

1. Discriminate direct and indirect interactions between species. Explain “apparent competition” and “competitive mutualism.”
- 2 . Graphically analyze the “paradox of enrichment.”
3. *Last Friday*. Let  $S_t$ ,  $I_t$  and  $R_t$  represent, respectively, the number of susceptible, infective, and removed individuals in a general epidemic.  $S_0$  is the initial number of susceptibles, and  $I_0 = 1$ . Consider the dynamics:

$$dS/dt = - 0.001 S_t I_t$$

$$dI/dt = 0.001 S_t I_t - 0.5 I_t$$

$$dR/dt = 0.5 I_t$$

Note that the “infection rate” appears in the first two equations, and that the removal rate appears in the second and third equations. Find the relative removal rate. If the initial population of susceptibles is  $S_0 = 501$  individuals, do we expect the infection to advance to an epidemic?

Relative removal rate =  $\gamma/\beta = 0.5/0.001 = 500$   
 Critical population size for pathogen invasion = 500

4. *Last Friday*. Consider the predator-prey system with dynamics

$$dH / dt = rH - \alpha HP - cH^2$$

$$dP / dt = \alpha HP - mP$$

where  $H$  is the prey density, and  $P$  is the predator density. How, biologically, does this interaction differ from the Lotka-Volterra model? What happens in the absence of predation? What is the equilibrium where predator and prey coexist?

5. *Last Friday*. Consider a metapopulation with the “internal colonization” dynamics of deme initiation (colonization) and local extinction. Let  $p_t$  represent the fraction of patches occupied at time  $t$ ;  $0 < p_t \leq 1$ . We have:

$$\frac{dp_t}{dt} = c p_t (1 - p_t) - e p_t$$

Let the colonization rate parameter  $c = 0.1$ , and let the extinction rate parameter  $e = 0.02$ . Find the immigration-extinction equilibrium  $p^*$ . What is the colonization rate when  $p = p^*$ ?

$$p^* = 1 - (e/c) = 1 - (0.02/0.1) = 0.8$$

$$\text{Col}(0.8) = 0.1(0.8)(1 - 0.8) = 0.1(0.8)(0.2) = 0.016$$

$$\text{Ext}(0.8) = 0.02(0.8) = 0.016$$

6. *last Friday*. Again consider the metapopulation model:

$$\frac{dp_t}{dt} = c p_t (1 - p_t) - e p_t$$

At dynamic equilibrium,  $p^* = 0.6$ . Then, what is the ratio of the colonization rate parameter  $c$  to the extinction rate parameter  $e$ ?

7 . Species 1 and 2 compete for common resource(s). The dynamics has the form of Lotka-Volterra competition.  $N_i$  is the density of species  $i$  ( $i = 1, 2$ ). We have:

$$\frac{dN_1}{dt} = \frac{r_1 N_1}{50} (50 - N_1 - 0.5N_2)$$

$$\frac{dN_2}{dt} = \frac{r_2 N_2}{40} (40 - N_2 - 0.4N_1)$$

where  $r_i$  is the intrinsic rate of increase for species  $i$ . Show that the 0-isocline for sp. 1 is:

$$N_1 = 50 - 0.5N_2$$

Find the 0-isocline for sp. 2. In < 4 sentences, define the isoclines.

0-isocline for sp. 1:  $dN_1/dt = 0$   
 $r_1 > 0$ ; if  $N_1 > 0$ , then  $50 - N_1 - 0.5 N_2 = 0$

Therefore,  $N_1 = 50 - 0.5 N_2$  when  $dN_1/dt = 0$

**8.** Summarize the general invasion-analysis approach to interspecific competition, competitive exclusion and coexistence.

**9 .** Character displacement:

- A) reduces inter-specific competition through "niche differentiation"
- B) is a co-evolutionary response to inter-specific competition
- C) implies that competing species will differ more in allopatry than in sympatry
- D) A and B, but not C, are correct      E) A, B and C are correct

**10.** Compare rates of colonization in an “internal” meta-population model with the epidemic curve for an SI process.

Near extinction: low growth rate, sources rare  
Near saturation: low growth rate; unoccupied patches, susceptible hosts rare  
Maximal growth near equality of unoccupied and occupied, susceptible and infectious

**11.** Consider an island where species number  $S$  increases through immigration from a mainland, and decreases due to local extinction on the island. Let the immigration rate  $I(S) = P - S$ ;  $P$  is pool size. Let the extinction rate  $E(S) = mS$ . Find the equilibrium species number  $S^\wedge$ .

$I(S^\wedge) = E(S^\wedge)$ ; therefore,  $P - S^\wedge = m S^\wedge$ . Then  $P = S^\wedge (m + 1)$ , and

$$S^\wedge = P/(m + 1)$$

**12.** How might one ask experimentally if two co-occurring species compete? Consider two small, similar plant species.

**13.** Given a meta-population with internal colonization dynamics, how might one promote persistence of the of the subject species? Assume only an occupation (presence, absence) dynamics.