Temporal Environmental Variation:

Among-Generation Variance in Growth Rate

Environmental Stochasticity:

Abiotic Variation at Generational Scale

All Same-generation Individuals Experience

Same Environment

Demographic Stochasticity:

Variation Among Individuals, Same Generation

Density-Independence & Temporal Dependence

Generation, Time *t*: Growth Rate $\lambda(t)$

$\lambda(t)$ Varies Randomly

N_t Population Density, time *t*, Varies Randomly

Given N_0, Initial Population Size

$\overline{N}(t)$ **Expected (Average) Population at time $t > 0$**

Geometric Mean Growth

$$\overline{N}_t = N_0 \left(\tilde{\lambda}\right)^t$$

Growth Rates λ_i $i = 1, 2, \ldots;$ $\Pr[\lambda_i] = p_i$

$$\tilde{\lambda} = \prod_i \left(\lambda_i\right)^{p_i}$$

Geometric-Mean Growth Rate

$\tilde{\lambda} > 1 \Rightarrow$ Persistence

Growth Rates λ_i

Arithmetic Mean $\mu = \sum_i p_i \lambda_i$

Variance $\sigma^2 = \sum_i p_i (\lambda_i - \mu)^2$

How does $\tilde{\lambda}$ depend on μ; on σ^2 ?

1. Fix Variance, then $\partial\tilde{\lambda}/\partial\mu > 0$

Larger Arithmetic Mean Favored

2. Fix Arithmetic Mean, then $\partial\tilde{\lambda}/\partial\sigma^2 < 0$

Reduced Variance Favored

For $\tilde{\lambda} \approx 1$ $\tilde{\lambda} \approx \mu - \frac{\sigma^2}{2}$

Select traits reducing growth-rate variability?

example?

Given $\tilde{\lambda} \approx \mu - \frac{\sigma^2}{2}$, and $\tilde{\lambda} < 1$ implies extinction

$\sigma > \sqrt{2\mu}$ assures extinction

Too-variable populations gone?

What environments increase (limit) temporal variation in growth rate for given organism?

Consequences of climate change for among-generation environmental variability?

When does temporal variation reduce/increment species diversity?

Text p.14: *Continuous Time*:

Exponential Growth with **Environmental Stochasticity**

Instantaneous growth rate r varies temporally

Each individual at time *t* experiences same *r*(*t*)

r has *arithmetic mean* ρ

Expected population size $\overline{N}(t)$

$$\boxed{\overline{N}(t) = N_0 \, e^{\rho t}}$$

Recall: $r = \ln \lambda$, and

$$\tilde{\lambda} = \prod_i (\lambda_i)^{p_i}$$

$$\ln \tilde{\lambda} = \sum_i p_i \ln \lambda_i$$

Hence, have arithmetic mean of r

Geometric mean growth and exponential growth under environmental stochasticity yield same general lessons/predictions.

Demographic Stochasticity

Even in constant environment, individuals will survive/reproduce differentially, due to "chance."

Fundamental discreteness of individuals implies that birth and death not continuous processes.

Demographic stochasticity: **among-individual variation** (*wrt* birth and death) **within same generation**.

Fundamentally important to dynamics of rarity:

Small population of invaders

Resident population fallen to extinction brink

Important for prediction:

Rates/probabilities of birth and death

Initial population size