power on subcarrier i is P_i
- fading on that subcarrier is α_i
- received SNR in subcarrier i is $\gamma_i = P_i \alpha_i^2 / (N_0 B)$
- where N_0 is the noise power and B is the bandwidth

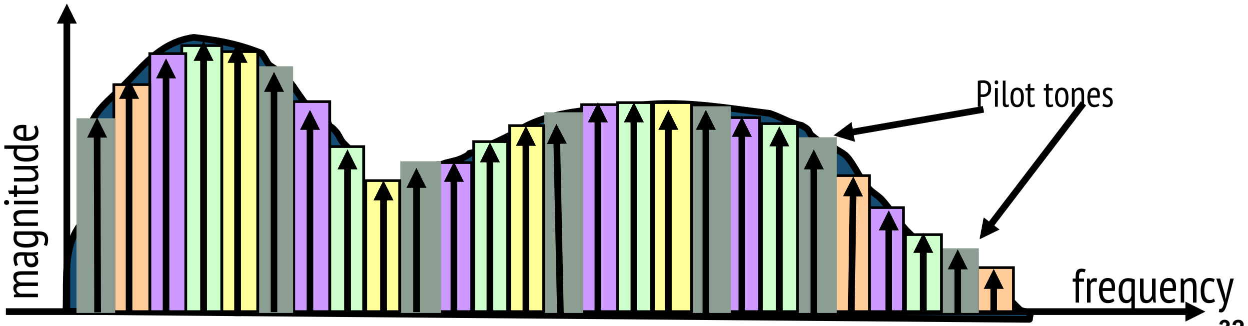
- Received SNR depends on α_i
- α_i varies with time in wireless channels

Frequency Equalization

- The fading α_i is inverted in the receiver
- Received signal is multiplied by $1/\alpha_i$
- Received signal power $\frac{P_i {\alpha_i}^2}{{\alpha_i}^2} = P_i$
- Pros: removes the impact of fading
- Cons: it enhances the noise (incoming noise gets multiplied by $1/\alpha_i$)

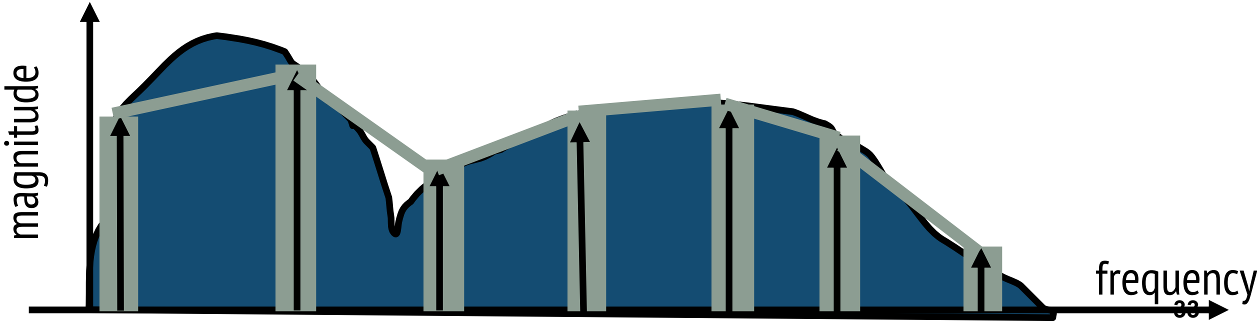
Ideal Channel Estimation

- Wireless channels change frequently ~ 10 ms
- Require frequent channel estimation
- Many systems use pilot tones – known symbols
  - Given s_k, for $k = k_1, k_2, k_3, \ldots$ solve $x_k = \sum_{l=0}^{L} h_l e^{-j2\pi k l/N} s_k$ for h_l
  - Find $H_k = \sum_{l=0}^{L} h_l e^{-j2\pi k l / N}$ (significant computation)
- More pilot tones
  - Better noise resilience
  - Lower throughput



Channel Estimation Via Interpolation

- More efficient approach is interpolation
- Algorithm
  - For each pilot k_i find $H_{ki} = x_{ki} / s_{ki}$
  - Interpolate unknown values using interpolation filter
  - $H_m = a_{m,1} H_{k1} + a_{m,2} H_{k2} + \dots$
- Comments
  - Longer interpolation filter: more computation, timing sensitivity
  - Typical 1dB loss in performance in practical implementation



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Precoding or Pre-equalization

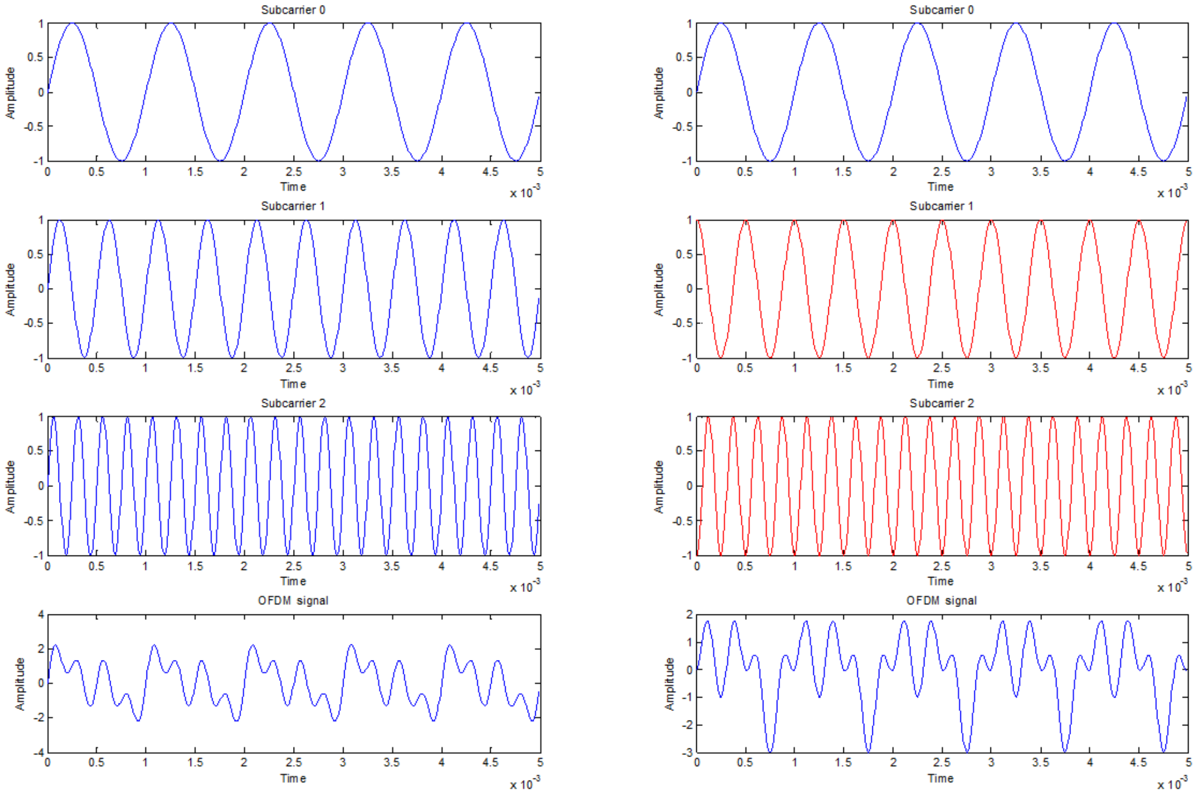
- Opposite of frequency equalization
- If the transmitter have *knowledge* of the subchannel fading α_i
- Transmitter transmits i-th subcarrier signal with power $P_i/{\alpha_i}^2$
- Channel gain α_i
- Received signal power $\frac{P_i {\alpha_i}^2}{{\alpha_i}^2} = P_i$
- Noise power is not multiplied

Adaptive Loading

- vary the data rate and power assigned to each subchannel relative to that subchannel gain
- Variable rate variable power can be assigned to subchannel to receive maximum capacity given a power budget.

Peak to Average Power Ratio (PAPR)

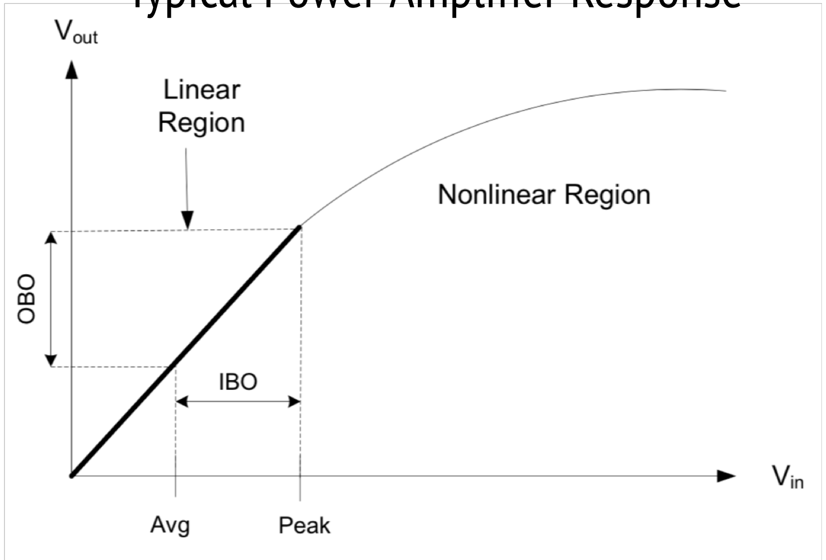
- In time domain, OFDM is a sum of multiple narrowband signals.



High PAPR: Implementation challenges of OFDM

- $PAPR = 10log_{10}\frac{P_{peak}}{P_{avg}}$
- generates out-of-band energy (spectral regrowth)
- in-band distortion (constellation tilting and scattering)

Typical Power Amplifier Response

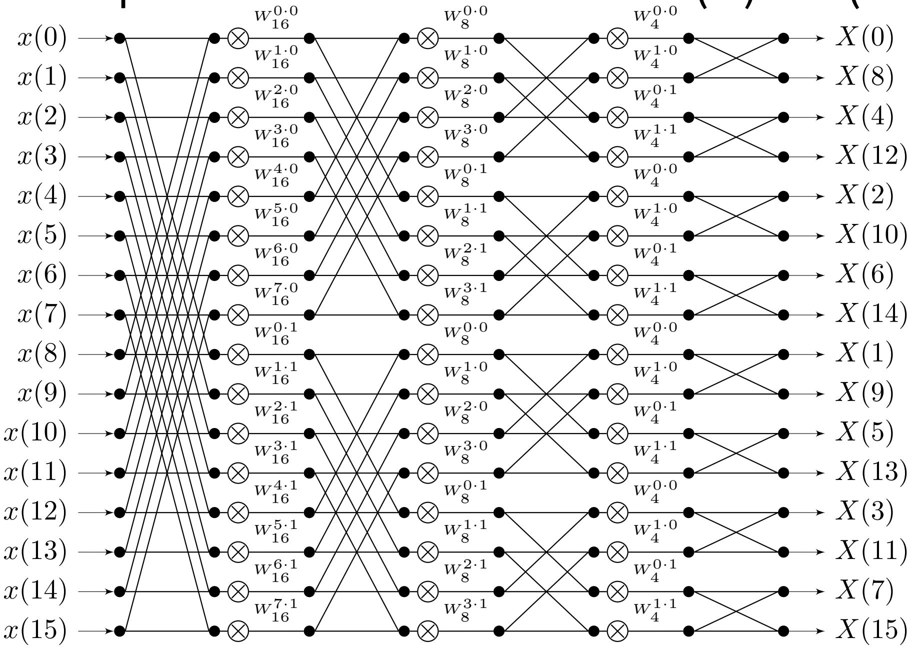


PAPR Reduction Techniques

- clipping and filtering
- selected mapping
- coding techniques

FFT Implementation

- The key to making OFDM realizable in practice is the utilization of the FFT algorithm for computing the DFT and the IFFT algorithm for computing the IDFT, which reduces the number of required multiplications and additions from $\mathcal{O}(L^2)$ to $\mathcal{O}(L \log L)$, which is extremely significant.



Brief History of OFDM

- Although OFDM has become widely used only recently, the concept dates back some 40 years.
  - **1958:** The "Kineplex" system was developed, which was a multicarrier modem for the HF bands (3 to 30MHz). This is widely considered the first ever multicarrier system—it actually used multiple HF radios as the FFT was not re-discovered[9] until 1954.
  - **1966:** Chang shows in the Bell Labs technical journal that multicarrier modulation can solve the multipath problem without reducing data rate. This is generally considered the first theoretical publication on multicarrier modulation, although there were naturally precursory studies, including Holsinger's 1964 MIT dissertation and some of Gallager's work on waterfilling.

Brief History of OFDM

- **1971:** Weinstein and Ebert show that multicarrier modulation can be accomplished using a "Discrete Fourier Transform" (DFT).
- **1985:** Cimini at Bell Labs identifies many of the key issues in OFDM transmission and does a proof of concept design.
- **1993:** DSL adopts OFDM, also called "Discrete Multitone," following successful field trials/competitions at Bellcore vs. equalizer-based systems.
- **1999:** IEEE 802.11 committee on wireless LANs releases 802.11a standard for OFDM operation in 5GHz UNI band.
- **2002:** IEEE 802.16 committee releases OFDM-based standard for wireless broadband access for metropolitan area networks under revision 802.16a.

Brief History of OFDM

- **2003:** IEEE 802.11 committee releases 802.11g standard for operation in the 2.4GHz band.
- **2003:** The “multiband OFDM” standard for ultrawideband is developed, showing OFDM’s usefulness in low-SNR systems.
- **2005:** 802.16e standard is ratified, supporting mobile OFDMA for WiMAX.
- **2006:** First commercial LTE demonstrations by Siemens (now Nokia Siemens Networks).
- **2008:** Qualcomm, the primary backer of Ultramobile Broadband (UMB), the main future competition to LTE and WiMAX and also OFDM/OFDMA-based, announces it will end UMB development and transition to LTE, solidifying LTE as the leading beyond 3G cellular standard.

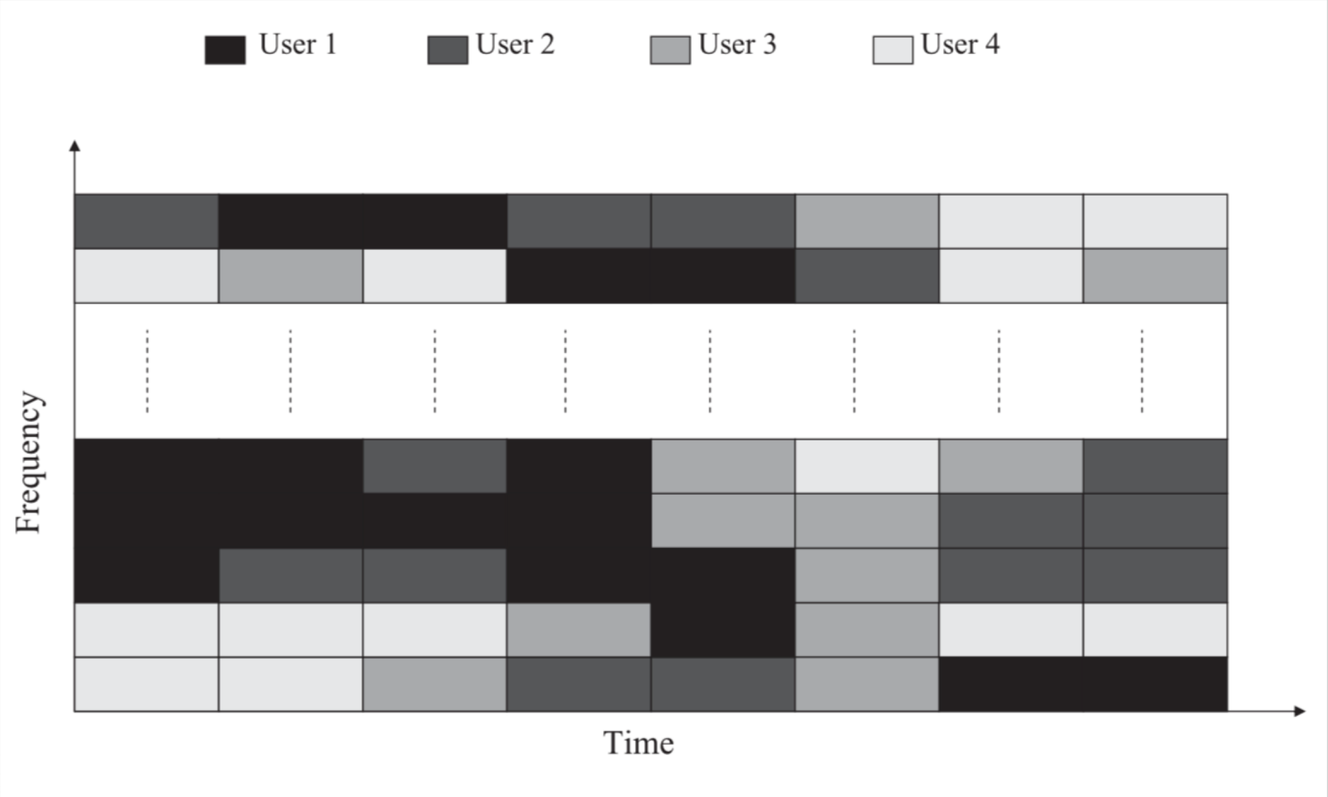
Brief History of OFDM

- **2009:** 3GPP Release 8 LTE/SAE specifications completed and released.
- **2009:** 802.11n standard is ratified, which performs MIMO-OFDM for wireless LANs for peak data rates of 600 Mbps.

S. B. Weinstein, "The history of orthogonal frequency-division multiplexing [History of Communications]," in IEEE Communications Magazine, vol. 47, no. 11, November 2009.

OFDMA

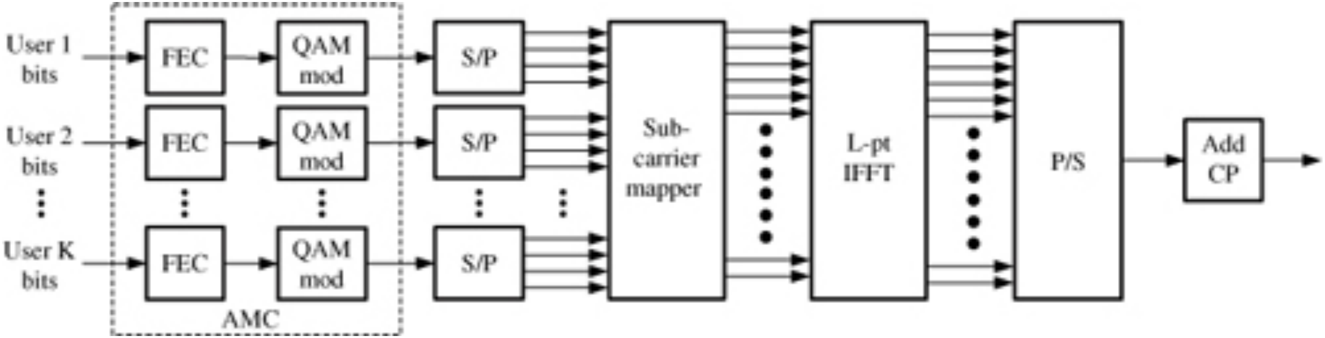
- Multiuser communication using OFDM in downlink LTE/5G

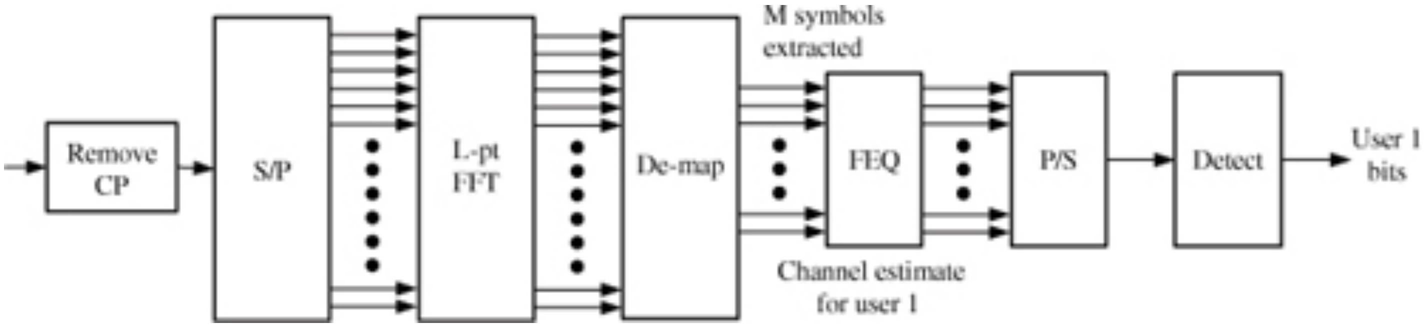


OFDMA

- Resource (OFDM subcarriers) can be allocated based on the application, data rate and QoS requirements
- Allocate subcarriers based on user channel fading
  - Requires user feedback
- Subcarriers are modulated at different rates based on received SNR at each UE

OFDMA Tx and Rx for Downlink



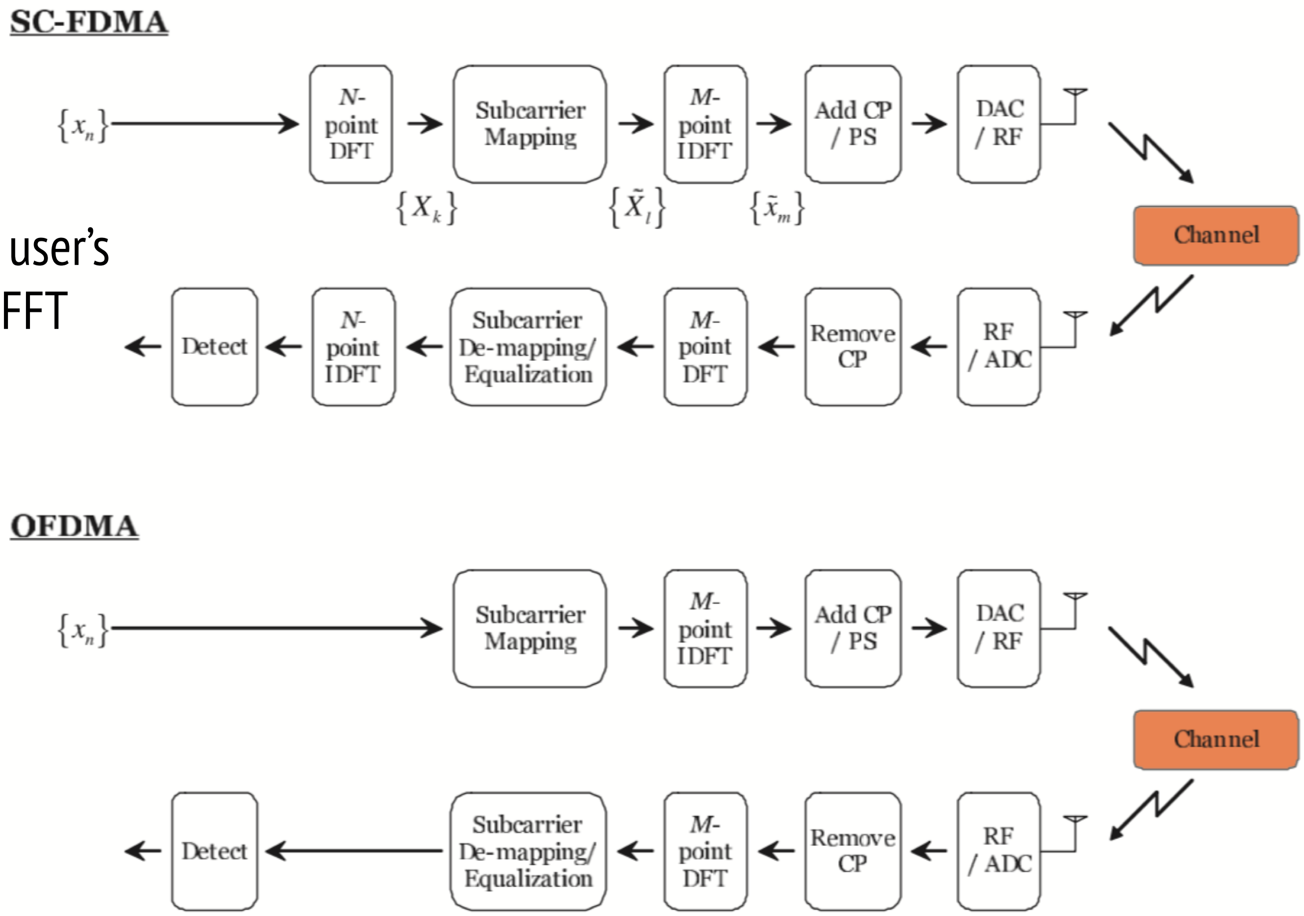


OFDMA unsuitable for uplink

- Uplink is naturally *asynchronous* - inevitable time/frequency offsets from different UEs that transmit simultaneously
- OFDMA: PAPR is a significant issue
- SC-FDMA (Single-Carrier Frequency Division Multiple Access) is used for uplink
- Often called as DFT-coded OFDM
- Significantly lower PAPR than OFDMA

SC-FDMA

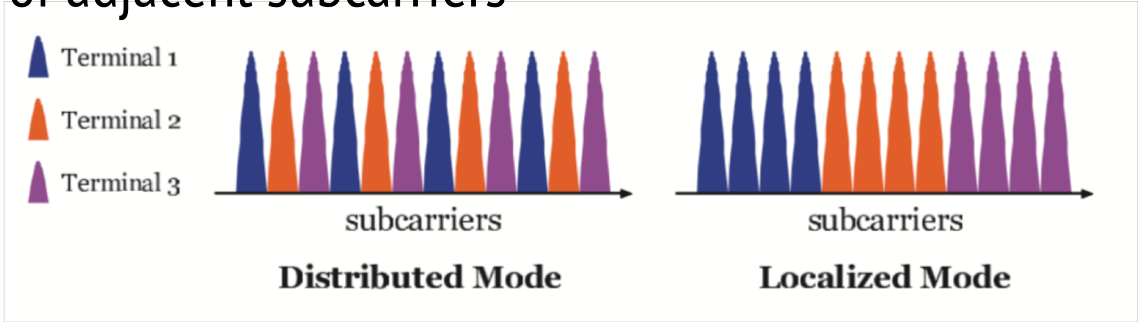
In SC-FDMA, frequency domain equalization is applied to each user’s signal independently after the FFT



* CP: Cyclic Prefix, PS: Pulse Shaping

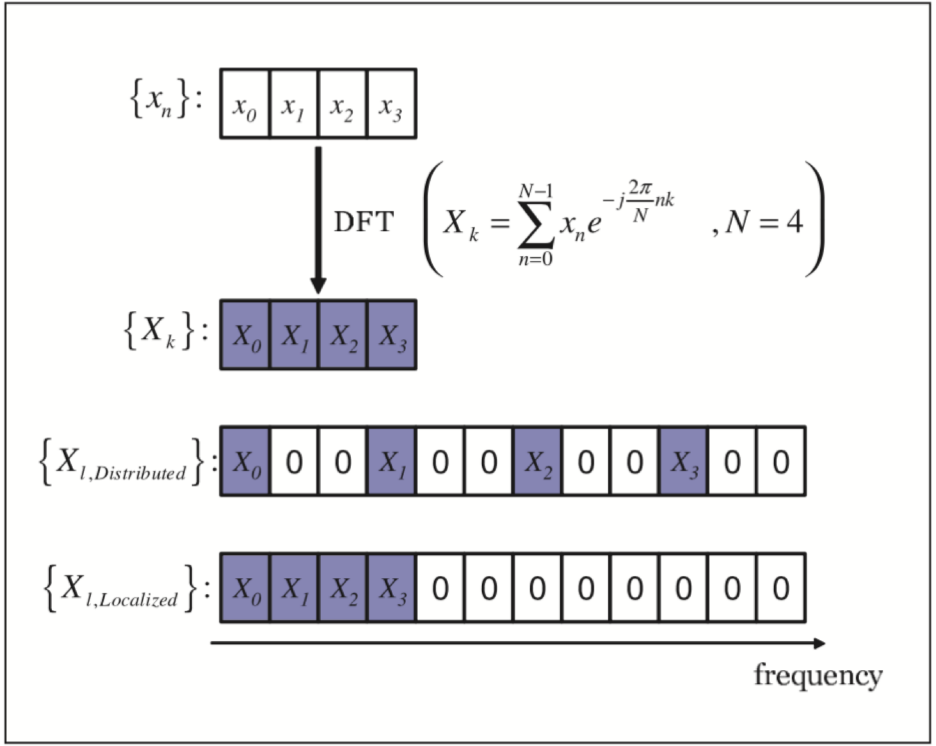
Modes of SC-OFDMA

- Interleaved SC-FDMA (IFDMA)
  - Subcarriers are equidistantly distributed
- Localized SC-FDMA (LFDMA)
  - Set of adjacent subcarriers

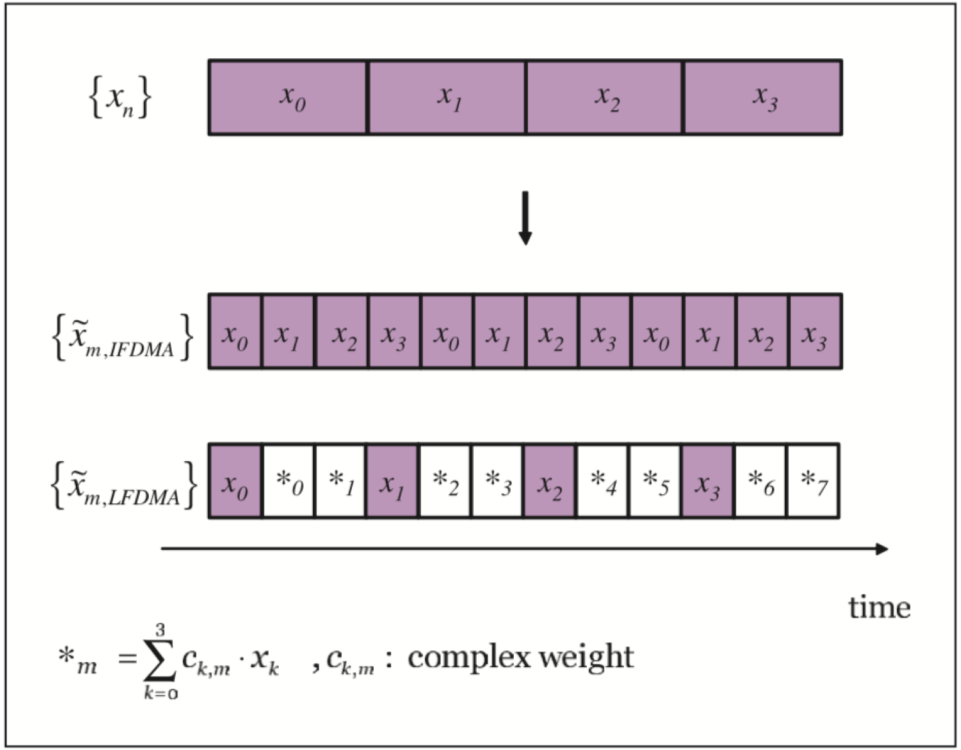


- IFDMA is less prone to transmission errors, channel dependent scheduling of subcarriers can be done.

Time/Frequency Representation of SC-FDMA



$$\left(X_k = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}nk} \quad , N = 4 \right)$$



$$*_m = \sum_{k=0}^{3} c_{k,m} \cdot x_k \quad , c_{k,m} : \text{complex weight}$$

Wireless Channel

- In discrete time, it is represented as a tap delay line

$$h[k,t] = h_0\delta[k,t] + h_1\delta[k-1,t] + \cdots + h_v\delta[k-v,t]$$

- $(v+1)$ channel taps
- Channel is sampled at $f_s = 1/T, T$ is symbol duration
- If channel is static over $(v+1)T$ seconds, output is

$$y[k,t] = \sum_{j=-\infty}^{\infty} h[j,t]x[k-j] = h[k,t] * x[k], *\text{ denotes convolution}$$

- In vector form, channel can be represented as a time-varying $(v+1)\times 1$ column vector

$$\mathbf{h}(\mathbf{t}) = [h_0(t)h_1(t) \ldots h_v(t)]^T$$

Key Attributes of Channel

- What is the value for the total received power? In other words, what are the relative values of the h_i terms?
    - A number of different effects cause the received power to vary over long (*path loss*), medium (*shadowing*), and short (*fading*) distances.
- How quickly does the channel change with the parameter t?
    - The *channel coherence time* specifies the period of time over which the channel's value is correlated. The coherence time depends on how fast the transmitter and receiver are moving relative to each other.
- What is the approximate value of the channel duration v?
    - This value is known as the *delay spread*, and is measured or approximated based on the propagation distance and environment.

Free Space Path Loss

- Free space loss, ideal isotropic antenna

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{\lambda^2} = \frac{(4\pi f d)^2}{c^2}$$

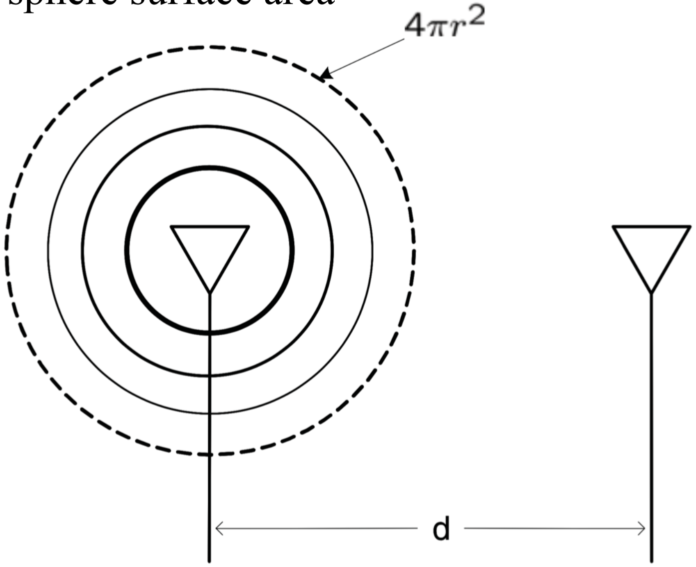
  - ○ P_t = signal power at transmitting antenna
  - ○ P_r = signal power at receiving antenna
  - ○ λ = carrier wavelength
  - ○ d = propagation distance between antennas
  - ○ c = speed of light (3×10^8 m/s)

  where d and λ are in the same units (e.g., meters)

- With antenna gains

$$P_r = P_t \frac{\lambda^2 G_t G_r}{(4\pi d)^2}$$

energy received at an antenna distance d away is inversely proportional to the sphere surface area



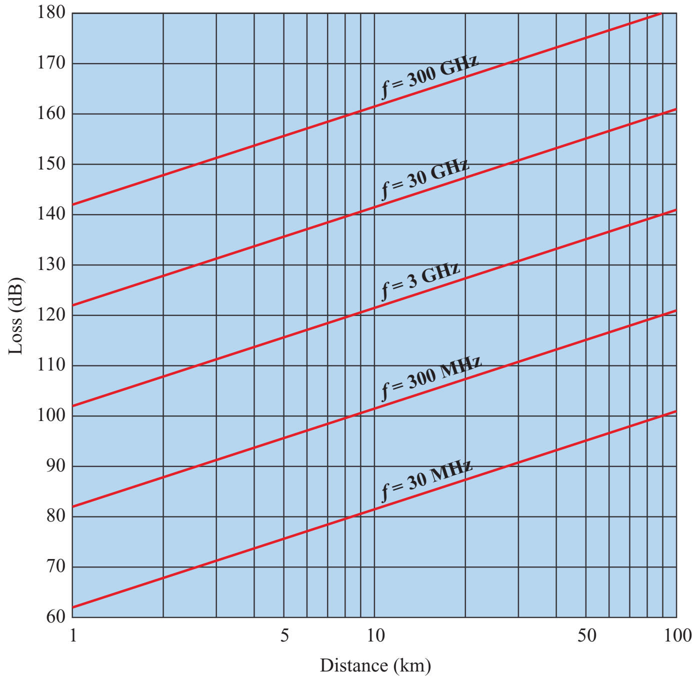
Free Space Loss

- Free space loss equation can be recast:

$$L_{dB} = 10\log\frac{P_t}{P_r} = 20\log\left(\frac{4\pi d}{\lambda}\right)$$

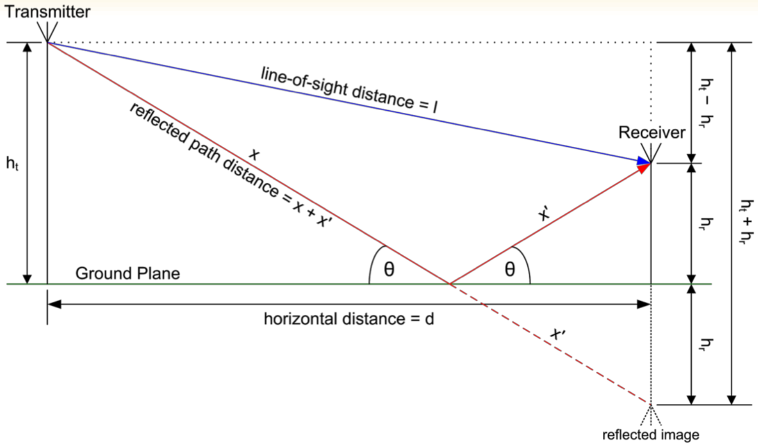
$$= -20\log(\lambda) + 20\log(d) + 21.98\,\text{dB}$$

$$= 20\log\left(\frac{4\pi f d}{c}\right) = 20\log(f) + 20\log(d) - 147.56\,\text{dB}$$



Two Ray Ground Reflection

$$P_r = P_t \frac{G_t G_r h_t^2 h_r^2}{d^4}$$



- Empirical Pathloss Formula

$$P_r = P_t P_o \left(\frac{d_o}{d}\right)^{\alpha}$$

$\alpha = Pathloss\ exponent$
$d_0 = 1m$
$P_0 = Received\ power\ at\ d_0$

Path Loss Exponent in practical systems

Table 6.5 Path Loss Exponents for Different Environments [RAPP02]

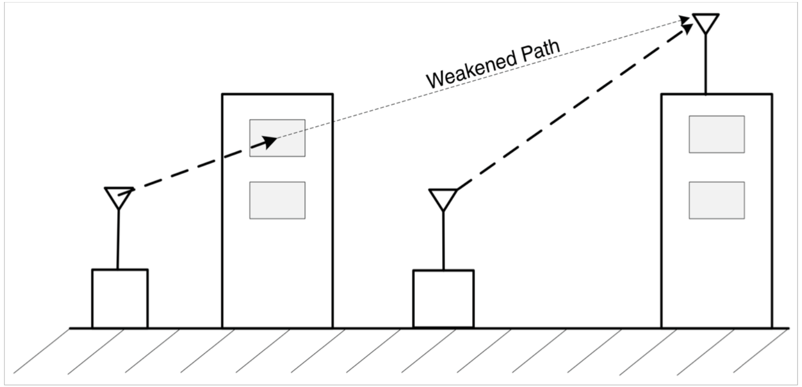
Environment	**Path Loss Exponent, *n***
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Shadowing

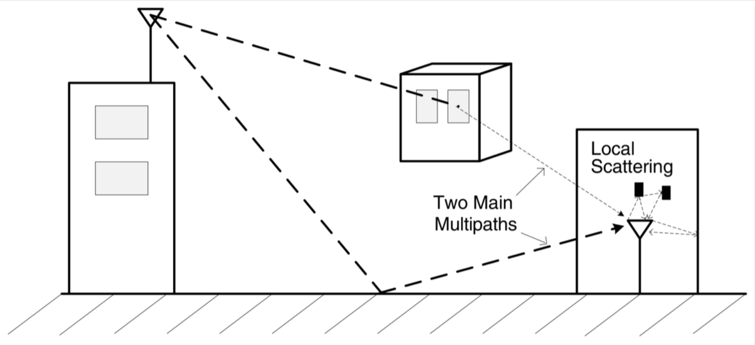
- Trees and buildings may be located between the transmitter and the receiver and cause degradation in received signal strength
- Shadowing is a random process

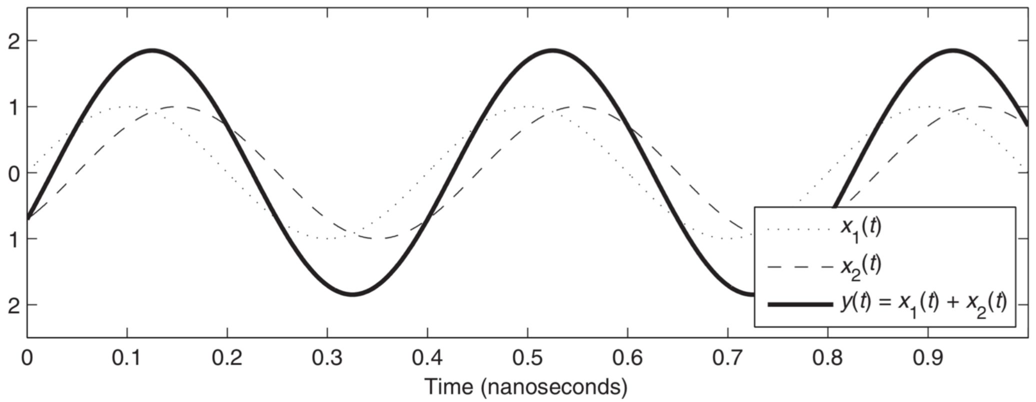
$$P_r = P_t P_o \chi \left(\frac{d_o}{d}\right)^{\alpha}$$

$$\chi = 10^{x/10}, \text{where } x \sim N(0, \sigma_s^2)$$

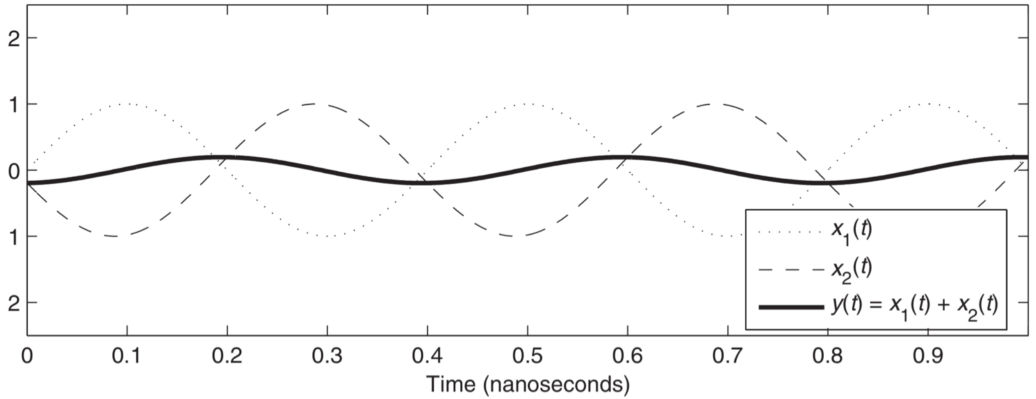


Fading





(a)

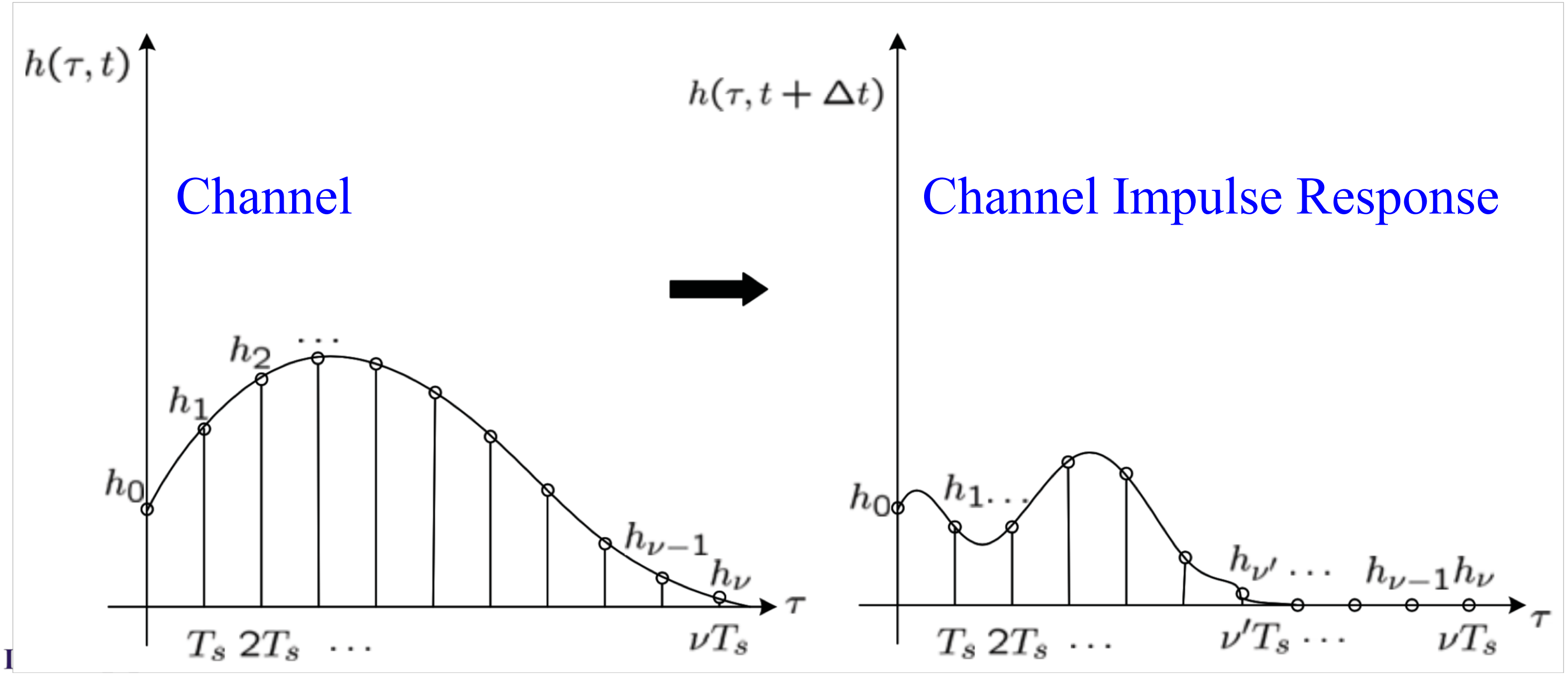


(b)

- Multipath
- Local Scattering
- Constructive & Destructive Interference

Channel Impulse Response

- The channel is time varying, so the channel impulse response is also a function of time and can be quite different at time $t + \Delta t$ than it was at time t



Doppler Spread

- Doppler power spectrum is caused by *motion* between the transmitter and receiver
- *Doppler power spectrum* gives the statistical power distribution of the channel versus frequency for a signal transmitted at one exact frequency
- Doppler spread is

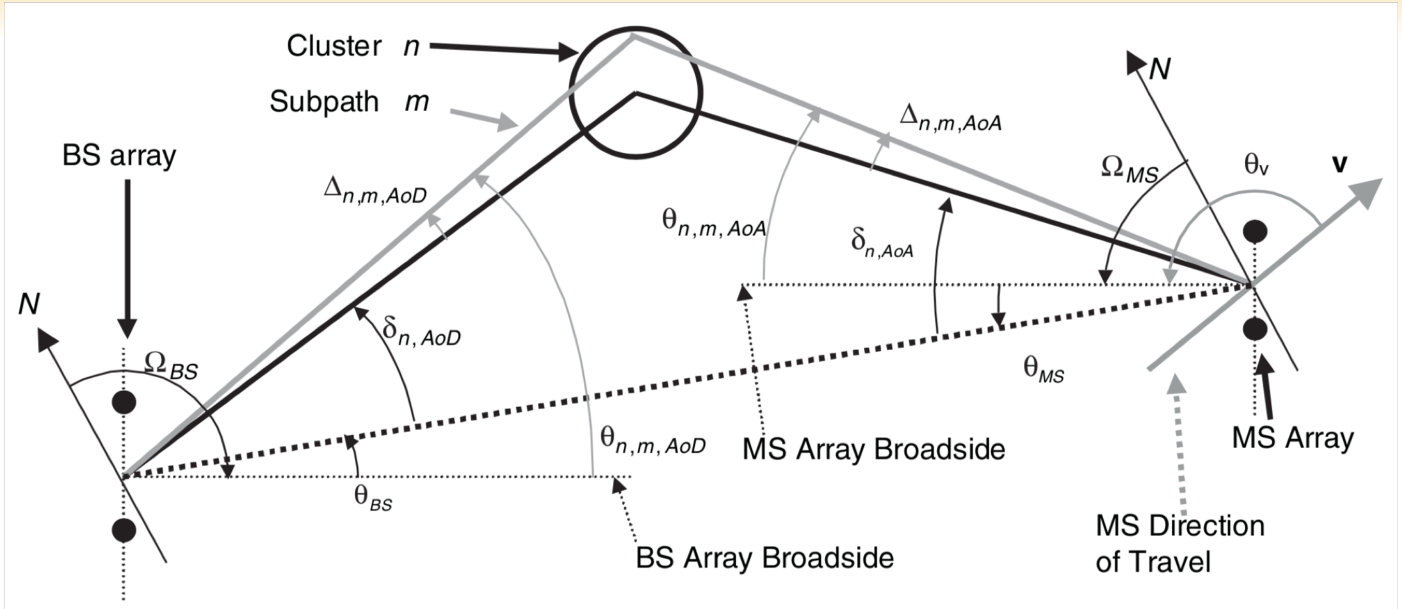
$$f_D = \frac{v f_c}{c}$$

where v is the maximum speed between the transmitter and the receiver, f_C is the carrier frequency, and c is the speed of light

- Doppler varies with f_c. If communication bandwidth $B \ll f_c$, f_D can be treated as approximately constant.
- The coherence time and Doppler spread are also inversely related

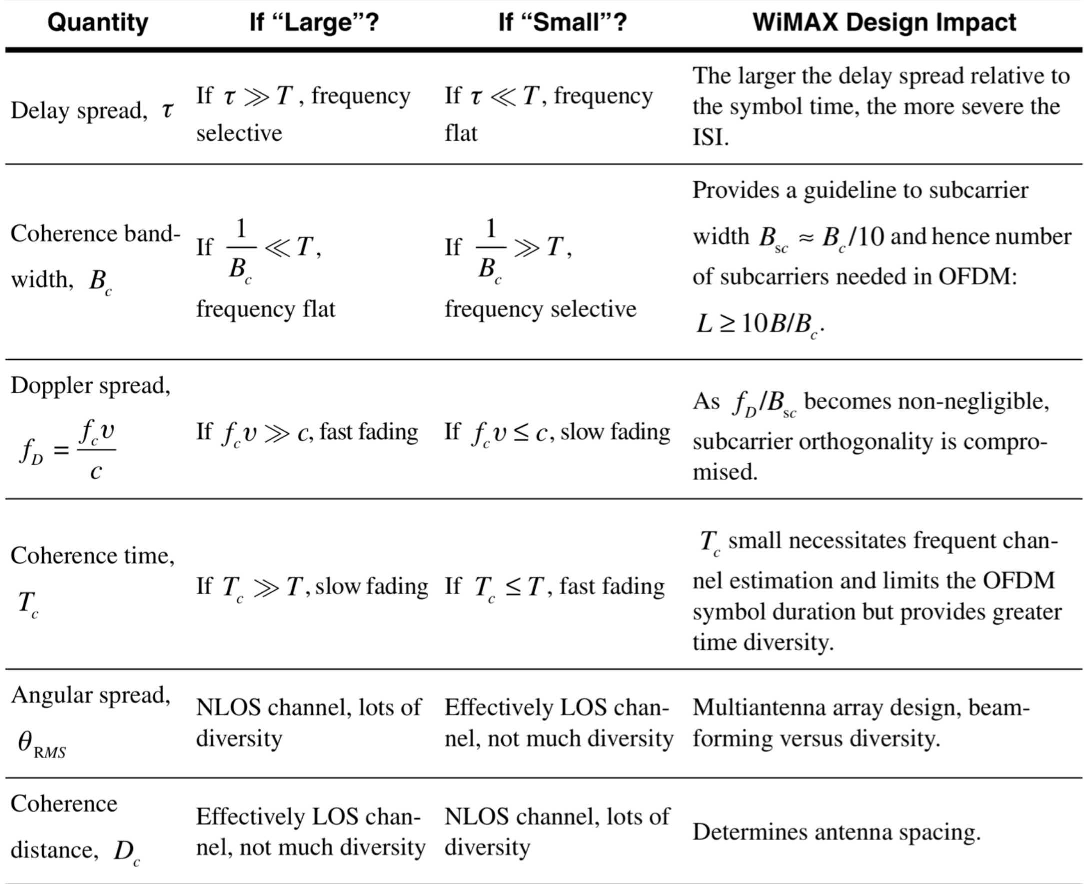
$$T_c \approx \frac{1}{f_D}$$

3GPP Channel Model



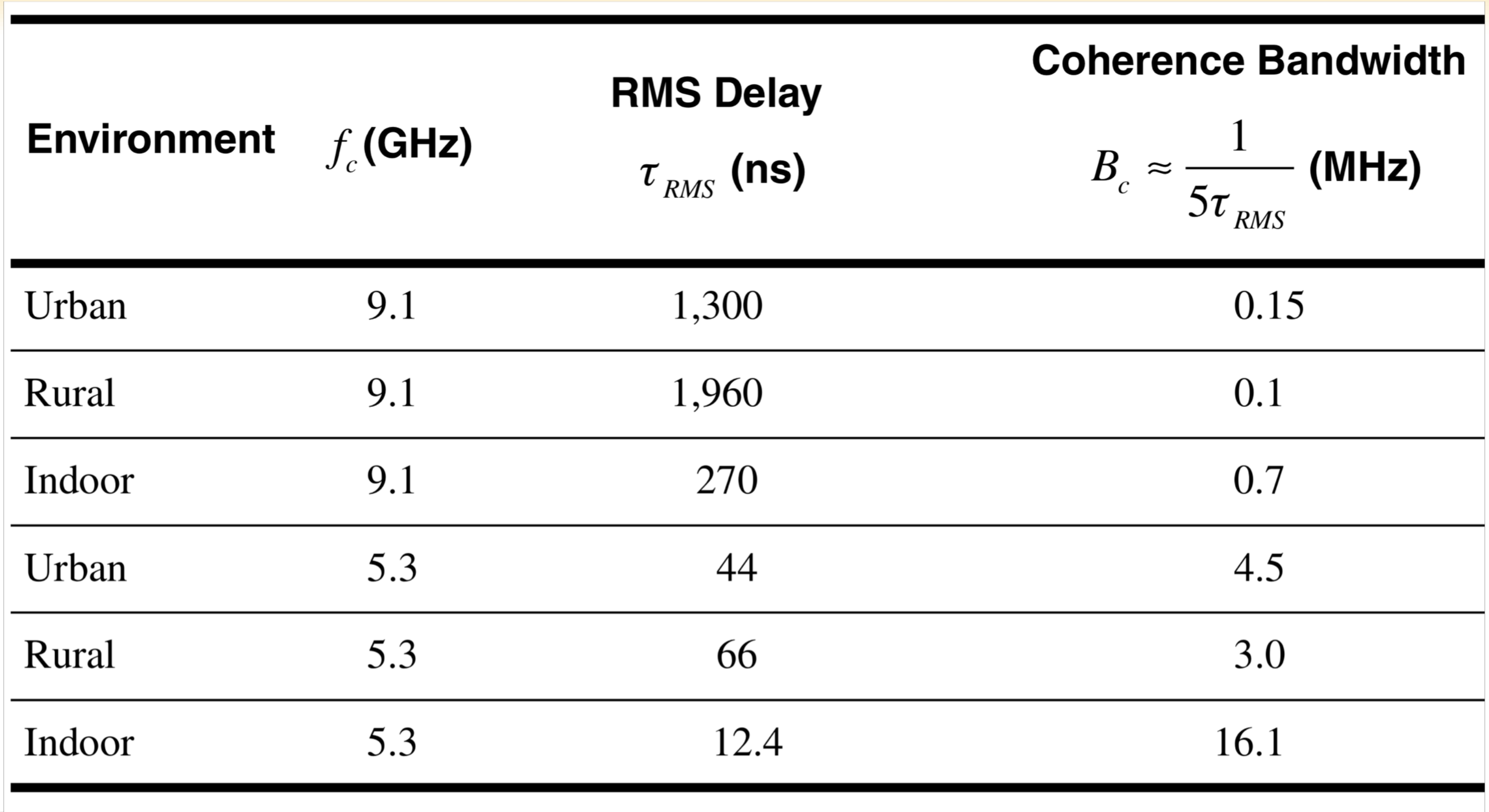
$$h_{u,s,n}(t) = \sqrt{\frac{P_n \sigma_s}{M}} \sum_{m=1}^{M} \left(\begin{array}{l} \sqrt{G_{BS}\left(\theta_{n,m,AoD}\right)} \exp\left(j\left[kd_s \sin\left(\theta_{n,m,AoD} + \Phi_{n,m}\right)\right]\right) \times \\ \sqrt{G_{BS}\left(\theta_{n,m,AoA}\right)} \exp\left(jkd_u \sin\left(\theta_{n,m,AoA}\right)\right) \times \\ \exp\left(jk \, \| \mathbf{v} \| \cos\left(\theta_{n,m,AoA} - \theta_v\right) t\right) \end{array} \right)$$

Channel Parameters



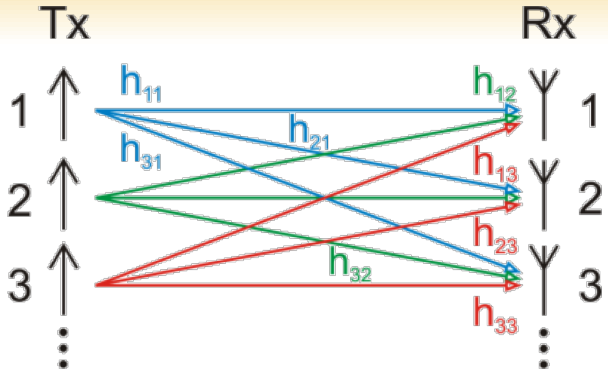
Quantity	If "Large"?	If "Small"?	WiMAX Design Impact
Delay spread, τ	If $\tau \gg T$, frequency selective	If $\tau \ll T$, frequency flat	The larger the delay spread relative to the symbol time, the more severe the ISI.
Coherence bandwidth, B_c	If $\frac{1}{B_c} \ll T$, frequency flat	If $\frac{1}{B_c} \gg T$, frequency selective	Provides a guideline to subcarrier width $B_{sc} \approx B_c/10$ and hence number of subcarriers needed in OFDM: $L \geq 10B/B_c$.
Doppler spread, $f_D = \frac{f_c v}{c}$	If $f_c v \gg c$, fast fading	If $f_c v \leq c$, slow fading	As f_D/B_{sc} becomes non-negligible, subcarrier orthogonality is compromised.
Coherence time, T_c	If $T_c \gg T$, slow fading	If $T_c \leq T$, fast fading	T_c small necessitates frequent channel estimation and limits the OFDM symbol duration but provides greater time diversity.
Angular spread, θ_{RMS}	NLOS channel, lots of diversity	Effectively LOS channel, not much diversity	Multiantenna array design, beamforming versus diversity.
Coherence distance, D_c	Effectively LOS channel, not much diversity	NLOS channel, lots of diversity	Determines antenna spacing.

RMS Delay Spread



Environment	f_c (GHz)	RMS Delay τ_{RMS} (ns)	Coherence Bandwidth $B_c \approx \frac{1}{5\tau_{RMS}}$ (MHz)
Urban	9.1	1,300	0.15
Rural	9.1	1,960	0.1
Indoor	9.1	270	0.7
Urban	5.3	44	4.5
Rural	5.3	66	3.0
Indoor	5.3	12.4	16.1

Categories of Multiple Antenna Tx & Rx



- Spatial Diversity
  - a number of different versions of the signal to be Tx/Rx
  - provides resilience against fading
- Interference suppression
  - uses the spatial dimensions to reject interference from other users
  - through the physical antenna gain pattern or through other forms of array processing such as linear precoding, postcoding, or interference cancellation
- Spatial multiplexing
  - allows multiple independent streams of data to be sent simultaneously in the same bandwidth, and hence is useful primarily for increasing the data rate

Spatial Diversity – Array Gain

- Coherently combines energy of each antenna (channels can be correlated if LOS and closely spaced antenna)
- Noise is uncorrelated and do not add coherently
- In correlated flat fading channel, received SNR increases linearly with the number of receive antennas, N_r

$y_i = h_i x + n_i = hx + n_i, h$ is correlated flat fading channel

SNR at antenna i is $\gamma_i = |h^2|/\sigma^2$

Resulting Signal from all antennas $y = \sum_{i=1}^{N_r} y_i = N_r hx + \sum_{i=1}^{N_r} n_i$

Combined SNR is $\gamma = {}^{|N_r h|^2}/_{N_r \sigma^2} = \frac{N_r |h^2|}{\sigma^2} = N_r \gamma_i$

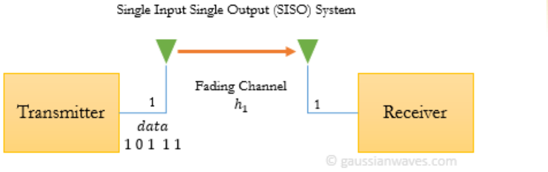
Spatial Diversity – Diversity Gain

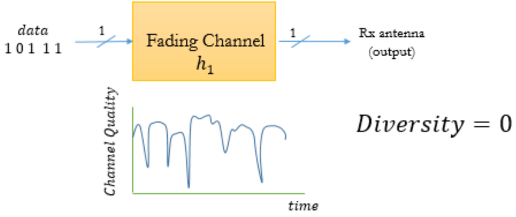
- Channel varies over space
- rms angular spread of a channel = θ_{rms}= statistical distribution of the angle of the arriving energy
- Dual of angular spread is coherence distance, D_c
- A coherence distance of *d* means that any physical positions separated by *d* have an essentially uncorrelated received signal amplitude and phase
- $D_C \approx .2\lambda/\theta_{rms}$, in Rayleigh fading, $D_C \approx 9\lambda/16\pi$
- coherence distance increases with the carrier wavelength λ, so higher-frequency systems have shorter coherence distances

Spatial Diversity – Diversity Gain

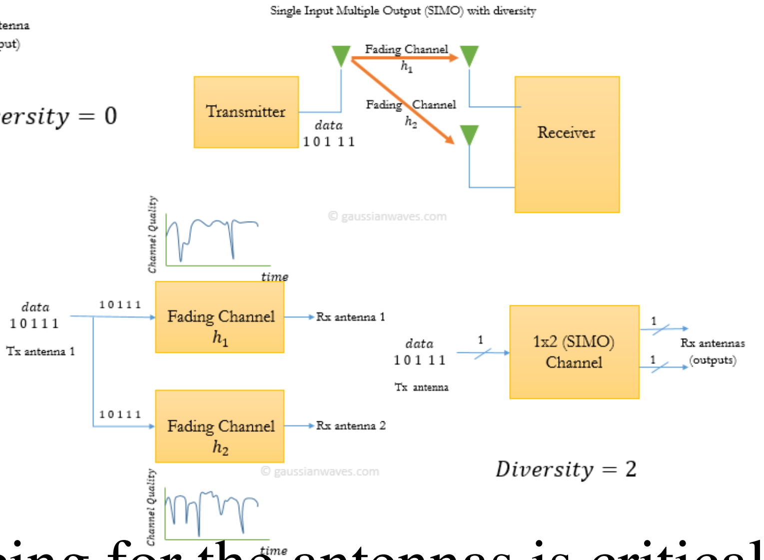
- If N_t transmit antennas and N_r receive antennas that are sufficiently spaced are added to the system
- the *diversity order* is $N_d = N_r N_t$
- N_d is the number of uncorrelated channel paths between the transmitter and receiver
- probability of all the N_d uncorrelated channels having low SNR is very small
- bit error probability improves dramatically

Spatial Diversity – Diversity Gain

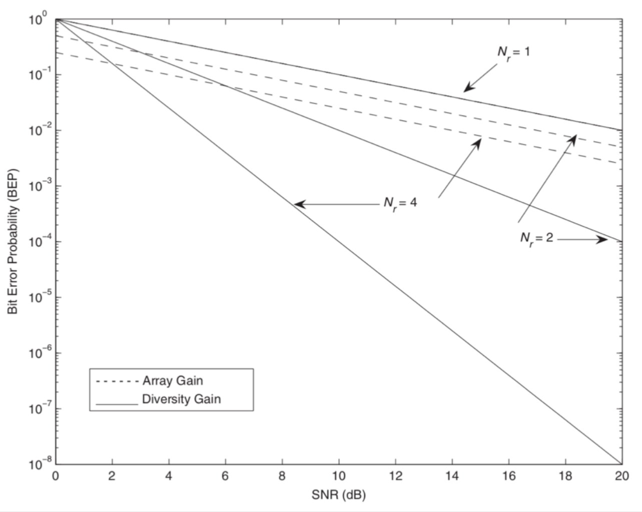

Single Input Single Output (SISO) System

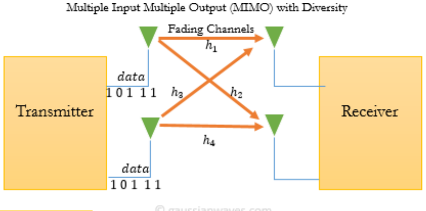


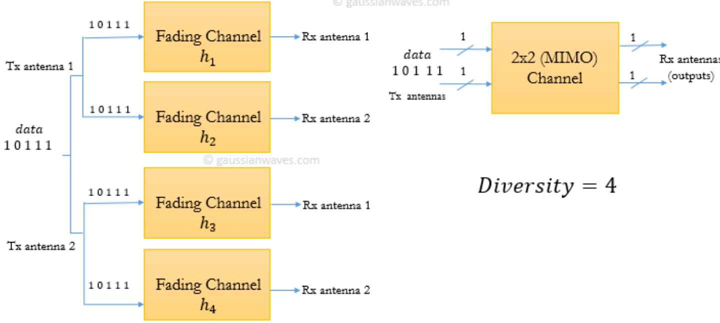
Diversity = 0


Single Input Multiple Output (SIMO) with diversity

Diversity = 2




Multiple Input Multiple Output (MIMO) with Diversity



Diversity = 4

Sufficient spacing for the antennas is critical for increasing the system reliability

Benefits of Spatial Diversity

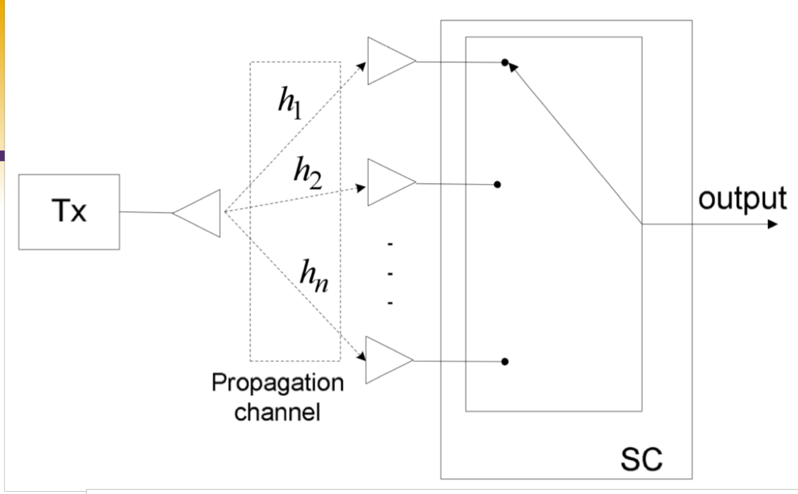
- Increased data rate
  - Antenna diversity increases SNR linearly
  - Receiver techniques increase capacity logarithmically wrt #antennas
  - data rate benefit rapidly diminishes as antennas are added
  - Multiple independent streams increase aggregate data rate
- Increased coverage or reduced Tx power
  - With only array gain, increase in SNR is $N_r \gamma_i$
  - Increase in SNR increases coverage range
  - transmit power can be reduced by $10 log_{10} N_r dB$

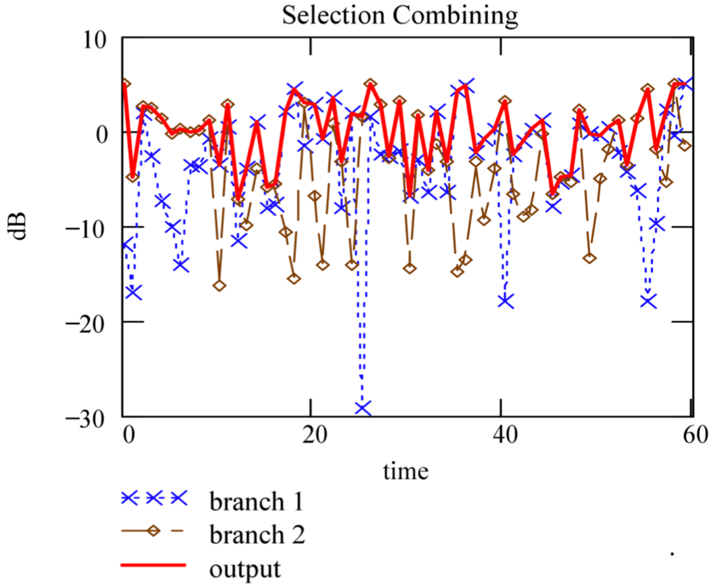
Receive Diversity

- Receive multiple streams and combine them
  - Selection Combining
  - Maximal Ratio Combining
  - Equal Gain Combining
  - Hybrid Combining

Selection Combining

- estimates the instantaneous strengths of each of the N_r streams and selects the highest one
- Since it ignores the useful energy on the other streams, SC is suboptimal
- Its simplicity and reduced hardware requirements make it attractive in many cases





$$
\begin{aligned}
\bar{\gamma}_{sc} &= \bar{\gamma}\sum_{i=1}^{N_r}\frac{1}{i}, \\
&= \bar{\gamma}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N_r}\right)
\end{aligned}
$$

Maximal Ratio Combining

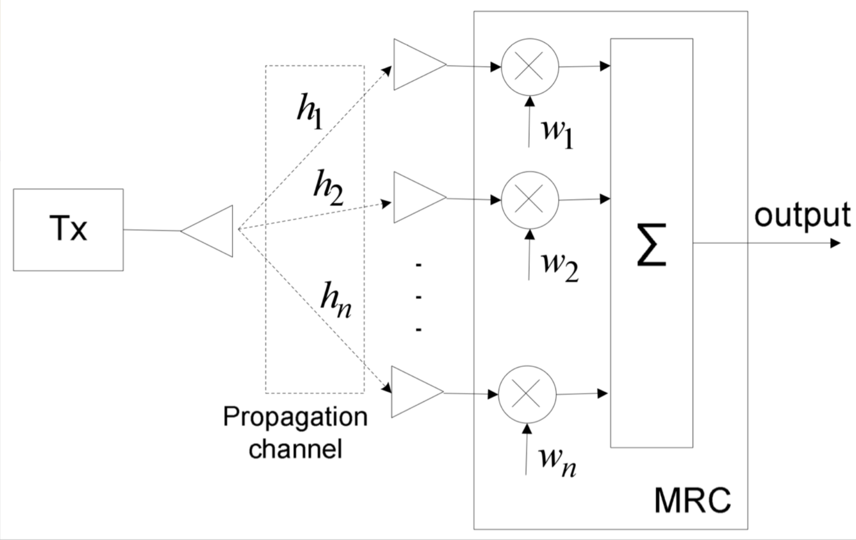
- use linear coherent combining of branch signals so that the output SNR is maximized
- Individual branch signal:

$$x_n = A \cdot h_n + \xi_n$$

- Output of the combiner:

$$x_{out} = \sum_{n=1}^{N} w_n x_n = \underbrace{A \sum_n w_n h_n}_{signal} + \underbrace{\sum_n w_n \xi_n}_{noise}$$

- coherent technique, i.e., signal's phase has to be estimated



$$\gamma_{MRC} = \frac{\varepsilon_x \sum_{i=1}^{N_r} |h_i|^2}{\sigma^2} = \sum_{i=1}^{N_r} \gamma_i$$

- Best performance
- Lot of circuitry for individual receivers

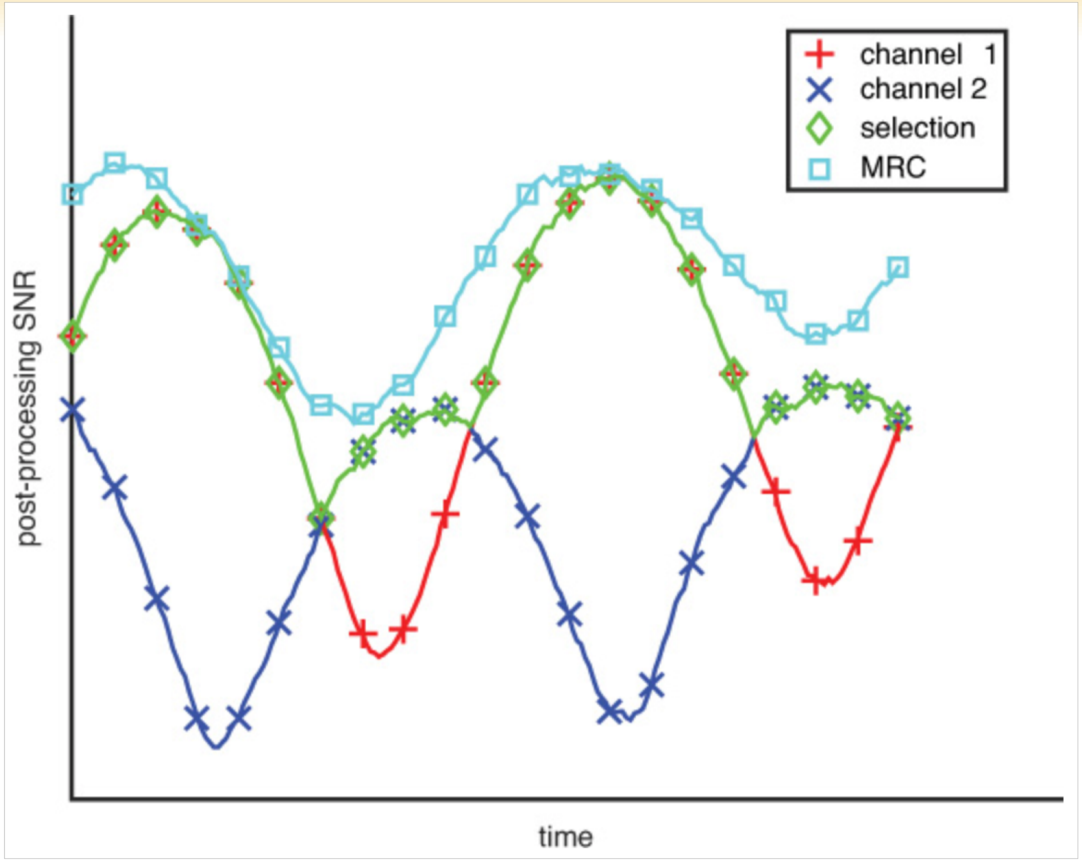
Equal Gain Combining

- corrects only the phase
- Simpler than MRC, easier to implement

$$\gamma_{\mathrm{EGC}} = \frac{\varepsilon_x \sum_{i=1}^{N_r} |h_i|^2}{N_r \sigma^2}$$

- Hybrid Combining
  - Combination of multiple of combining techniques

Comparing Receiver Diversity

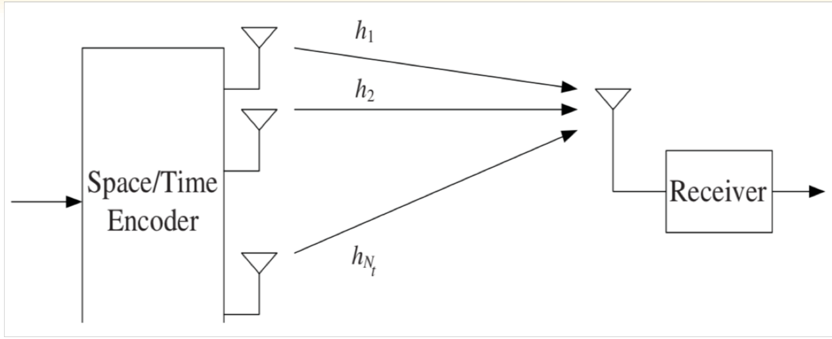


Transmit Diversity

- signals sent from different transmit antennas interfere with one another
- processing is required at both the transmitter and the receiver
- goal is to achieve diversity while removing or attenuating the spatial interference
- used for the downlink of infrastructure-based systems
- Mobile stations may not need to use it due to size, power constraints
- Can be open loop or closed loop

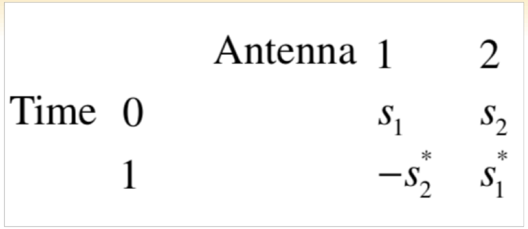
Open Loop Transmit Diversity

- Space Time Block Codes (STBC)
- Alamouti code is a type of STBC
- ease of implementation—linear at both the transmitter and the receiver



Alamouti Code

- If two symbols to be transmitted



	Antenna	1	2
Time	0	s_1	s_2
	1	$-s_2^*$	s_1^*

- Received Signal, (flat fading channel & $h_1\,(t=0) = h_1\,(t=T) = h_1$)

$$\begin{aligned} r(0) &= h_1 s_1 + h_2 s_2 + n(0), \\ r(T) &= -h_1 s_2^* + h_2 s_1^* + n(T), \end{aligned}$$

- Linear diversity Combining (channel known to receiver)

$$\begin{aligned} y_1 &= h_1^* r(0) + h_2 r^*(T), \\ y_2 &= h_2^* r(0) - h_1 r^*(T). \end{aligned}$$

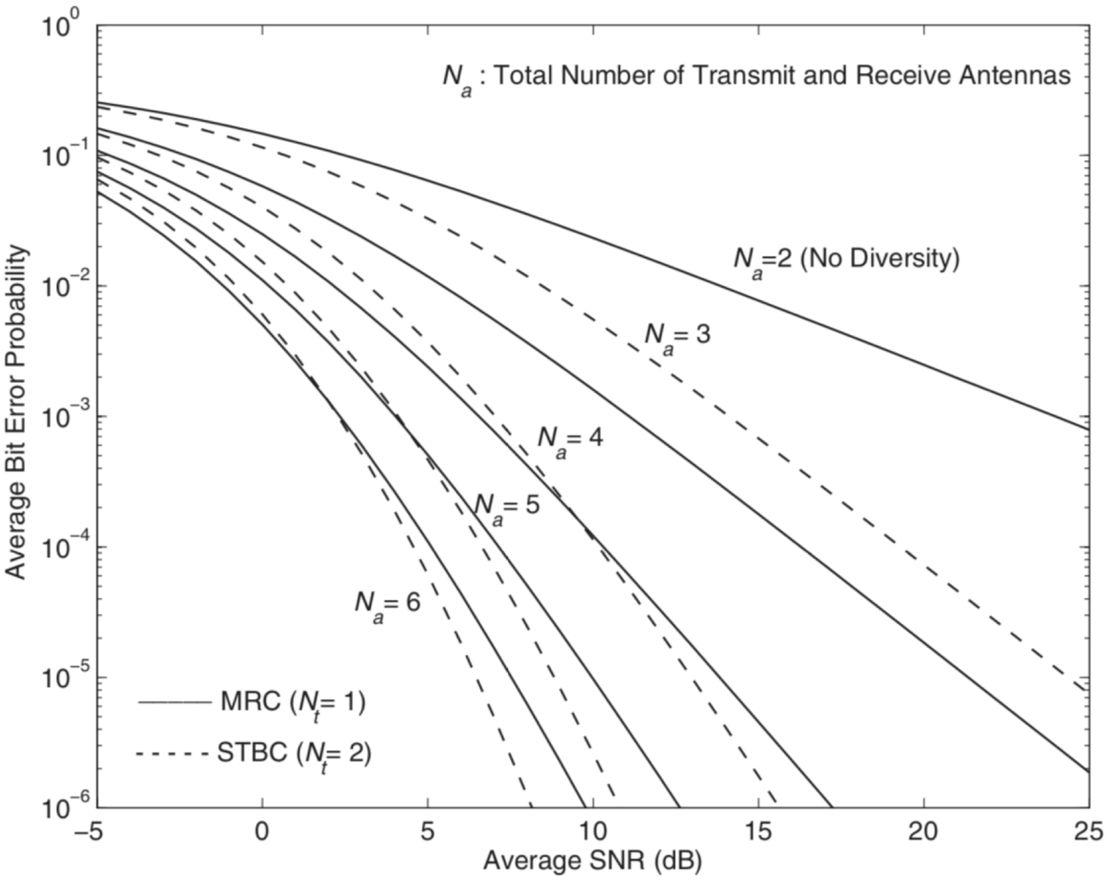
- Eliminates spatial interference

STBC in OFDM

- Owing to the flat-fading assumption, the STBC in an OFDM system is performed in the *frequency domain*, where each subcarrier experiences flat fading
- Space/time trellis codes introduce memory and achieve better performance (about 2dB) than orthogonal STBCs
- Trellis code decoding complexity $O\left(M^{min\{N_t,N_r\}}\right)$
- STBC complexity $O(min\{N_t, N_r\})$

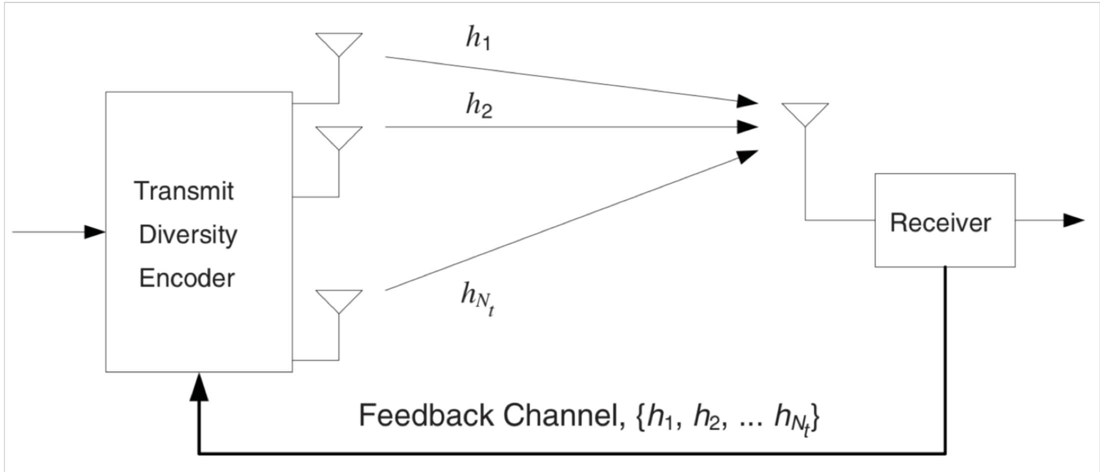
Alamouti STBC vs MRC

- Alamouti STBC outperforms MRC at high SNR owing to the diversity order
- MRC has better BEP performance than Alamouti STBC at low SNR owing to the array gain



Closed loop Transmit Diversity

- Feedback needs to be added to the system
- channel changes quickly in a highly mobile scenario
- closed-loop transmission schemes feasible primarily in fixed or low-mobility scenarios



Transmit Selection Diversity

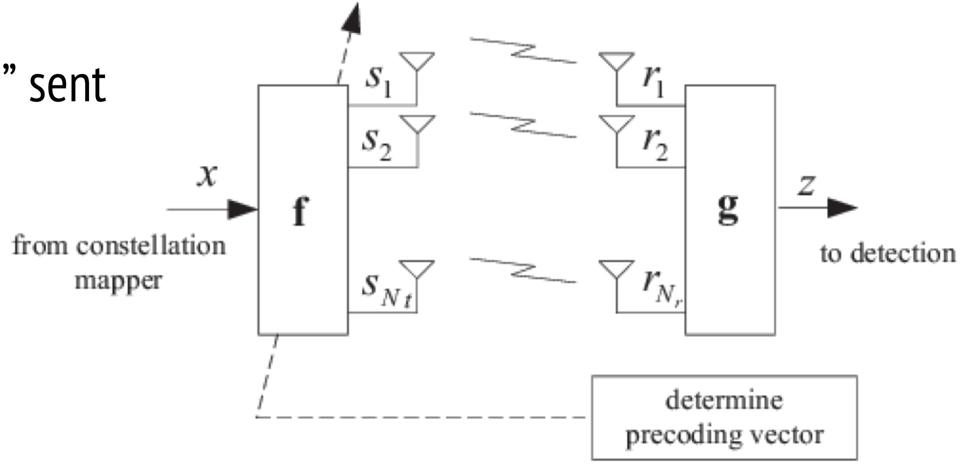
- A subset of all available antennas used
- Subset corresponds to the best channels between the transmitter and the receiver
- Advantages:
    - significantly reduced hardware cost and complexity
    - reduced spatial interference, since fewer transmit signals are sent
    - reaches $N_t N_r$ diversity order, even though only a subset of all antennas are used

Linear Diversity Precoding

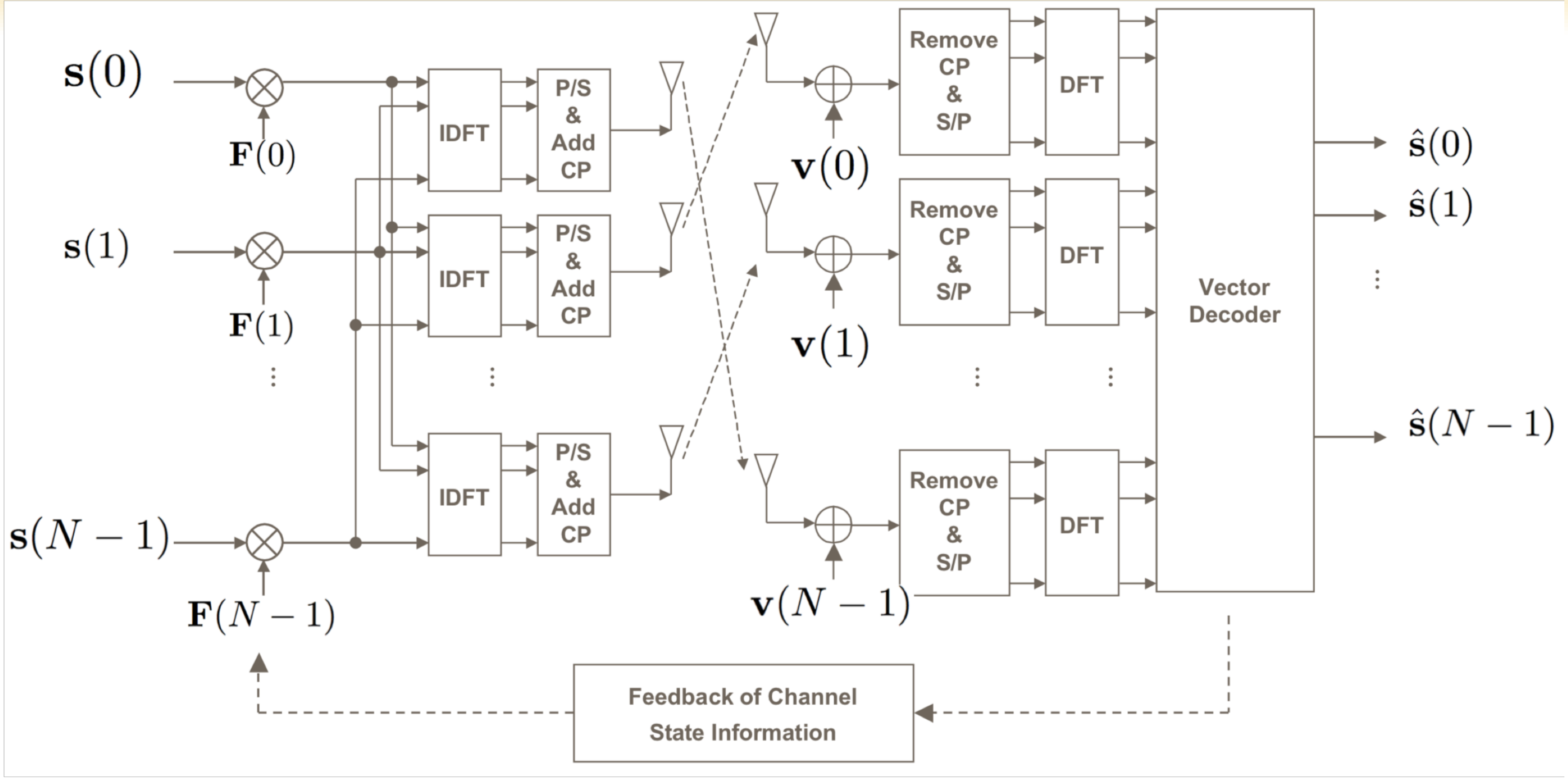
- general technique for improving the data rate by exploiting the CSI at the transmitter
- *diversity* precoding, a special case of linear precoding, where data rate is unchanged
- linear precoder at the transmitter and a linear postcoder at the receiver

Received Data Vector

- $z = Gy = G(HFx + n)$
  - M is the number of spatial data “streams” sent
  - Transmitted vector x is $M \times 1$
  - Received vector y is $N_r \times 1$
  - Postcoder matrix G is $M \times N_r$
  - Channel matrix H is $N_r \times N_t$
  - Precoder matrix F is $N_t \times M$
  - M = 1 is known as maximal ratio transmission (MRT)



Precoding in MIMO OFDM



Interference Cancellation Suppression

- Suppress undesired signals and/or enhance the power of the desired signal
- In MIMO, channel is multidimensional
  - the dimensions of the channel can be applied to null interference in a certain direction, while amplifying signals in another direction
  - Contrast to transmit diversity (statistical diversity of the total signal is increased)
- Types:
  - DOA-Based Beamsteering
  - Linear Interference Suppression: Complete Knowledge of Interference Channels

Beamsteering (Physically steering)

- Electromagnetic waves can be physically steered to create beam patterns at either the transmitter or the receiver
- Static pattern-gain beamsteering : called sectoring
  - Example: in a three-sector cell, a strong beam is projected over 120 degrees, while very little energy is projected over the remaining 240 degrees

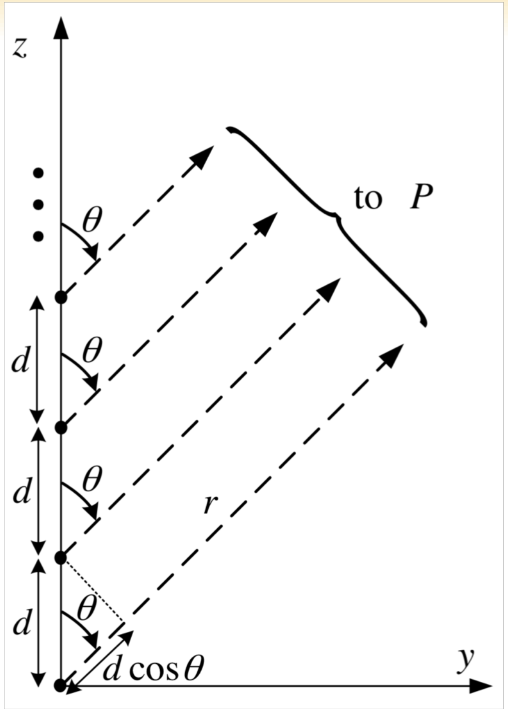
DOA based Beamsteering

- Incoming signal may consist of
  - desired energy + interference energy (other users or multipath)
- Signal processing techniques are used to identify angle of arrival (AoA) of these signals
  - MUSIC, ESPIRIT, JADE, MLE
- These AoAs are used by a beamformer to calculate weighting vector of the antenna elements

Uniform Linear Array

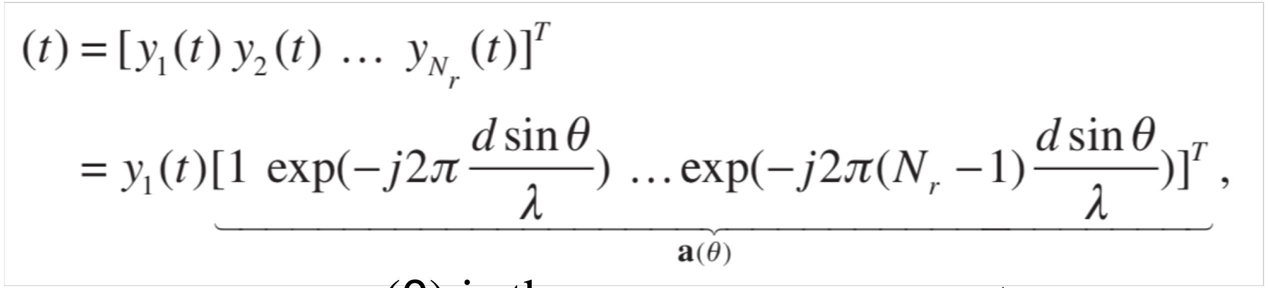
- wave at the first antenna element travels an additional distance of *d* sin θ to arrive at the second element
- difference in propagation distance between the adjacent antenna elements results in arrival-time delay, τ = *d*/*c* sin θ
- signal arriving at the second antenna can be expressed in terms of signal at the first antenna element

$$\begin{aligned} y_2(t) &= y_1(t)\exp(-j2\pi f_c \tau), \\ &= y_1(t)\exp(-j2\pi \frac{d\sin\theta}{\lambda}). \end{aligned}$$



Uniform Linear Array

- For an antenna array with N_r elements all spaced by d, the resulting received signal vector is



$$
\begin{aligned}
(t) &= [y_1(t)\ y_2(t)\ \ldots\ y_{N_r}(t)]^T \\
&= y_1(t)\underbrace{[1\ \exp(-j2\pi\frac{d\sin\theta}{\lambda})\ \ldots \exp(-j2\pi(N_r-1)\frac{d\sin\theta}{\lambda})]^T}_{\mathbf{a}(\theta)},
\end{aligned}
$$

$\mathbf{a}(\theta)$ is the *array response vector*

Weight vector Calculation

- Example:
  - a three-element ULA with $d = \lambda/2$
  - desired signal is received at $\theta_1 = 0$, two interfering signals at $\theta_2 = \pi/3$ and $\theta_3 = -\pi/6$

$$\mathbf{a}(\theta_1) = [1 \;\; 1 \;\; 1]^T, \quad \mathbf{a}(\theta_2) = \left[1 \;\; e^{-j\frac{\sqrt{3}}{2}\pi} \;\; e^{-j\sqrt{3}\pi}\right]^T, \quad and \quad \mathbf{a}(\theta_3) = \left[1 \;\; e^{j\frac{\pi}{2}} \;\; e^{j\pi}\right]^T$$

- Objective:
  - The beamforming weight vector $\mathbf{w} = [w_1 \;\; w_2 \;\; w_3]^T$ should increase the antenna gain in the direction of the desired user while minimizing the gain in the directions of interferers.

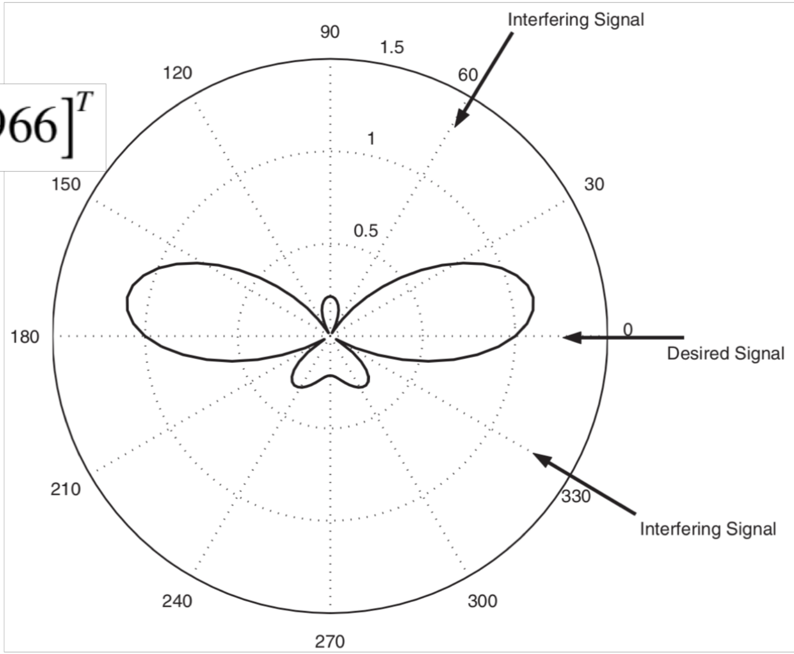
Weight vector Calculation

- weight vector **w** should satisfy the following criterion

$$\mathbf{w}^{*}\left[\mathbf{a}\left(\theta_1\right)\ \mathbf{a}\left(\theta_2\right)\ \mathbf{a}\left(\theta_3\right)\right]=\left[1\ \ 0\ \ 0\right]^T$$

- Solution for weight vector

$$\mathbf{w}=\left[0.3034+j0.1966\ \ 0.3932\ \ 0.3034-j0.1966\right]^T$$



Null-steering Beamformer

- number of nulls is less than the number of antenna elements.
- the antenna gain is not maximized at the direction of the desired user
- trade-off between interference nulled and desired gain lost
- May exist several unresolved components coming from significantly different angles
- DOA-based beamformer is viable only in
  - LOS environments or
  - in environments with limited local scattering around the transmitter

Linear Interference Suppression

- Received signal vector

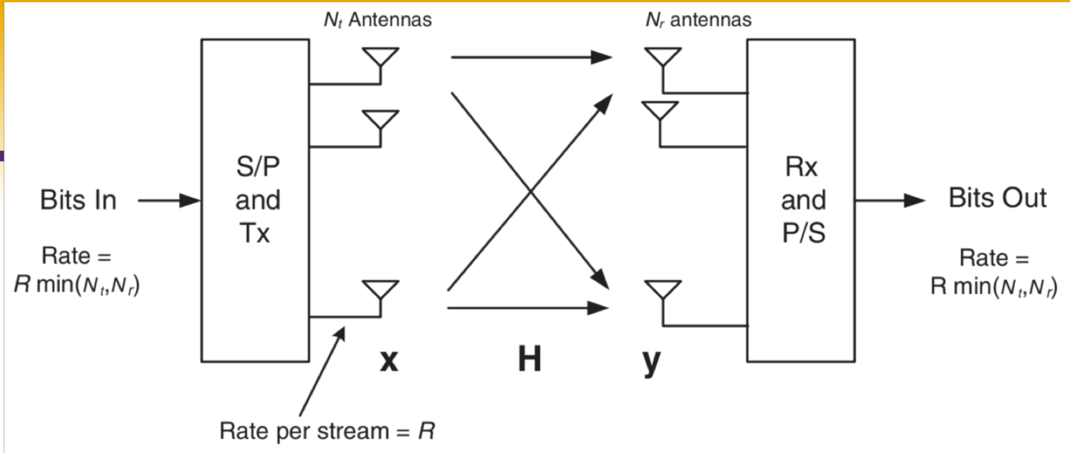
$$\mathbf{y} = \mathbf{H}\mathbf{w}_t x + \mathbf{H}_I x_I + \mathbf{n}$$

- where
  - $\mathbf{w}_t$ is the $N_t x1$ weighting vector at the desired user's transmitter,
  - x is the desired symbol
  - $\mathbf{x}_I = [x_1\ x_2\ \ldots\ x_L]^T$ is the interference vector
  - $\mathbf{n}$ is the noise vector
  - $\mathbf{H}$ is the N_r x N_t channel gain matrix for the desired user
  - $\mathbf{H}_I$ is the N_r x L channel gain matrix for the interferers

Linear Interference Suppression

- With statistical knowledge of channel:
  - In order to maximize the output SINR at the receiver, joint optimal weighting vectors at both the transmitter and the receiver can be obtained
- This is termed optimum eigenbeamformer, or interference-aware beamforming, or optimum combiner (OC)
- interference-aware beamformer is conceptually similar to the linear diversity precoding
- difference is that the eigen-beamformer takes interfering signals into account

Spatial Multiplexing



- $N_t <= N_r$
- Split the incoming high rate-data stream into N_t independent data streams
- decoding N_t streams is theoretically possible when there exist at least N_t nonzero eigenvalues in the channel matrix, that is rank(**H**) $\geqslant N_t$
- *Assuming* that the streams can be successfully decoded, the nominal spectral efficiency is thus increased by a factor of N_t

Spatial Multiplexing: Key Points

- When the SNR is high, spatial multiplexing is optimal.
  - The capacity, or maximum data rate, grows as $\min(N_t, N_r)\log(1 + SNR)$ when the SNR is large.
- When the SNR is low, the capacity-maximizing strategy is to send a single stream of data using diversity precoding.
  - Although the capacity is much smaller than at high SNR, it still grows approximately linearly with $\min(N_t, N_r)$ since capacity is linear with SNR in the low-SNR regime.

Spatial Multiplexing: Key Points

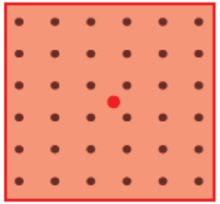
- Both of these cases are superior in terms of capacity to space-time coding, where the data rate grows at best logarithmically with N_r
- The average SNR of all N_t streams can be maintained without increasing the total transmit power relative to a SISO system
  - each transmitted stream is received at $N_r \geqslant N_t$ antennas and hence recovers the transmit power penalty of N_t due to the array gain.
- Note: even a single low eigenvalue in the channel matrix can dominate the error performance.

Open Loop Spatial Multiplexing

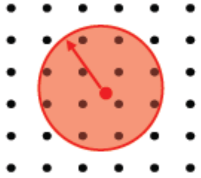
- Optimal Receiver:
  - Maximum likelihood: finds input symbol most likely to have resulted in received vector
  - Exponentially complex with # of streams and constellation size
- Sphere Decoder:
  - Only considers possibilities within a sphere of received symbol.
    - If minimum distance symbol is within sphere, optimal, otherwise null is returned

$$\hat{x} = \arg\min_{x} |y - Hx|^2$$

ML Decoding

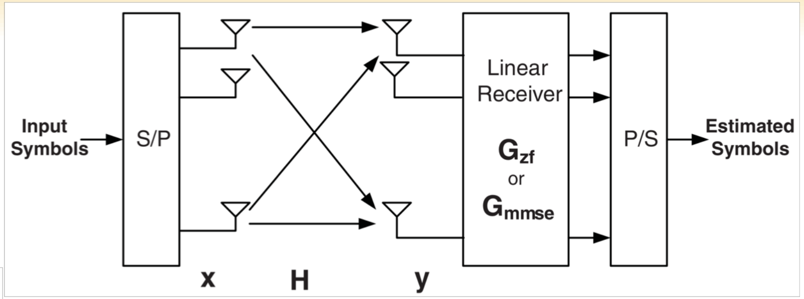


Sphere Decoding



$$\hat{x} = \arg\min_{x:|Q^H y - Rx| < r} |Q^H y - Rx|^2$$

Linear Detectors : Zero Forcing Detector



- sets the receiver equal to the inverse of the channel $\mathbf{G}_{zf} = \mathbf{H}^{-1}$

$$\mathbf{G}_{zf} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*$$

$$\hat{\mathbf{x}} = \mathbf{G}_{zf}\mathbf{y} = \mathbf{G}_{zf}\mathbf{H}\mathbf{x} + \mathbf{G}_{zf}\mathbf{n} = \mathbf{x} + (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{n}$$

- zero-forcing detector removes the spatial interference from the transmitted signal
- As $\mathbf{G}_{zf}$ inverts eigenvalues of H, poor subchannels can severely amplify noise
- Not practical in interference-limited MIMO (LTE)

Linear Detectors : MMSE Receiver

- MMSE receiver attempts to strike a balance between spatial-interference suppression and noise enhancement by minimizing the distortion

$$\mathbf{G}_{mmse} = \arg\min_{\mathbf{G}} \mathrm{E}\,\|\mathbf{G}\mathbf{y} - \mathbf{x}\|^2$$

$$\Downarrow$$

$$\mathbf{G}_{mmse} = (\mathbf{H}^{*}\mathbf{H} + \frac{\sigma_z^2}{P_t}\mathbf{I})^{-1}\mathbf{H}^{*}$$

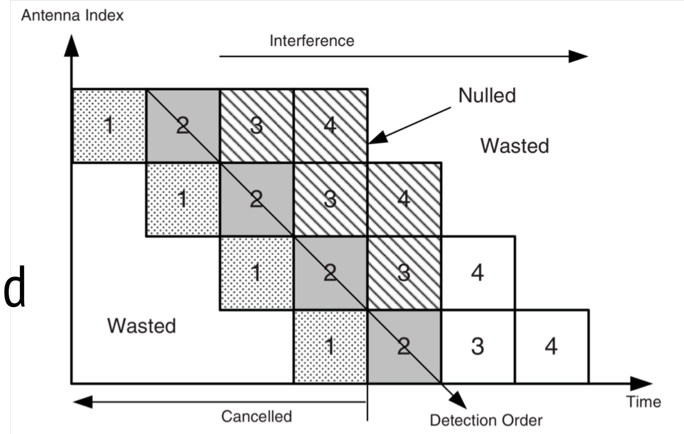
- As the SNR grows large, the MMSE detector converges to the ZF detector
- At low SNR, it prevents the worst eigenvalues from being inverted

Interference Cancellation: BLAST

- *Bell labs LAyered Space-Time* (BLAST) : invented and prototyped in Bell Labs
- BLAST consists of parallel “layers” supporting multiple simultaneous data streams
- The layers (substreams) in BLAST are separated by interference-cancellation techniques that decouple the overlapping data streams
- two most important techniques are
  - the original *diagonal BLAST* (D-BLAST)
  - its subsequent version, *vertical BLAST* (V-BLAST)

D-BLAST

- in each layer's data is transmitted in a *diagonal* of space and time
  - groups the symbols into "layers" that are then coded in time independently of the other layers
  - these layers are then cycled to the various transmit antennas in a cyclical manner
- one layer decoded at a time
- Each successive layer is detected by
  - nulling the layers that have not yet been detected
  - canceling the layers that have already been detected

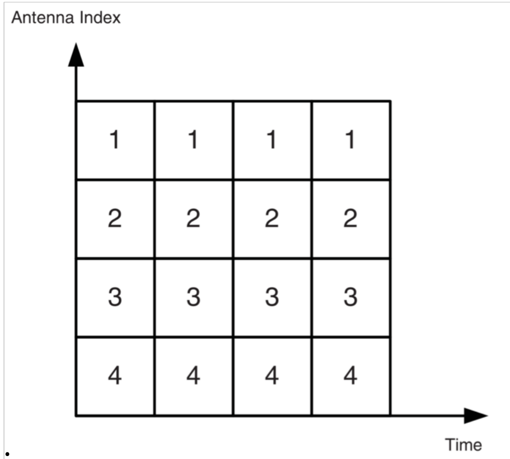


D-BLAST Pros & Cons

- Pro: each symbol stream achieves diversity
  - in time via coding and
  - in space by it rotating among all the antennas
- Cons:
  - Decoding process is iterative and complex
  - wastes space/time slots at the begin- ning and end of a D-BLAST block

V-BLAST

- each antenna transmits an independent symbol stream—for example, QAM symbols
- different techniques can be used at the receiver to separate the various symbol stream from one another
  - Including ZF, MMSE
  - the strongest symbol stream is detected, using a ZF or MMSE receiver
  - subtracted out from the composite received signal



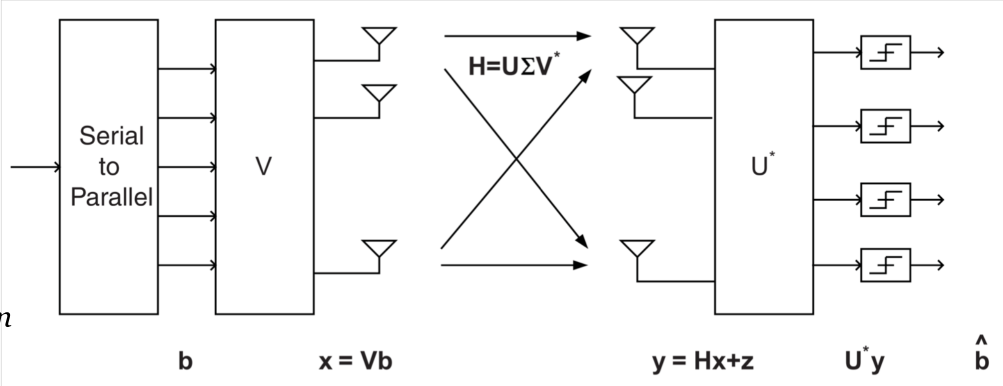
- Pros:
  - ordered successive interference cancellation lowers the block error rate by a factor of ten relative to a purely linear receiver
- Cons:
  - error propagation when initial layers are detected incorrectly leads to huge penalty
  - depends on high SNR (not available in cell edge)

Closed Loop Spatial Multiplexing

- The advantage of channel knowledge
- SVD Precoding and Postcoding
  - Channel expressed as singular-value decomposition (SVD, or generalized eigenvalue decomposition)
  - **U** and **V** are complex unitary matrices, Σ is a diagonal matrix of singular values (non-negative real numbers)

Impractical, but promising results compared to open loop approach

complexity of finding the SVD of an $N_t x N_r$ matrix is on the order of $O(N_r, N_t^2)$ if $N_r >= N_t$

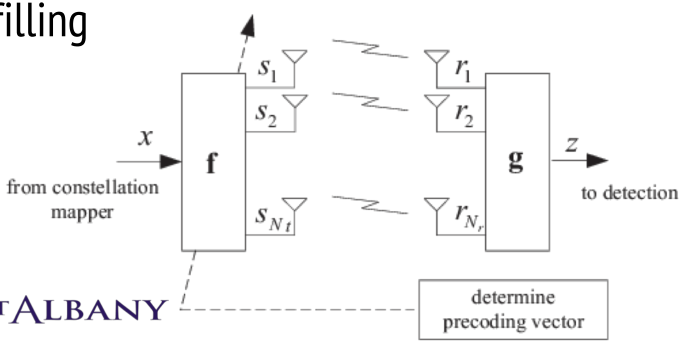


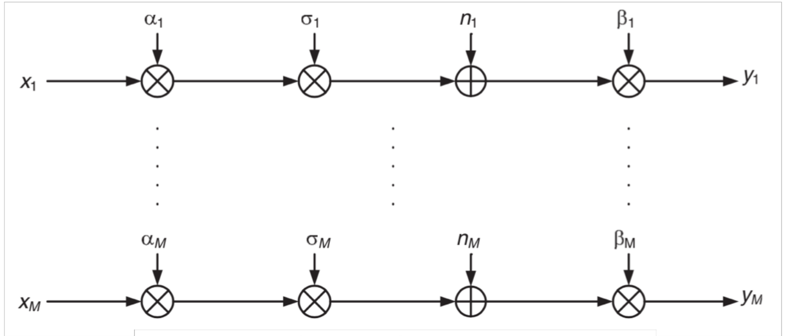
https://en.wikipedia.org/wiki/Singular_value_decomposition

Linear Precoding and Postcoding

- decomposes the MIMO channel into a set of parallel subchannels
- the precoder and the postcoder can be jointly designed based on
  - information capacity, error probability, detection MSE, or received SNR
- precoder weights are used to maximize the total capacity by distributing more transmission power to subchannels with larger gains and less to the others - waterfilling

- $z = Gy = G(HFx + n)$
  - M is the number of spatial data “streams” sent
  - Transmitted vector x is $M \times 1$
  - Received vector y is $N_r \times 1$
  - Postcoder matrix G is $M \times N_r$
  - Channel matrix H is $N_r \times N_t$
  - Precoder matrix F is $N_t \times M$
  - $1 \leq \mathrm{M} \leq \min(N_r, N_t)$





$$y_i = \alpha_i \sigma_i \beta_i x_i + \beta_i n_i, \quad i = 1, \cdots, M$$

σ_i are the singular values of $\mathbf{H}$,

How to choose MIMO Techniques?

- https://ieeexplore.ieee.org/document/5374062
- Due March 25 after Spring break

Channel Estimation for MIMO OFDM

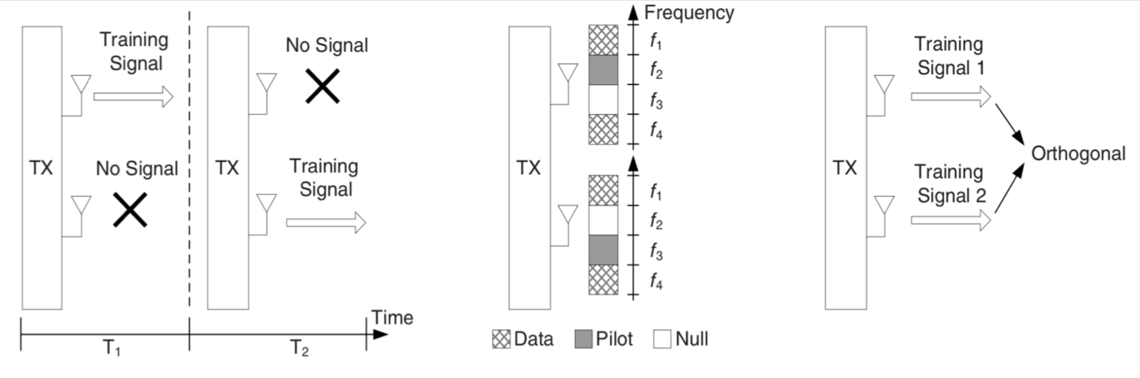
- Channel estimation required
  - At the receiver in order to
    - coherently detect the received signal
    - for diversity combining
    - spatial-interference suppression
  - At the transmitter
    - For closed loop MIMO
- Types:
  - Training based – known symbols (preambles, pilots) transmitted, reliable, mostly used
  - Blind – no training, no overhead, low convergence speed, lower estimation accuracy

Training Symbols

- Two ways to transmit training symbol:
  - Preambles : send a certain number of training symbols prior to the user data symbols
  - Pilot tones : insert a few known (time, frequency, phase, amplitude) pilot symbols among the subcarriers
- Channel estimation typically done by using
  - the preamble for synchronization and initial channel estimation
  - the pilot tones for tracking the time-varying channel in order to maintain accurate channel estimates

Pilot Insertion Patterns

- received signal at each antenna is a superposition of the signals transmitted from N_t transmit antennas
- the training signals for each transmit antenna should not interfere with one another
- Independent: orthogonality achieved in time domain, requires N_t training signal times
- Scattered: orthogonality achieved in frequency domain
- Orthogonal: orthogonality achieved using orthogonal codes



(a) Independent Pattern (b) Scattered pattern (c) Orthogonal pattern

Time Domain Channel Estimation

- Preamble based with cyclic prefix

$$
\begin{aligned}
\mathbf{y} &= \begin{bmatrix} h(0) & \cdots & h(v) & 0 & \cdots & 0 \\ 0 & h(0) & \cdots & h(v) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h(1) & \cdots & h(v) & 0 & \cdots & h(0) \end{bmatrix} \begin{bmatrix} x(L-1) \\ \vdots \\ x(0) \end{bmatrix} + \mathbf{n} \\
&= \begin{bmatrix} x(0) & x(L) & x(L-1) & \cdots & x(L-v+1) \\ x(1) & x(0) & x(L) & \cdots & x(L-v+2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x(L) & x(L-1) & \cdots & \cdots & x(L-v) \end{bmatrix} \begin{bmatrix} h(0) \\ \vdots \\ h(v) \end{bmatrix} + \mathbf{n} \\
&= \mathbf{X}\mathbf{h} + \mathbf{n},
\end{aligned}
$$

$x(l)$ is the l^{th} time sample of the transmitted OFDM symbol, and $h(i)$ is the i^{th} time sample of the channel impulse response

$$\hat{\mathbf{h}} = (\mathbf{X}^*\mathbf{X})^{-1}\mathbf{X}^*\mathbf{y}$$

X is deterministic and hence known *a priori* by the receiver

Frequency Domain Channel Estimation

- simpler in the frequency domain than in the time domain

$$Y(l) = H(l)X(l) + N(l)$$

$$\hat{H}(l) = X(l)^{-1}Y(l)$$

Equalization

- Linear Equalization
  - runs the received signal through a filter that models the inverse of the channel
- Non-linear Equalization
  - uses previous symbol decisions made by the receiver to cancel out their subsequent interference and so are often called *decision-feedback equalizers* (DFEs)
- Maximum-likelihood sequence detection (MLSD)