

Aggregation Economy

Replace general fitness function $w_i(\textit{Group size})$

Probability of energetic failure

Risk-sensitivity affects solitary vs group foraging?

Affects equilibrium group size?

Recall **risk-sensitivity** as hypothesized currency of fitness,

then

Apply to social economy, foraging for clumps of food

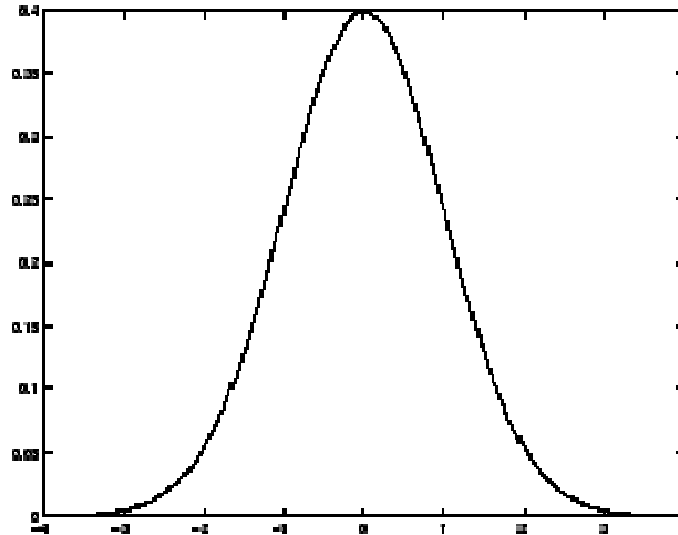
Theory, elements

Test, Ekman & Hake (1988, *Behav Ecol Sociobiol* 22:91)

Forager's total food/energy intake varies **randomly**

Sum of gains & losses over time T

(Approximately) normal density



Abscissa: Food/energy intake (X , RV)

Ordinate: Pr density of intake [$X(T)$]

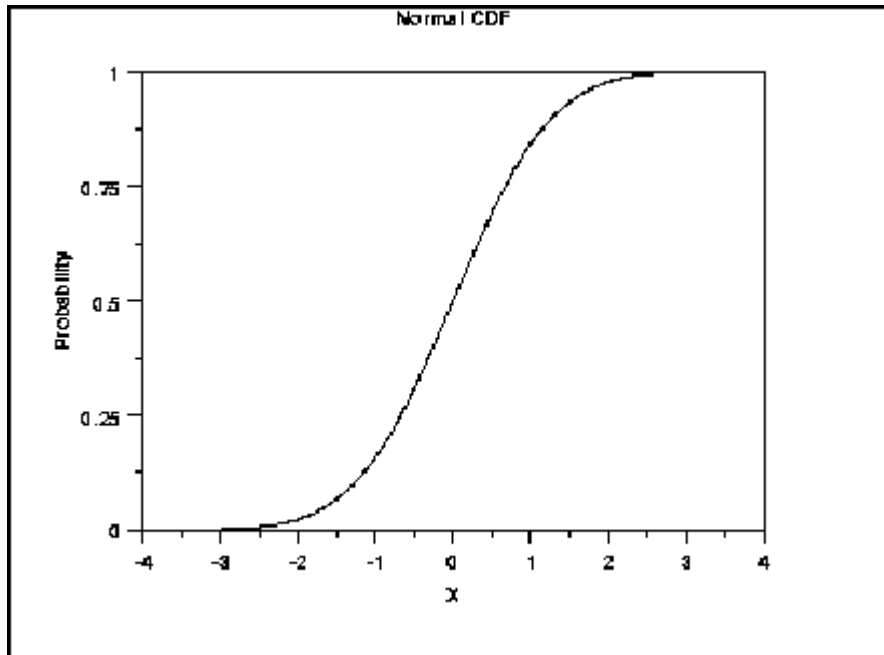
Intake exceeds physiological requirement, forager survives

Intake fails to exceed requirement, forager starves

Requirement = R

Currency of fitness: $\Pr[X(T) \leq R]$

Minimize $\Pr[X(T) \leq R]$



Different behavioral strategies

Different mean intake, intake variance

Different $\Pr[X(T) \leq R]$

Apply to solitary/group foraging

Group has G members; $G = 1, 2$; all search for food

Encounter divisible food clumps rate λG^α ($\lambda, \alpha > 0$)

Solitary's ($G = 1$) encounter rate λ

$\alpha < 1$, the average rate at which food is discovered *per group member* decreases with group size

$\alpha < 1$ either inefficiency (*e.g.*, groups alert their prey) or competitive interference among foragers

If $\alpha > 1$, the average rate at which food is discovered *per group member* increases with group size

$\alpha > 1$ mutualism: enhanced searching efficiency or group members forage cooperatively

$\alpha = 1$, independence during search

Clumps divided equally; handling time negligible

Number clumps discovered: random variable

$$\text{mean} = \text{variance} = \lambda TG^\alpha \quad (\text{Poisson})$$

Food per clump: random variable

Individual's energy intake X(T): random variable

Expected intake (mean, average):

$$\begin{aligned} & (\text{Expected no. clumps}) (\text{Expected clump size})/G \\ & = \lambda TG^{\alpha-1} (\text{Expected clump size}) \end{aligned}$$

Intake variance:

$$\begin{aligned} & (\text{Expected no. clumps}) (\text{Clump size variance})/G^2 \\ + & (\text{Variance no. clumps}) (\text{Expected clump size})/G^2 \\ & = \lambda TG^{\alpha-2} [\text{Clump size variance} + \text{Expected clump size}] \end{aligned}$$

Mean intake declines with G when $\alpha < 1$

Interference during search

Variance declines with G when $\alpha < 2$

Special case, independence: $\alpha = 1$

Expected intake = $\lambda T G^{\alpha-1}$ (Expected clump size)

= λT (Expected clump size)

Same mean for all group sizes

Intake variance =

$(\lambda T/G)$ [Clump size variance + Expected clump size]

Variance declines as G increases

$\alpha = 1$, individual's clump-discovery rate independent of G

Pr[starvation]

increases with group size if $R > \textit{Expected intake}$

requirement exceeds expected intake

Forage as solitary to **Minimize Pr[X(T) ≤ R]**

Risk-prone, since highest intake variance

Pr[starvation]

declines with group size if $\textit{Expected intake} > R$

expected intake exceeds requirement

Forage as group member to **Minimize Pr[X(T) ≤ R]**

Risk-averse, since lower intake variance

Demonstrate

Ekman & Hake (1988):

Greenfinches (*Carduelis chloris*)

Aviary, choose to feed alone or in pairs

Expected intake

Solitary: Mean seeds/day = 79.5

Group: Mean seeds/day = 77.8

No significant difference

Pairs found food twice as often as solitaries

Pairs 83% trials, food divided equally

Solitaries 41% trials

Close to $\alpha = 1$

Intake variance

Significantly greater for solitaries

Requirement: 80 seeds/day (10 seeds/hour)

Fed 20 seeds/hour

Expected intake exceeds requirement

80 % choice trials - group

20% choice trials - solitary

Significantly risk-averse

Fed 5 seeds/hour

Requirement exceeds expected intake

35 % choice trials - group

65% choice trials - solitary

Significantly risk-prone



Above: fixed time T available; Energy intake random

Dual: Fix number of food clumps; Time random

Suppose each individual:

Must consume energy = 1 clump by time τ (tau)

Otherwise, individual starves

Solitary, encounters food at rate λ ;

Must find 1 clump by time τ

Group of G , encounters food at rate $G\lambda$;

Must find G clumps by time τ

Food clumps divided equally

Waiting time problem

Individual **Min Pr[time to find G clumps $\leq \tau$]**

Solitary ($G = 1$)

Waiting time t exponential

Probability density $f_1(t) = \lambda e^{-\lambda t}$

$$\Pr[t \leq \tau] = 1 - e^{-\lambda \tau}$$

$$\text{Mean time} = \frac{1}{\lambda} \quad \text{Variance} = \frac{1}{\lambda^2}$$

Group ($G > 1$)

Waiting time t gamma (sum G exponentials)

Probability density $f_G(t) = \frac{(\lambda G)^G t^{G-1} e^{-\lambda G t}}{(G-1)!}$

$$(G-1)! = (G-1)(G-2) \dots (2)(1)$$

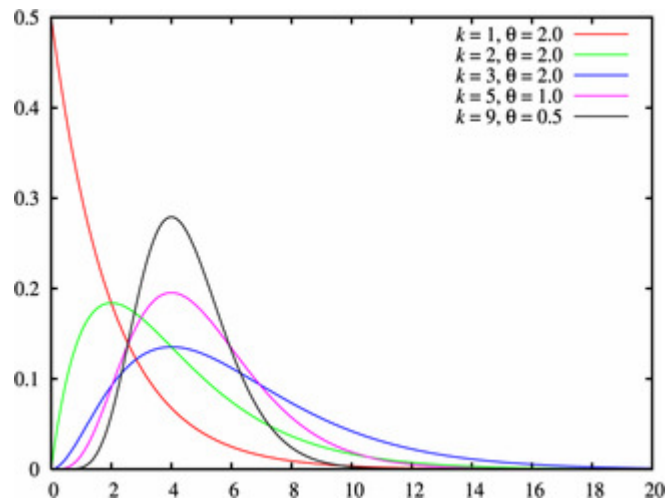
$$\text{Mean time} = \frac{G}{G} \frac{1}{\lambda} = \frac{1}{\lambda}$$

$$\text{Variance} = \frac{1}{G} \frac{1}{\lambda^2}$$

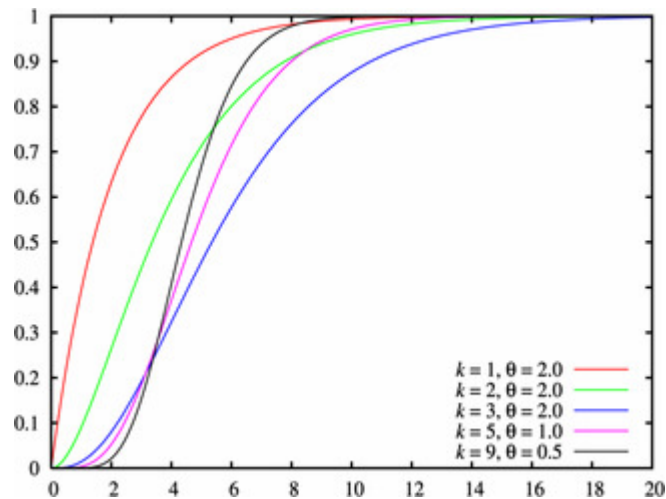
Same mean time to find required food

Variance in search-time declines as group size increases

en.wikipedia.org/wiki/Gamma_distribution



Abscissa: Time t
Ordinate: density $f(t)$



Abscissa: Time t
Ordinate: $\Pr[t \leq \tau]$