

# DISEASE ECOLOGY

Simple Epidemic, No Births, No Deaths

$$S_t \rightarrow 0 \text{ as } t \rightarrow \infty$$

General Epidemic, No Births, No Deaths

$$R_0 > 1: S_t \rightarrow \delta > 0 \text{ as } t \rightarrow \infty$$

Susceptibles decay until Removals > New Infections

$$\gamma I_t > \beta S_t I_t; \gamma > \beta S_t$$

$R_0 < 1$ : Pathogen Fails to Invade Susceptible Population

Critical population Size  $S_{\text{crit}}$

Threshold Susceptible-Host Density  $S_T$

Today: SIS Model, Host Birth and Death

Endemic Equilibrium

$$N_t \rightarrow N^* \text{ and } I_t \rightarrow I^*$$

## SIS Model

Allstadt's SI Model with Density Dependent Transmission;

(pp 120, 121 errors)

Epidemiological Transitions (no immune memory)

$S \rightarrow I$ : Infection,  $\beta$        $I \rightarrow S$ : Recovery,  $\nu$

Population Dynamics

Birth,  $b$       Death,  $\mu$  or  $(\alpha + \mu)$        $r = b - \mu$

$$\frac{dS}{dt} = b(S + I) + \nu I - \mu S - \beta SI$$

birth, recovery, death, infection

$$\frac{dI}{dt} = \beta SI - (\alpha + \mu) I - \nu I$$

infection, death (note virulence  $\alpha$ ), recovery

$$N_t = S_t + I_t \quad \text{Therefore} \quad \frac{dN}{dt} = rN - \alpha I$$

Important Question: Can disease biotically regulate host growth?

## Pathogen $R_0$

New Infections per Infection When Pathogen Rare

$$\left(\frac{dI}{dt}\right)_{I=0} \quad \text{Can pathogen invade population of susceptibles?}$$

Duration of Infection

Host Infectious Until Dead or Recovered

$$\text{Rate} \quad (\alpha + \mu) \quad \nu$$

$$\text{Time Infectious} = 1/(\alpha + \mu + \nu)$$

Rate of Infection, per unit time, while infectious =  $\beta S_0$

$$R_0 = \frac{\beta S_0}{\alpha + \mu + \nu}$$

At  $S_{\text{crit}}$ ,  $R_0 = 1$

$$S_{\text{crit}} = S_T = \frac{\alpha + \mu + \nu}{\beta}$$

Pathogen Invades  $S_0 > S_{\text{crit}}$

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt}$$

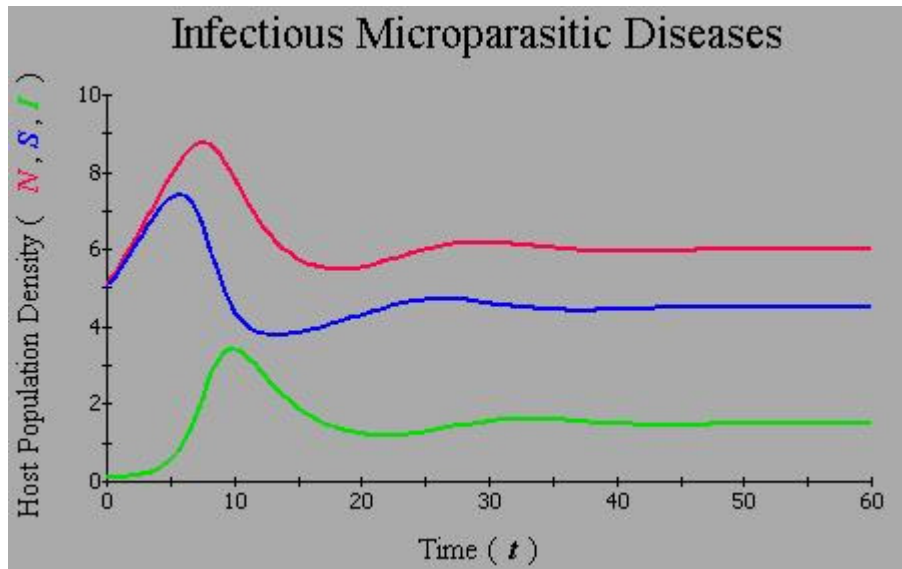
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$$N^* = \frac{\alpha}{\alpha - r} \left( \frac{\alpha + \mu + \nu}{\beta} \right) = \frac{\alpha}{\alpha - r} S_{crit}$$

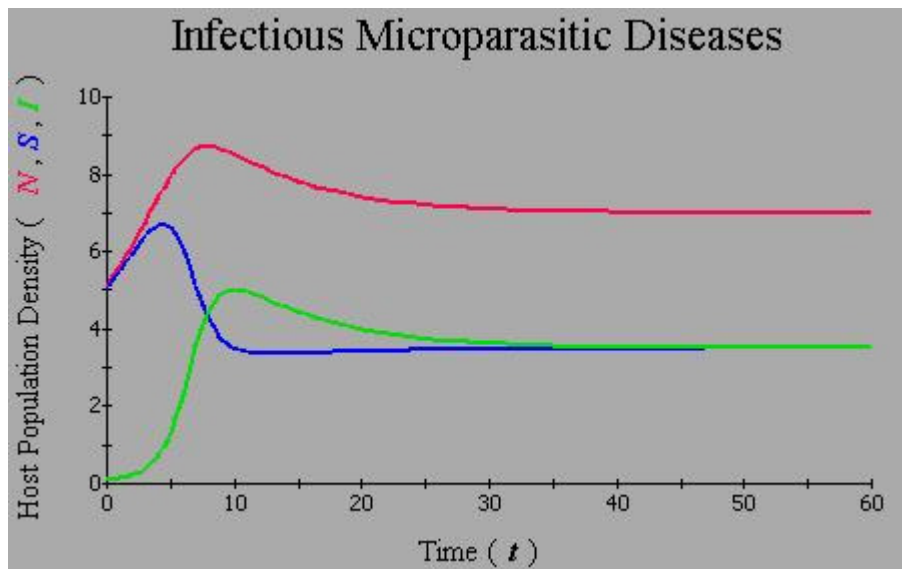
$$I^* = \frac{r}{\alpha - r} \left( \frac{\alpha + \mu + \nu}{\beta} \right) = \frac{r}{\alpha - r} S_{crit}$$

$$S^* = N^* - I^* = S_{crit}$$

Endemic Equilibrium: Each Infection Generates 1 New Infection



Lower Virulence



Greater Virulence (Less Recovery)