Programming for Engineers

Number System

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Types of Numbers

- **Natural Numbers**
  - The number 0 and any number obtained by repeatedly adding a count of 1 to 0
- **Negative Numbers**
  - A value less than 0
- **Integer**
  - A natural number, the negative of a natural number, and 0.
  - So an **integer number system** is a system for ‘counting’ things in a simple systematic way

Exponent Review

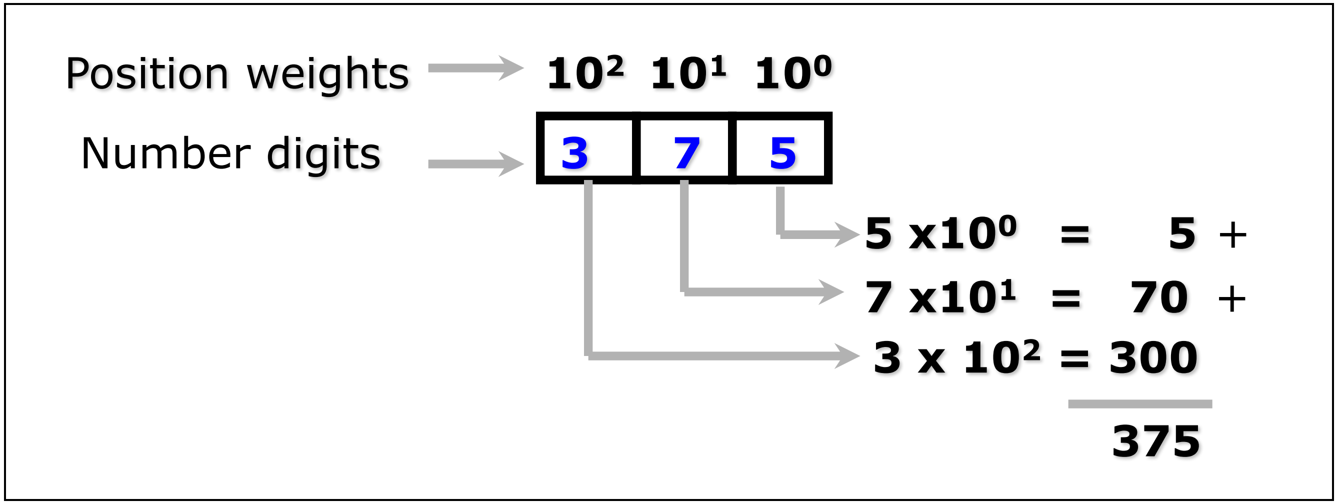
- An exponent (power) tells you how many times to multiply the base by itself:
  - $2^1 = 2$
  - $2^2 = 2 \times 2 = 4$
  - $2^3 = 2 \times 2 \times 2 = 8$
- $2^0 = 1$ (ANY number raised to power 0 is 1)
- $1 / x^2 = x^{-2}$

Decimal Number System

- How is a **positive integer** represented in decimal?
- Let's analyze the decimal number **375**:

$$375 = (3 \times 100) + (7 \times 10) + (5 \times 1)$$

$$= (3 \times 10^2) + (7 \times 10^1) + (5 \times 10^0)$$



Bits

- In a computer, information is stored using digital signals that translate to binary numbers
- A single binary digit (0 or 1) is called a bit
  - A single bit can represent two possible states, on (1) or off (0)
- Combinations of bits are used to store values

Data Representation

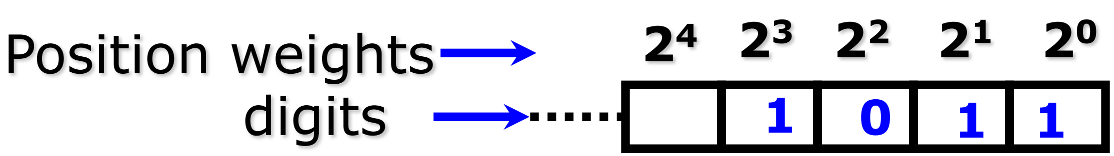
- Data representation means encoding **data** into **bits**
  - Typically, **multiple bits** are used to represent the '**code**' of each **value** being represented

- Values being represented may be characters, numbers, images, audio signals, and video signals.

- Although a different scheme is used to encode each type of data, in the end the code is always a string of **zeros** and **ones**

Decimal to Binary

- So in a computer, the only possible digits we can use to encode data are **{0,1}**
  - The numbering system that uses this set of digits is the base 2 system (also called the Binary Numbering System)
- We can apply all the principles of the base 10 system to the base 2 system

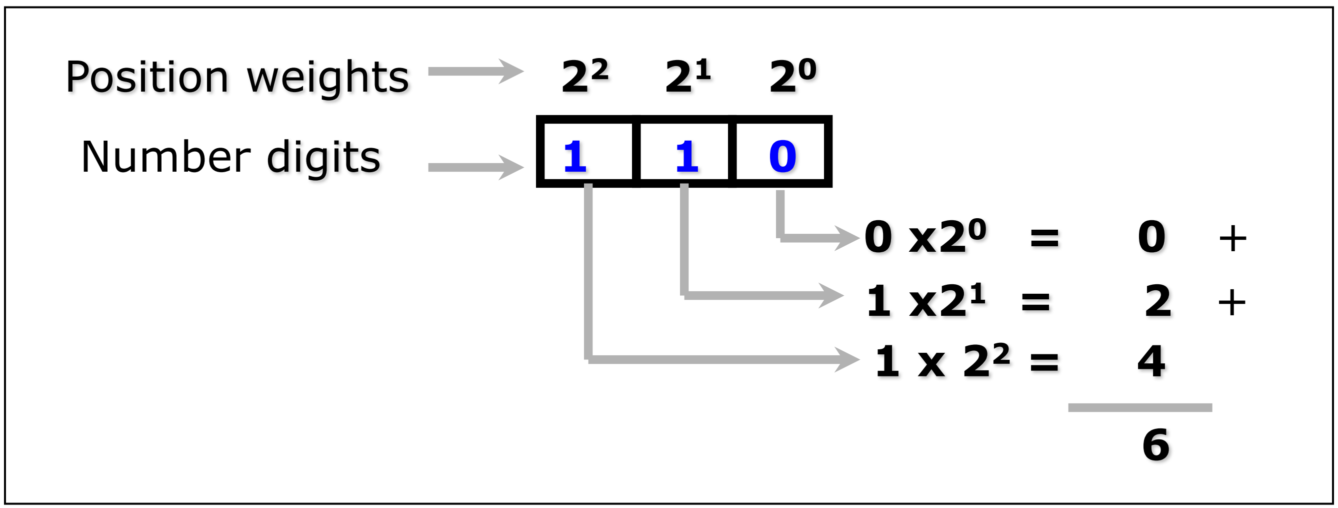


Binary Numbering System

- How is a **positive integer** represented in **binary**?
- Let's analyze the binary number **110**:

110 $= (\mathbf{1} \times 2^2) + (\mathbf{1} \times 2^1) + (\mathbf{0} \times 2^0)$

$= (\mathbf{1} \times 4) + (\mathbf{1} \times 2) + (\mathbf{0} \times 1)$



- So a count of **SIX** is represented in binary as **110**

Binary to Decimal Conversion

Example: **Convert binary 100101 to decimal**

(written $1\ 0\ 0\ 1\ 0\ 1_2$) =

$1*2^0 +$ → $1 +$

$0*2^1 +$

$1*2^2 +$ → $4 +$

$0*2^3 +$

$0*2^4 +$

$1*2^5$ → 32

37_{10}

Binary to Decimal Conversion

Example #2: 10111_2

positional powers of 2:	2^4	2^3	2^2	2^1	2^0
decimal positional value:	**16**	**8**	**4**	**2**	**1**
binary number:	1	0	1	1	1

$16 + 4 + 2 + 1 = 23_{10}$

Binary to Decimal Conversion

Example #3: 110010_2

positional powers of 2:	2^5	2^4	2^3	2^2	2^1	2^0
decimal positional value:	32	16	8	4	2	1
binary number:	1	1	0	0	1	0

$32 + 16 + 2 = 50_{10}$

Decimal to Binary Conversion

Using the **Division** Method:

Divide decimal number by 2 until you reach zero, and then collect the **remainders** in reverse.

Example 1: 22_{10} = 10110_2

Division	Rem:
2) 22	
2) 11	0
2) 5	1
2) 2	1
2) 1	0
0	1

Decimal to Binary Conversion

Using the **Division** Method

Example 2: 56_{10} = 111000_2

2) 56	Rem:	
2) 28	0	
2) 14	0	↑
2) 7	0	
2) 3	1	
2) 1	1	
0	1	

Octal Number

- Base: 8
- Digits: 0, 1, 2, 3, 4, 5, 6, 7
- Octal number: 357_8

$$= (3 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)$$

- To convert to base 10, beginning with the rightmost digit, multiply each nth digit by $8^{(n-1)}$, and add all of the results together.

Octal to Decimal Conversion

- Example 1: 357_8

	8^2	8^1	8^0
positional powers of 8:	8^2	8^1	8^0
decimal positional value:	64	8	1
Octal number:	3	5	7

$$(3 \times 64) + (5 \times 8) + (7 \times 1)$$

$$= 192 + 40 + 7 = 239_{10}$$

Decimal to Octal Conversion

Using the **Division** Method:

Example 1: $\mathbf{214_{10} = 326_8}$

8) 214		Rem:
8) 26		6
8) 3		2
0		3

Hexadecimal Number

- Base: 16
- Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Hexadecimal number: $1F4_{16}$

$$= (1 \times 16^2) + (F \times 16^1) + (4 \times 16^0)$$

Hex Values

Decimal Value	Hexadecimal Digit
10	**A**
11	**B**
12	**C**
13	**D**
14	**E**
15	**F**

Hex to Decimal Conversion

- Example 1: $1F4_{16}$

positional powers of 16:	16^3	16^2	16^1	16^0
decimal positional value:	4096	256	16	1
Hexadecimal number:		1	F	4

(1 x 256) + (F x 16) + (4 x 1)

= (1 x 256) + (15 x 16) + (4 x 1)

= 256 + 240 + 4 = 500_{10}

Decimal to Hex Conversion

Using The **Division** Method:

Example 1: $\mathbf{126_{10}} = \mathbf{7E_{16}}$

16) 126	Rem:	↑
16) 7	14=E	
0	7	