Geometric Mean Growth

Discrete time: t, $(t + 1)$, $(t+2)$, …

N_t Population density at time t

Non-overlapping *generations*

Consecutive generations may experience different environments

Assume:

$$N_{t+1}/N_t = \lambda(t)$$

Growth rate at time t depends on current environment

Growth rate density-independent

Time-dependent growth rate

$\lambda(t)$ varies among generations

Environmental stochasticity

Random, exogenous signal; varies temporally

Within a generation, individuals same environment

Among generations, different environments

Temporal variation density-independent

Usually abiotic; can be biotic

Simple example:

Understand significance of temporal variation for population dynamics

Time: years = generations

$$\lambda(t) \in \{\lambda_B, \lambda_G\}; \qquad \lambda_G > \lambda_B > 0$$

G: good year *B*: bad year

Suppose that generations independent

λ_B and λ_G each occur with Pr = ½; (fair coin toss)

Initial population size N_0

Random process

What can we predict about average long-term growth?

Mean behavior of dynamics? Persist or extinct?

$N(t) = N_0$ [$\lambda(0)$ $\lambda(1)$ $\lambda(2)$ …. $\lambda(t-1)$]

Single quantity to characterize expected long-term growth

Geometric mean growth rate $\hat{\lambda}$

$N(\text{t}) = [\ \lambda(0)\ \ \lambda(1)\ \ \lambda(2)\ \ \ldots.\ \ \lambda(t-1)\ ]\ \ N_0$

$$\boxed{N_t \;=\; \left(\hat{\lambda}\right)^t N_0}$$

$t \rightarrow \infty$

$$\lambda(0)\ \ \lambda(1)\ \ \lambda(2)\ \ \ldots.\ \ \lambda(t-1) \rightarrow \lambda_B^{t/2}\ \lambda_G^{t/2}$$

Then

$$(\ \hat{\lambda}\ )^t = \lambda_B^{t/2}\ \lambda_G^{t/2} \quad \text{and} \quad \hat{\lambda}\ = (\lambda_B\ \lambda_G)^{1/2}$$

$\hat{\lambda}$ **Geometric mean of random growth rates**

In general

Suppose $\lambda(t) = \lambda_i$ with probability p_i; $Pr[\lambda_i] = p_i$

Then geometric mean of the λ_i growth rates $= \prod_i \lambda_i^{p_i}$

1. Raise each λ_i to power p_i
2. Multiply to obtain geometric mean

$$\tilde{\lambda} = \prod_i \lambda_i^{p_i}$$

$\lambda_1 = 1 \quad p_1 = 1/3$

$\lambda_2 = 8 \quad p_2 = 2/3$

$$\tilde{\lambda} = 1^{1/3} \cdot 8^{2/3} = 1 \cdot 2^2 = 4$$

Long-term persistence: require $\hat{\lambda}$ **> 1**

Predict extinction: $\hat{\lambda}$ **< 1**

Predict persistence? $\lambda = \begin{bmatrix} 1/2 \\ 2/3 \\ 6 \end{bmatrix} \quad p = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$

$$\hat{\lambda} = \mathbf{(1/2)^{1/3}\ (2/3)^{1/3}\ (6)^{1/3} = 2^{1/3} > 1}$$

Recall:

Arithmetic mean = $\sum_i p_i \lambda_i$

Temporal variation > 0

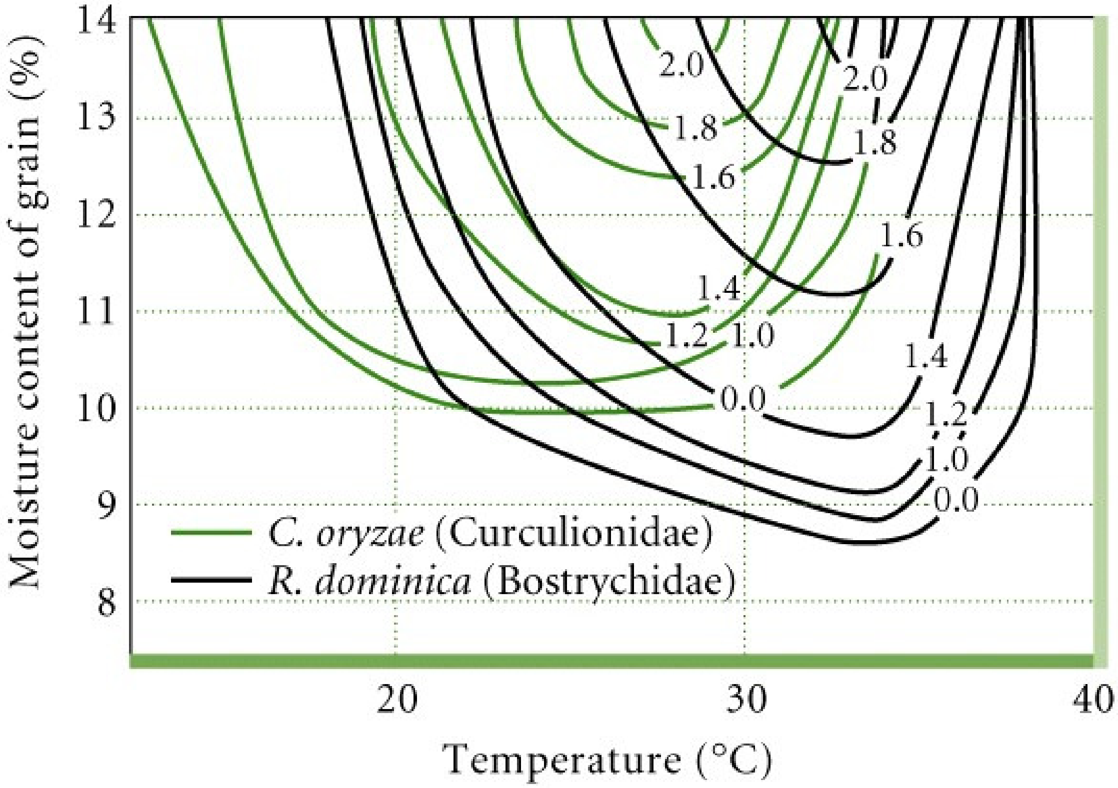
Geometric mean < Arithmetic mean

Temporal Niches

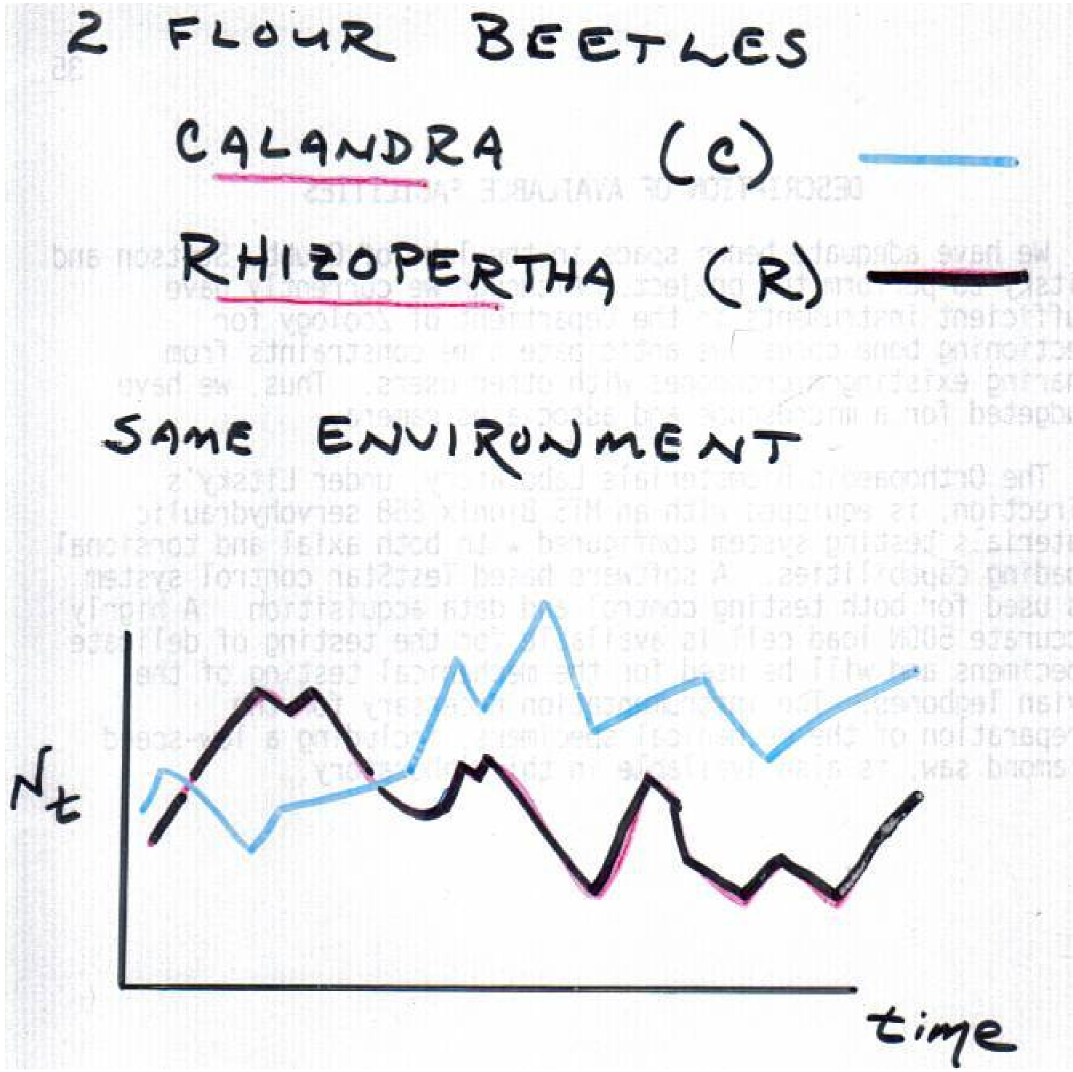
2 Spp: Use same biotic resource (food), but respond

differently to physical environment

Coexist? Will better competitor (for resource) exclude other?



Ricklefs and Miller (1999)



Growth negatively correlated

Spp coexist: temporal niches differ