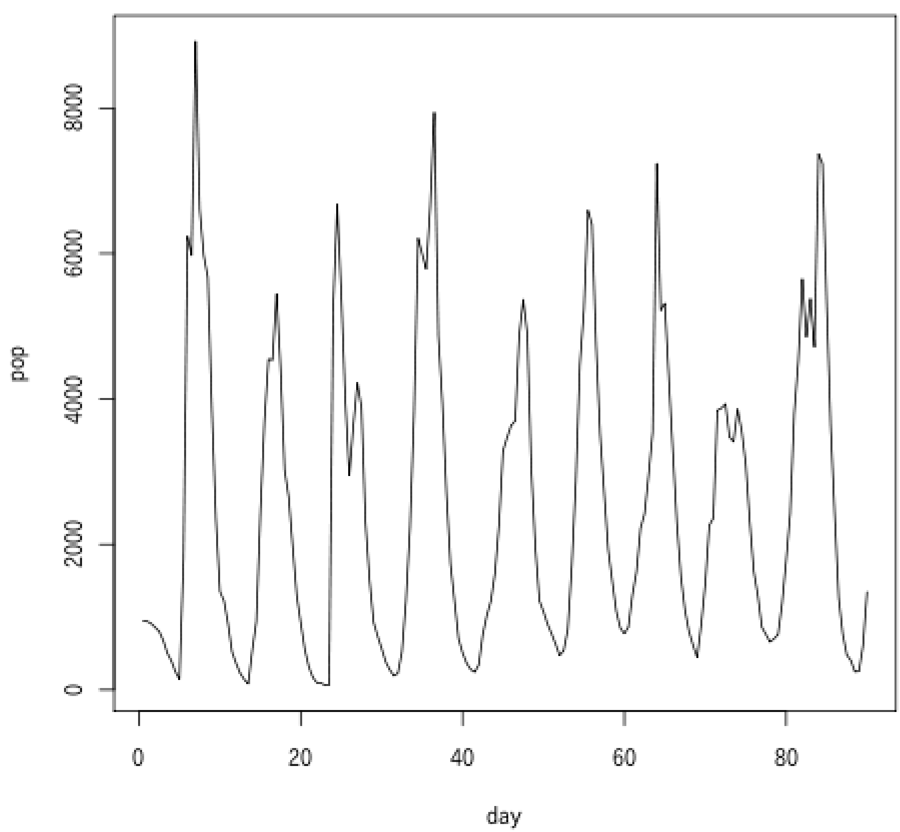
Logistic growth: Discrete-time dynamics

Self-regulation ⇔ Intra-specific competition

Discrete generations: inherent "time delay"

Population density over time:



Abiotic variation? Biotic effect?

1-D maps (one-dimension: population density)

Discrete time

$$N(t+1) \quad = \quad F[N(t)]\,N(t)$$

$F[N(t)]$ Individuals per individual; Density-dependent

Map from time t to time $(t + 1)$

Equilibrium?

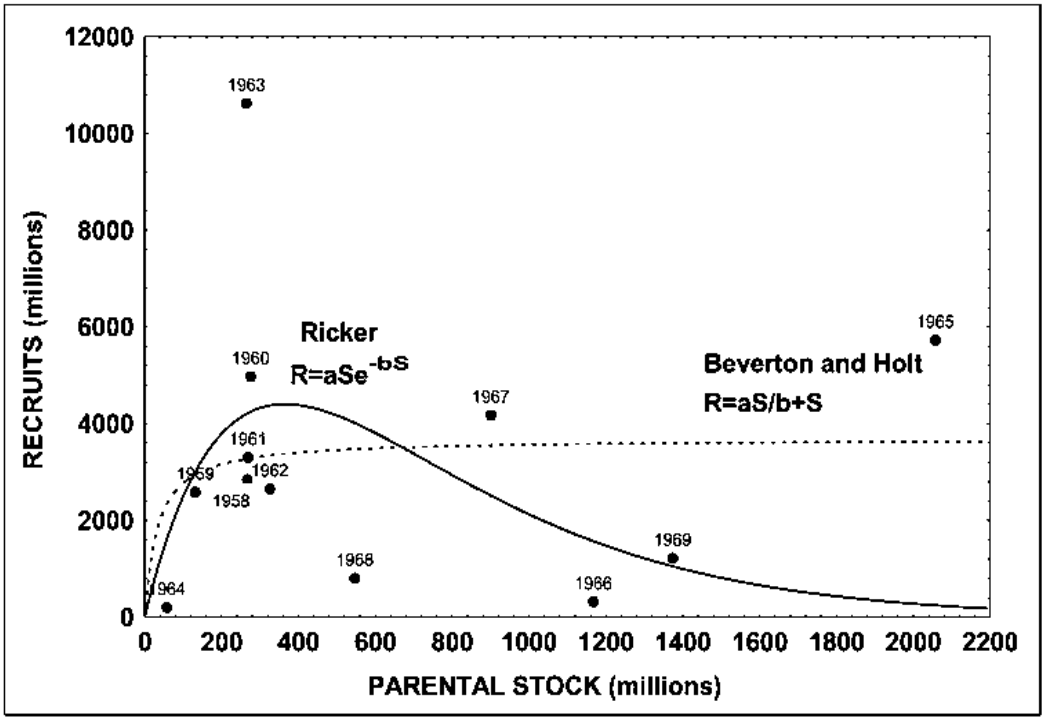
$$N(t+1) \quad = \quad N(t) \quad = \quad N^*$$

Therefore

$$N^* \quad = \quad F[N^*]\,N^*; \quad F[N^*] = 1$$

Each individual replaces itself at positive equilibrium

Fisheries: Stock-recruitment



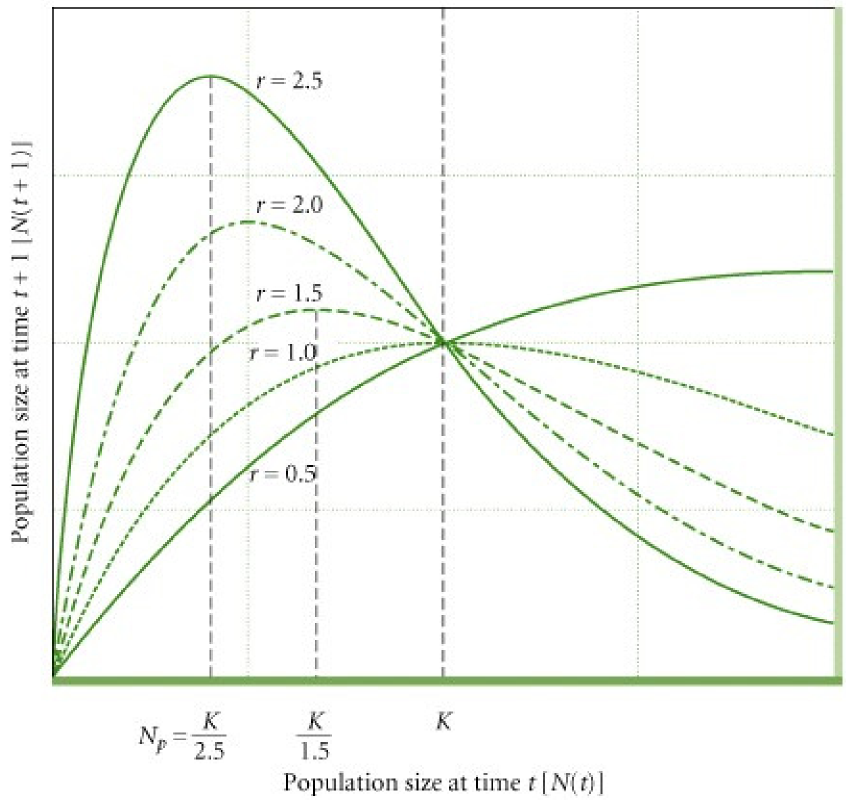
Normal compensation $N^* = 0,\, K$

Depensation $N^* = 0,\, K$

Critical depensation $N^* = 0,\, K_0, K$

Overcompensation $N^* = 0,\, K$

Overcompensation: Fast growth at low density, $N_t > K$



"Ricker curves" $$N_{t+1} = N_t \exp\left[r \left(1 - N_t / K \right) \right]$$ $r, K > 0$

$r \leq 1$ Normal compensation, Strong self-regulation

$r > 1$ Overcompensation, Weaker, delayed self-regulation

Alstad book: *Discrete-time logistic growth*

Equilibrium **nodes**: $N^* = 0, K$

Extinction unstable; carrying capacity?

Wednesday

Pierre Simon Laplace

Laplace's Demon: If an intelligence knew the position of all the bodies, their masses, and vector velocities (speed and direction), that "demon" could calculate the future positions of all objects ("know everything").

Premised by continuity, the calculus: $dX/dt = F[X(t), \Theta(t)]$

Continuous-time logistic growth

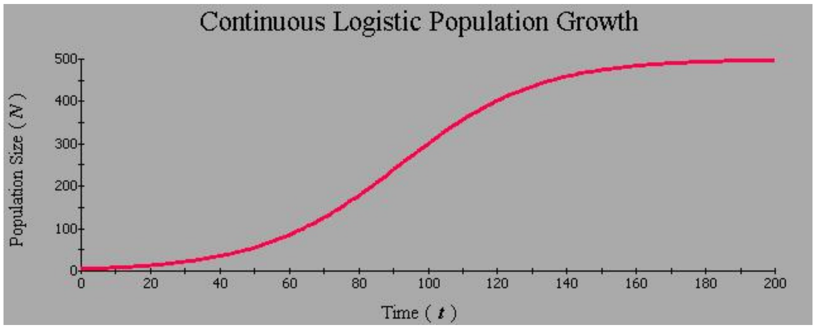
"*Small change in input produces small change in output*"

Deterministic = Predictable, Orderly, Understandable

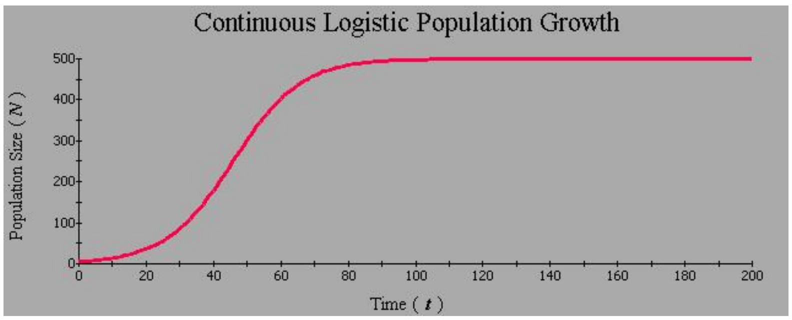
$$dN_t/dt = r N_t \left(1 - N_t/K\right)$$ *Instantaneous density-dependence*

Increase intrinsic rate *r*?

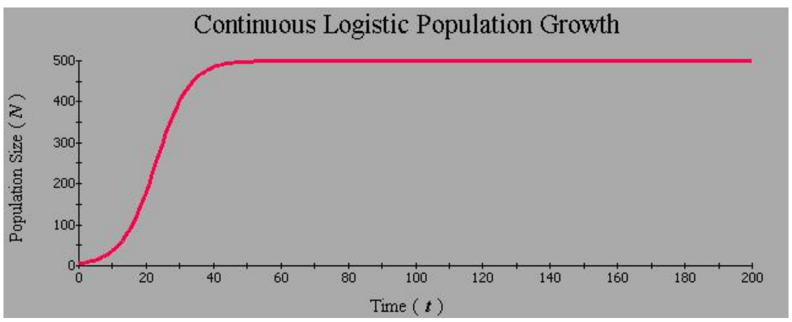
$r = 0.05$



$r = 0.1$



$r = 0.2$

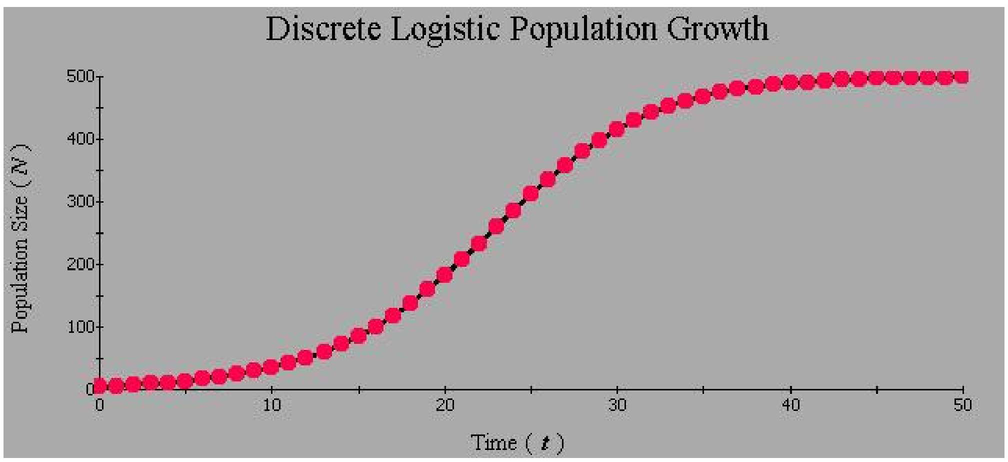


Discrete-time logistic $$N_{t+1} = N_t \exp\left[r\left(1 - \frac{N_t}{K}\right)\right]$$

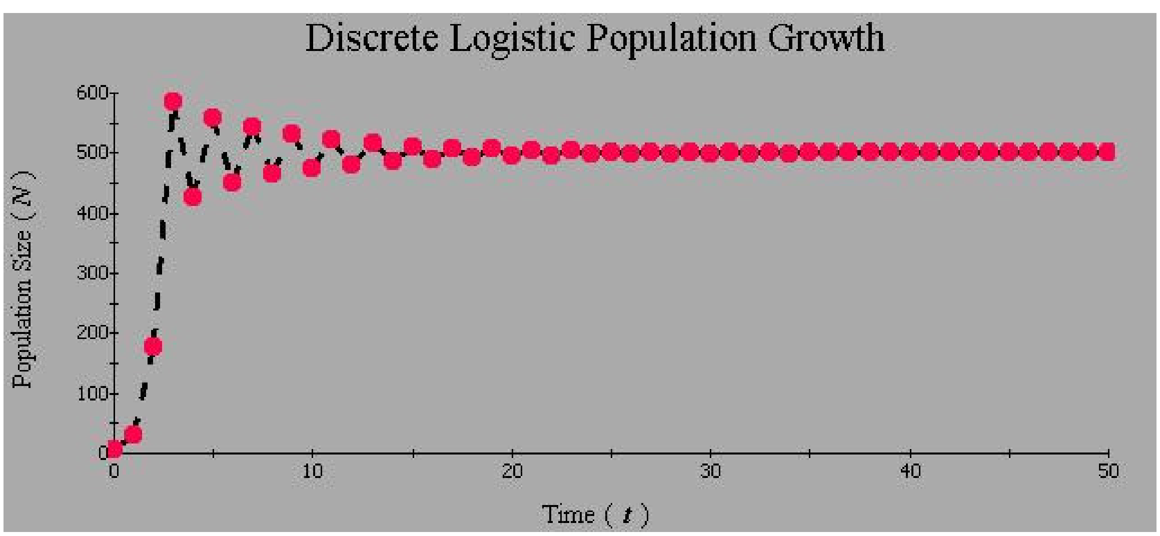
Increase *r* through family of Ricker curves

$N_0 = 10$; K = 500 held constant

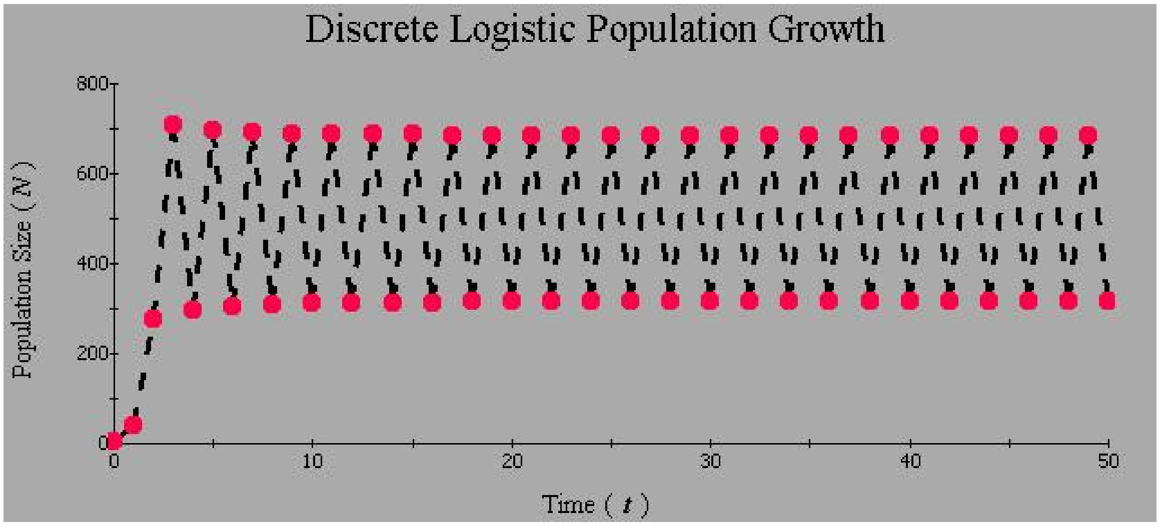
$r = 0.25$



$r = 1.9$

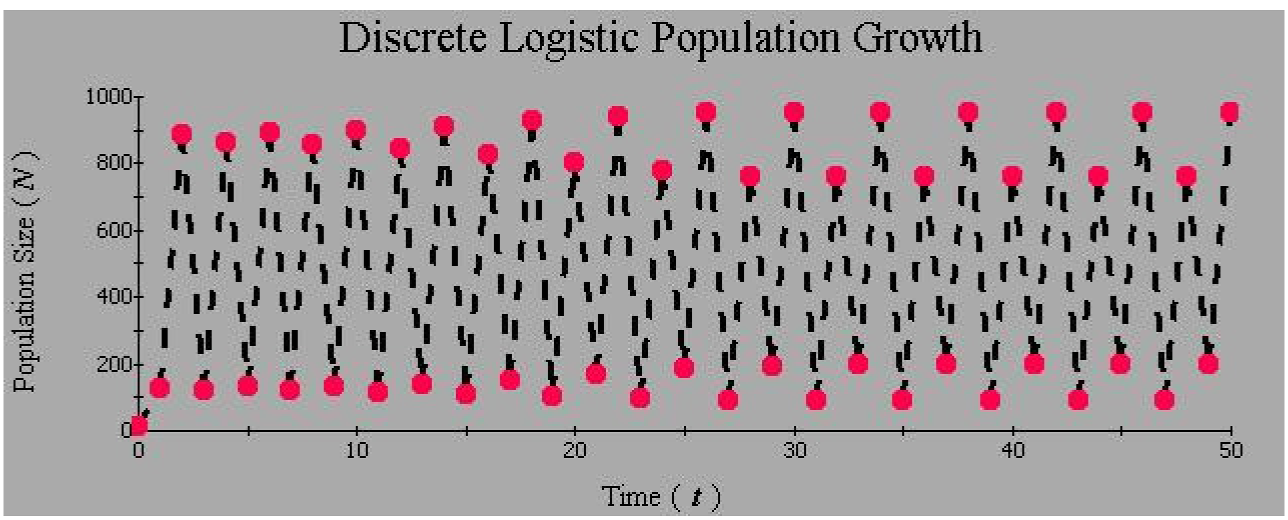


r = 2.2



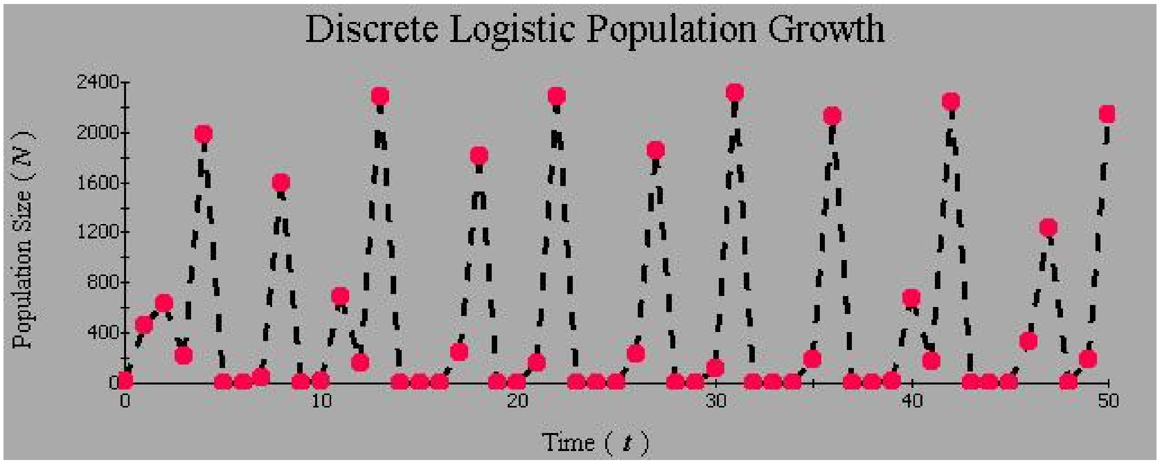
Pitchfork Bifurcation: Stable node splits into stable 2-cycle

r = 2.6



Stable 4-cycle

$r = 3.9$



Deterministic Chaos

Period-doubling route to chaos as self-regulation at low density relaxed; equivalently, as overcompensation increased

Increase r: 2^k cycles; $k = 0, 1, 2, 3, \ldots \infty$

Stable node ($k = 0$), stable cycles, chaos

Chaos

1. **Aperiodic**: no value *ever* repeats

   Not random, might look so

2. **Bounded**: max value, min value > 0

   May be *close to extinction*

3. **Sensitive to initial conditions**

   "Nearby trajectories diverge"

Strange attractor: each point repels!

Impossibility of prediction (sensu Laplace)

Small change input (r) produces large, qualitative change in output

Deterministic chaos "looks" like random process