THE ECONOMIC ANIMAL

Suppose Evolution by Natural Selection

    Selection Favors Efficiency

"Efficiency" $\Leftrightarrow$ Survival and/or Reproduction

    Functional Significance

Understand $\Leftrightarrow$ Predict Efficiency: Model Behavior

MATHEMATICAL MODELS: ADVANTAGES

Make Explicit Assumptions:

    Strategy Set, Constraints

    Clear Hypothesis: Currency of Fitness

Conceptual Integration: Generality

Precise Prediction; Falsifiable

Natural Selection: *Individuals*

Optimizing Selection: *Population* Adapts

Optimize Behavior

Extrema of Hypothesized Fitness Function

Calculus: Language of Science

DERIVATIVES

a: Action (Behavior), Continuous,

Subject to Selection

f: Currency of Fitness, *Hypothesis* re Function

f(a): Continuous function

"Map" from behavior to fitness currency

Gradient: *del*, *h* small,

How fast *f* changes as *a* changes

$$del = \frac{f(a+h) - f(a)}{h}$$

Limit of *del* as $h \to 0$: **first derivative** of *f* wrt *a*

Write $df/da = f'(a)$

Recall

$$IF \quad f(a) = ka^n \quad THEN \quad df/da = nka^{n-1}$$

Fitness increases with *a*: $f'(a) > 0$

Second Derivative

Write $df'/da = f''(a)$

$$f''(a) = (n-1)nka^{n-2}$$

MAXIMA AND MINIMA

Take first derivative of $f(a)$

Set $f'(a) = 0$

Solve for a^*

Take second derivative of $f(a)$

Evaluate $f''(a^*)$

If a^* Maximizes $f(a)$

Then $f''(a^*) < 0$

If a^* Minimizes $f(a)$

Then $f''(a^*) > 0$

EXAMPLES

Suppose $f(a) = 2a - a^2$

$df/da = 2 - 2a$ and $df^2/da^2 = -2$

Set First Derivative = 0

$2 - 2a = 0$; $a^* = 1$ Max or Min?

Evaluate Second Derivative at $a = a^*$

$-2 < 0$

Therefore, a^* maximizes $f(a)$

Foraging Time

Action: foraging time $t > 0$

Benefit: More Food, Increased Energy Intake,
Increased Survival, As t Increases

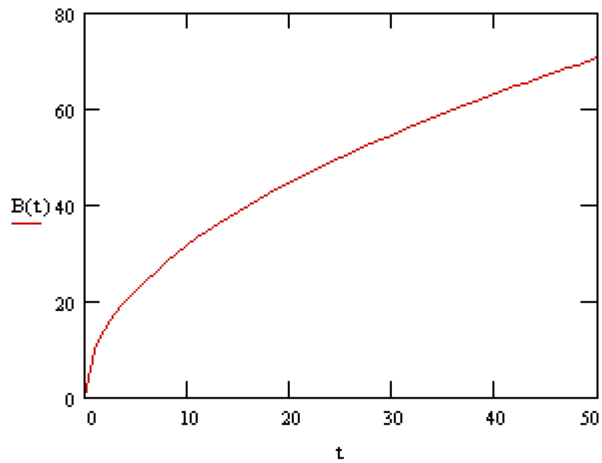
As Food Depleted, Rate of Intake Declines

Benefit: B(t) = k (t ^ 1/2) $k > 0$

$B(t) = k\, t^{1/2}$ k constant

k increases with food density

Benefit



Foraging Time t

Cost: Energy Expended Increases as t Increases

Assume Total Expenditure Linear in t

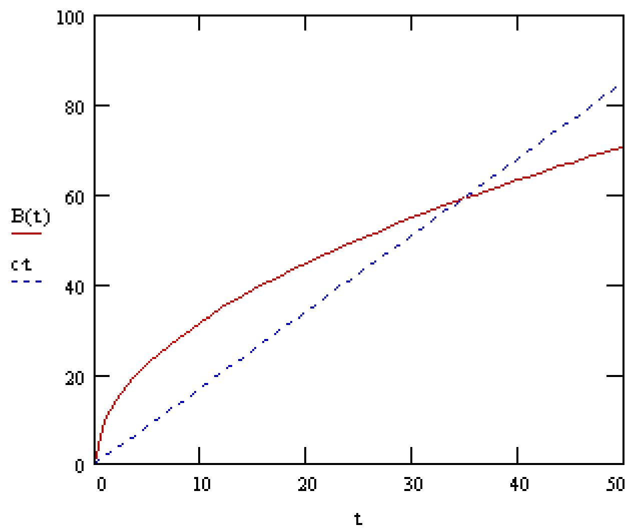
Cost: $C(t) = ct$

$c > 0$; c declines as temp increases

B(t)

and

C(t)



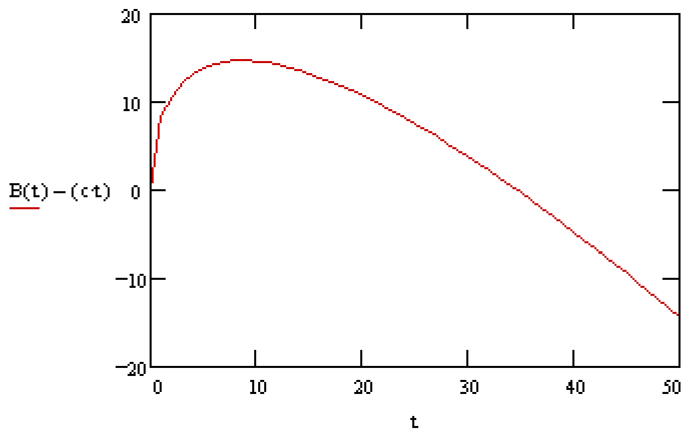
Foraging Time t

Hypothesis: Selection Maximizes Net Benefit

Currency of Fitness: Net Benefit = B(t) - C(t)

This is what we test; don't test animal's perfection

B(t) - C(t)



Foraging Time t

Objective function:

$f(t) = B(t) - C(t) = k\,t^{1/2} - c\,t$

$$f'(t) = \left(\frac{k}{2}\right) t^{-1/2} - c$$

Set $f'(t) = 0$, Obtain $c = \frac{k}{2t^{1/2}}$

$$t^{1/2} = k/2c$$

$$t^{*} = k^{2}/4c^{2}$$

Maximum, Minimum???

$$f''(t) = -\left(1/2\right)\left(k/2\right) t^{-3/2}$$

$$f''(t^{*}) = -\left(k/4\right)\left[\frac{k}{2c}\right]^{-3} < 0$$

Optimal foraging time maximizes net benefit.

Maximized (Hypothesized) Currency of Fitness

by Optimizing Behavior (Foraging Time)

$t^* = k^2 / 4c^2$ Optimal value of t

Do?

Recall: k increases with food density

Predict: Forage longer when food density greater

Quantify foraging time across natural food densities

Experimentally manipulate food density

Test Hypothesis: Natural Selection Acts on

Net Benefit as f(Foraging Time)

Don’t Test: Is Nature Optimal?