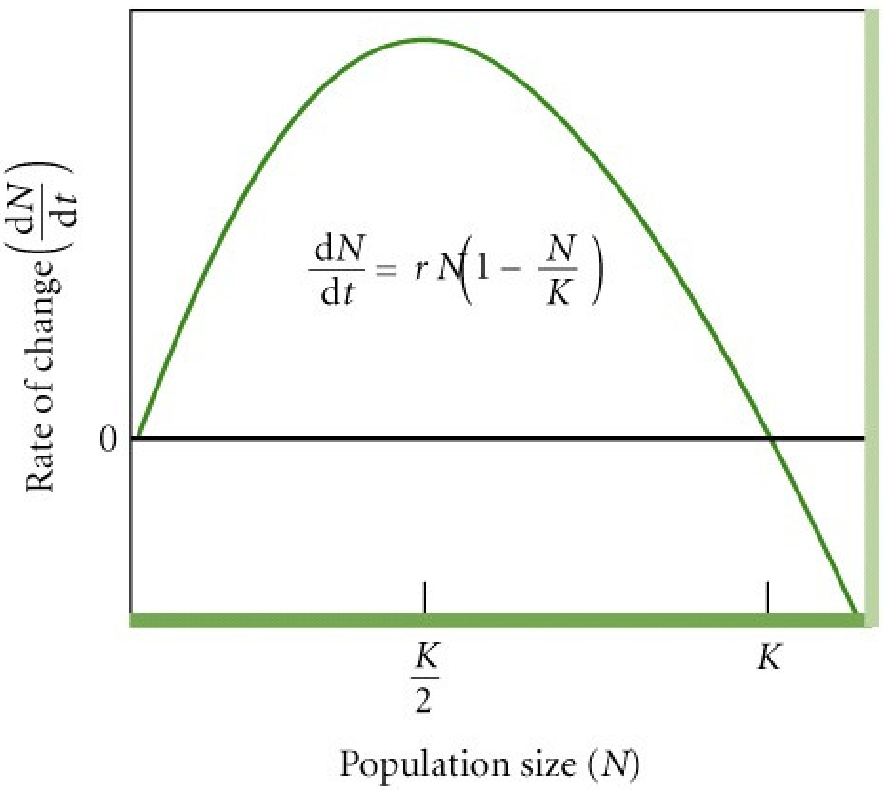
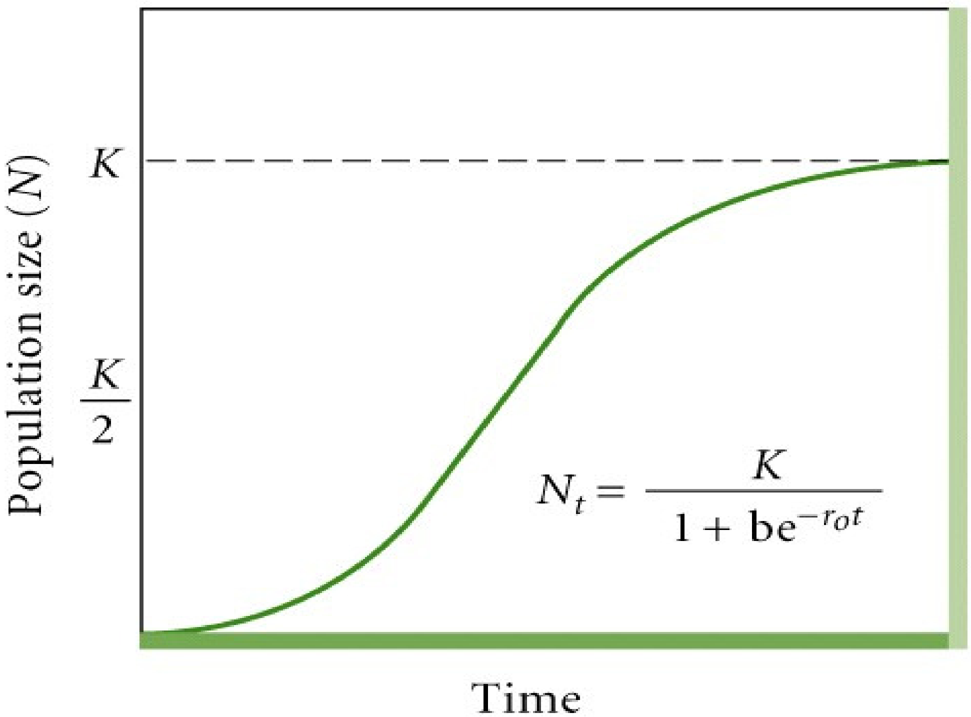


Logistic Growth: Quadratic, **No Time Delay**, K Constant



b = [K - N(0)]/N(0)

Logistic Population Growth

Simple Assumption: Instantaneous Feedback

$dN(t)/dt = F[N(t)]\,N(t)$

Each Individual Added Immediately Reduces $F[N(t)]$

Australian Sheep: maximal growth rate 25 yrs

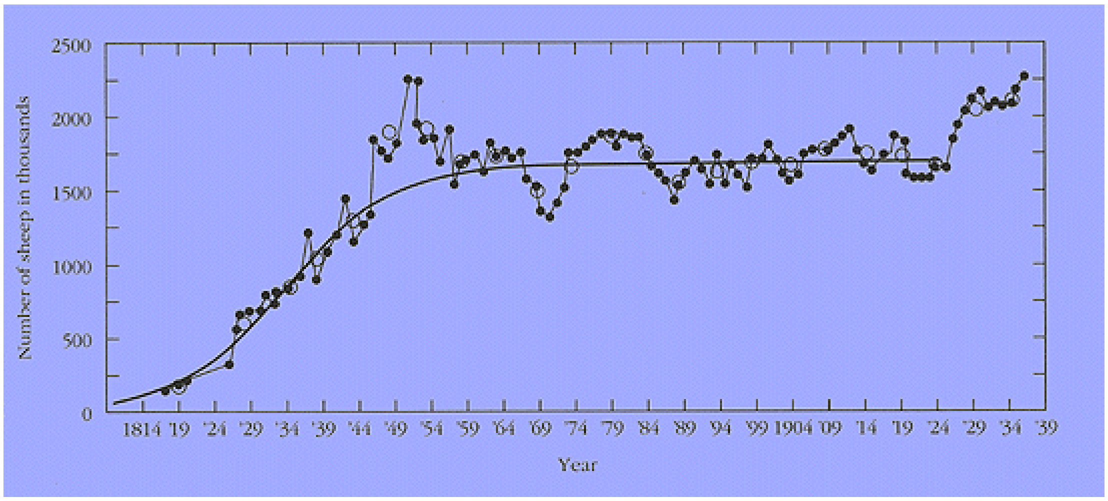
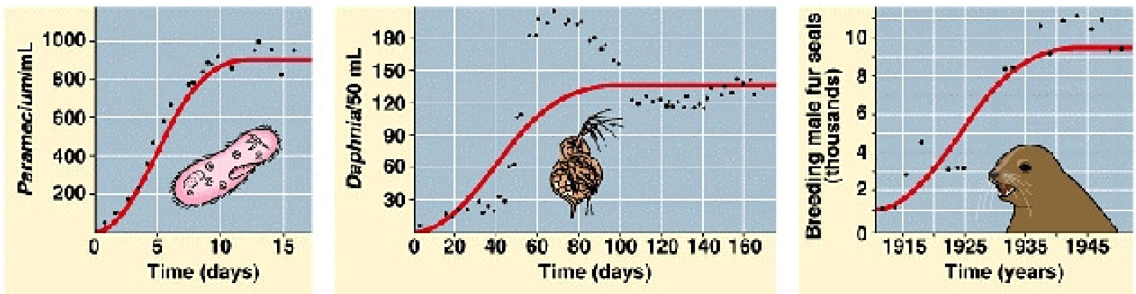


Figure 52.15 Examples of logistic population growth



(a) A *Paramecium* population in laboratory culture

(b) A *Daphnia* population in laboratory culture

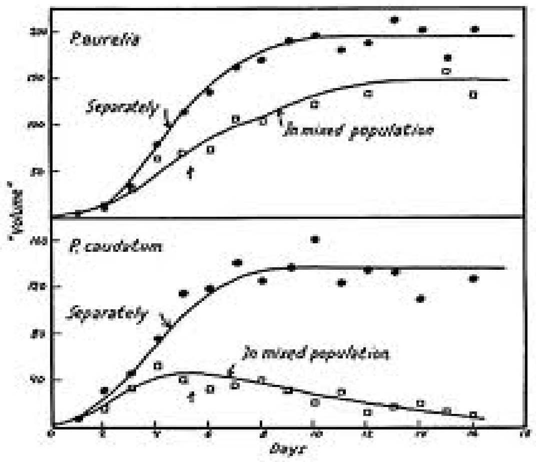
(c) A fur seal (*Callorhinus ursinus*) population on St. Paul Island, Alaska

Pearson Education, Benjamin-Cummings

Protozoan, small metazoan & large mammal:

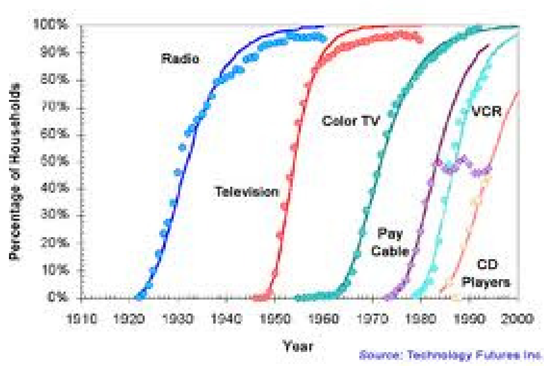
Logistic growth (app.), Increasing time to max dN/dt

Gause 1930's: *Paramecia* spp.



Sigmoid growth: arises from many dynamics

Ecology, chemistry, epidemiology, sociology, technology



How might logistic growth's assumptions fail biologically?

Density-dependent feedback delayed during maturation

Age (Time to Maturity), Size May *Delay*
Newborn Individual's Impact on Per-Capitum Growth

Alternative Assumption: Time Delay, Lagged Logistic

$$dN(t)/dt = F[N(t - \tau)]\, N(t)$$

τ Time Delay

At time *t*, Density Dependence Responds to

Population Size at time (t - τ); i.e. τ Time Units *Earlier*

Population Can Grow Beyond *K*

Time Lag May Destabilize *K*

Standard Logistic Modified with Time Delay

$$dN(t)/dt = rN(t)\left[1 - \frac{N(t-\tau)}{K}\right]$$

K: Carrying Capacity, Constant

r: Intrinsic Rate of Increase ~
Individual Growth Rate When Rare

τ: Time Delay

Population Reproduction:
Responds to Density τ Time Units Earlier

Predict Population Behavior?

Key: Delay ÷ "Response Time" = $r\tau$

$\tau \to 0$, Standard Stability Condition ($r > 0$)

$\tau > 0$:

$r\tau$ small enough ($r\tau < e^{-1} = 0.368$):

Standard Increase to Stable K

$r\tau$ "intermediate" ($e^{-1} = 0.368 < r\tau < 1.571 = \pi/2$):

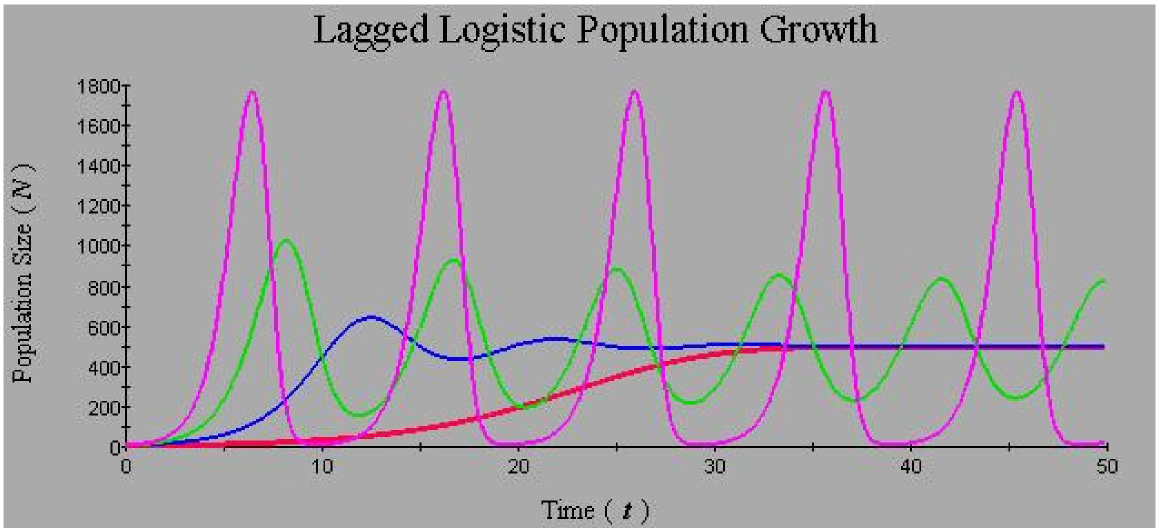
Damped Oscillations; K stable

$r\tau$ large enough ($r\tau > 1.57 = \pi/2$):

Stable Limit Cycles; K Unstable: Focus of Cycles

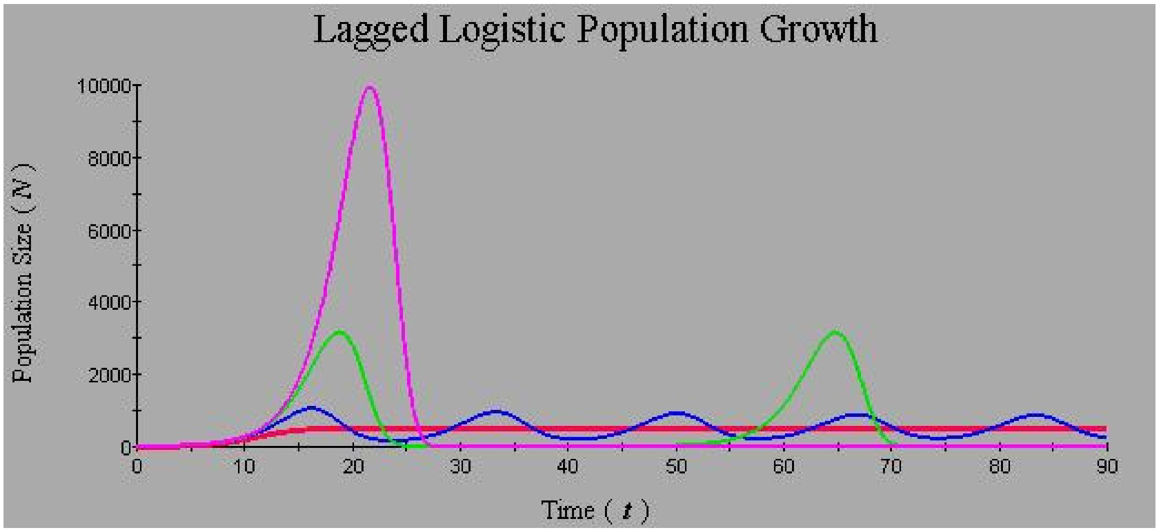
Fix time Lag ($\tau = 2$) $K = 500$

Increase *r*, Population Responds “Too Fast”



Greater *r*, Larger Population Fluctuation

Fix *r*, Increase Time Lag



Greater Lag, Larger Fluctuation, Lower Frequency

Period of Stable Limit Cycle = 4τ (Independently of *r*)

Frequency = 1/Period

Lesson: **Time delay** in density-dependent population regulation may **destabilize** population growth, and increase **dynamic complexity**.