

Arrays: Arrangements of Elements

1. Matrices: Multiple Rows and Columns

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

2 Rows, 2 Columns, 4 Elements m_{ij}

2. Column Vector: Multiple Rows; One Column

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

3. Row Vector: One Row; Multiple Columns

$$\begin{bmatrix} x & y \end{bmatrix}$$

POSTMULTIPLY MATRIX BY COL. VECTOR

Patches of Vegetation:

Count 2 Types, x and y , at time t :

$$s(t) = \begin{bmatrix} x(t) = 40 \\ y(t) = 60 \end{bmatrix}$$

Want to Predict Community Counts *Next Year*:

Successional Time Interval

Want to Predict:

$$s(t + 1) = \begin{bmatrix} x(t + 1) \\ y(t + 1) \end{bmatrix}$$

Transition Matrix A ; Elements a_{ij}

$$0 \leq a_{ij} \leq 1$$

a_{ij} : Proportion of j -patches at time t

That Become i -patches at time $(t + 1)$

$$a_{xx}, a_{xy}; a_{yx}; a_{yy}$$

$$A = \begin{bmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix}$$

Note: $a_{xx} + a_{yx} = 1 = a_{xy} + a_{yy}$
Columns sum to unity

Markov Chain Model for Succession

$$s(t+1) = As(t)$$

$$s(t+1) = \begin{bmatrix} x(t+1) \\ y(t+1) \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$\begin{bmatrix} x(t+1) \\ y(t+1) \end{bmatrix} = \begin{bmatrix} a_{xx}x(t) + a_{xy}y(t) \\ a_{yx}x(t) + a_{yy}y(t) \end{bmatrix}$$

Account for all $x(t)$ and all $y(t)$

Iterate: $(t+2), (t+3), \dots, \infty$

Example

$$s(t) = \begin{bmatrix} x_t = 40 \\ y_t = 60 \end{bmatrix}$$

Transition matrix:

$$\begin{bmatrix} 0.2 & 0.6 \\ 0.8 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} 0.2(40) + 0.6(60) \\ 0.8(40) + 0.4(60) \end{bmatrix} = \begin{bmatrix} 8 + 36 \\ 32 + 24 \end{bmatrix} = \begin{bmatrix} 44 \\ 56 \end{bmatrix}$$

New Community Pattern

Succession to Equilibrium?

At Equilibrium: $s(t + 1) = s(t) = s^*$

Find Equilibrium:

$$s^* = As^*$$

$$s^* = \begin{bmatrix} \frac{3}{7}(100) \\ \frac{4}{7}(100) \end{bmatrix}$$

s^* Independent of Initial State

Are Transition Probabilities Stationary?

Spatial Structure?