"Population projection"

"Leslie matrix model"

Age Structured Population Growth

Dynamics of Numbers in Each ***Age Class***

Age class i; $i = 1, 2, \ldots, k$
Discriminate **age class** i from **age** $x = 0, 1, \ldots$

$n_i(t)$: Number in age class i at time t

Time from t to $(t + 1)$, survivors move to age class $(i + 1)$

$n_1(t), \; n_2(t), \; \ldots, \; n_k(t)$

Life table: $L(x)$, $b(x)$, functions of **age *x***
Translate to P_i, F_i, vital ***rates for age class i***

Recall R_0 assumptions

Survive age 0 to **age x**: l_x

Given survival to age x $\Rightarrow$ births b_x

$$R_0 = \Sigma_x\, l_x\, b_x$$

Population projection (age classes):

Different assumptions *re* accounting

Live through (Survive) an age class

Reproduction at *transition* to next age class

Immediate census

$\boldsymbol{P_i}$ **Survival** probability for age class *i*

Pr [individual in age class *i* enters age class (*i* + 1)]

(*k* – 1) values > 0: *k* age classes

$\boldsymbol{F_i}$ **Fertility** (no. offspring) of individual in age class *i*

(*k* – 1) values ≥ 0; some may be zero

Dynamics: Use $\boldsymbol{P_i}$ and $\boldsymbol{F_i}$ to **project** $n_i(t)$ to $n_i(t + 1)$; all *i*

<u>Assume</u>

1. **Birth-pulse**: Individuals in **age class** *i* produce offspring for that age class on the day they enter age class (*i* + 1)

   Given survival, reproduction is b_i

   Must survive entire time in an age class to produce offspring for that age class

   Fraction surviving age class i = P_i

2. Census [count all $n_i(t)$] just after new offspring appear

   Post-breeding census

Particular assumptions: simplicity

Survival $\boldsymbol{P_i}$ Entering **age class** (i + 1) from **age class** i

$$P_i = \frac{l(i)}{l(i-1)}$$

l_i from survival to **age** $x = i$

Why? age **x** = 0, 1, … age class i = 1,2 …

Convert from age to age class; Page 60-62 of text

P_1 Survives first age class; just enters age class 2

Survives from age 0 to age 1; therefore:

$$P_1 = l_1 / l_0$$

As above

Given survival thru **age class** 1, reproduction b_1

Density-independence assumed

Fertility $\boldsymbol{F_i}$

$$F_i = b_i \, P_i$$

$b(i)$ maternity of life table for age $x = i$

Initial fecundity F_1

Density-independence assumed

Let **age** $x = 1 \Rightarrow$ Individual has survived 1 year from **age** 0

Then individual has survived **age class** i = 1

Enters **age class** $i = 2$

Birth pulse: b_1

Project age-structured population

Time t to time $(t + 1)$

For $\mathbf{1 < \boldsymbol{i} \leq \boldsymbol{k}}$, enter by survival

$$n_{i+1}(t+1) = P_i \, n_i(t)$$

Some age-class i survive, enter age-class $(i + 1)$, time $(t + 1)$

For $\boldsymbol{i} = \mathbf{1}$, first age class

$$n_1(t+1) = \sum_{i=1}^{k} F_i \, n_i(t)$$

Enter through birth

$$n_1(t+1) = F_1 \, n_1(t) + F_2 \, n_2(t) + \ldots + F_k \, n_k(t)$$

Sum births across all possible parental age classes

Life Table, $k = 3$, so three *age classes*

Age	No. Surviving N_x	Births b_x
0	1000	-
1	400	1.5
2	240	2
3	0	0

$l_0 = 1$ (definition) $l_1 = g_0 = 0.4$

$l_2 = (g_0)\, g_1 = (0.4)\, 0.6 = 0.24$ $l_3 = 0$

Time t, Age Structure: $n_1(t)$, $n_2(t)$, $n_3(t)$

$$n(t) = \begin{bmatrix} 100 \\ 60 \\ 40 \end{bmatrix}$$

Time $(t + 1)$?

$P_1 = l_1/\ l_0 = 0.4/1 = 0.4$

$P_2 = l_2/\ l_1 = 0.24/0.4 = 0.6$

$P_3 = l_3/l_2 = 0/0.24 = 0$

$F_1 = b_1\ P_1 = 1.5\ (0.4) = 0.6$

$F_2 = b_2\ P_2 = 2\ (0.6) = 1.2$

$F_3 = b_3\ P_3 = 0$

Population Projection

$$n_2(t + 1) = P_1\ n_1(t) = 0.4\ (100) = 40$$

$$n_3(t + 1) = P_2\ n_2(t) = 0.6\ (60) = 36$$

All 40 individuals in age-class 3 die

$n_1(t + 1) = F_1\ n_1(t) + F_2\ n_2(t) + F_3\ n_3(t)$

$n_1(t + 1) = P_1\ b_1\ \ n_1(t) + P_2\ b_2\ n_2(t) + P_3\ b_3\ n_3(t)$

$n_1(t + 1) = 0.4\ (1.5)\ (100) + 0.6\ (2)\ 60 + 0\ (40) =$

$60 + 72 = 132$

$$n(t) = \begin{bmatrix} 100 \\ 60 \\ 40 \end{bmatrix} \qquad n(t+1) = \begin{bmatrix} 132 \\ 40 \\ 36 \end{bmatrix}$$

F_i and P_i fixed:

Population grows to **Stable Age Distribution**

Relative proportions of each age class constant

Some exceptions: Observe in *Populus*

$$LM = \begin{bmatrix} 0.6 & 1.2 & 0 \\ 0.4 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix}$$