Mark-Recapture Problem

t_1 Capture, mark & release 30 *P. maniculatus*

t_2 Capture both marked & unmarked mice

36 unmarked individuals

Estimate population size 210 individuals

How many recaptures?

M Marked — N Estimated Population Size

x recaptures — $(n - x)$ Unmarked at t_2

Assume:

$$M/N = x/n$$

$$30/210 = x/(x + 36) \Rightarrow 210\,x = 30\,x + 1080$$

$$180\,x = 1080 \Rightarrow \mathbf{x = 1080/180 = 6} \quad (n = 42)$$

Exponential-Growth Problem

Let $r = 0.8$ and $N_5 = 180$

What was $\mathbf{N_{2.5}}$ (closest integer)?

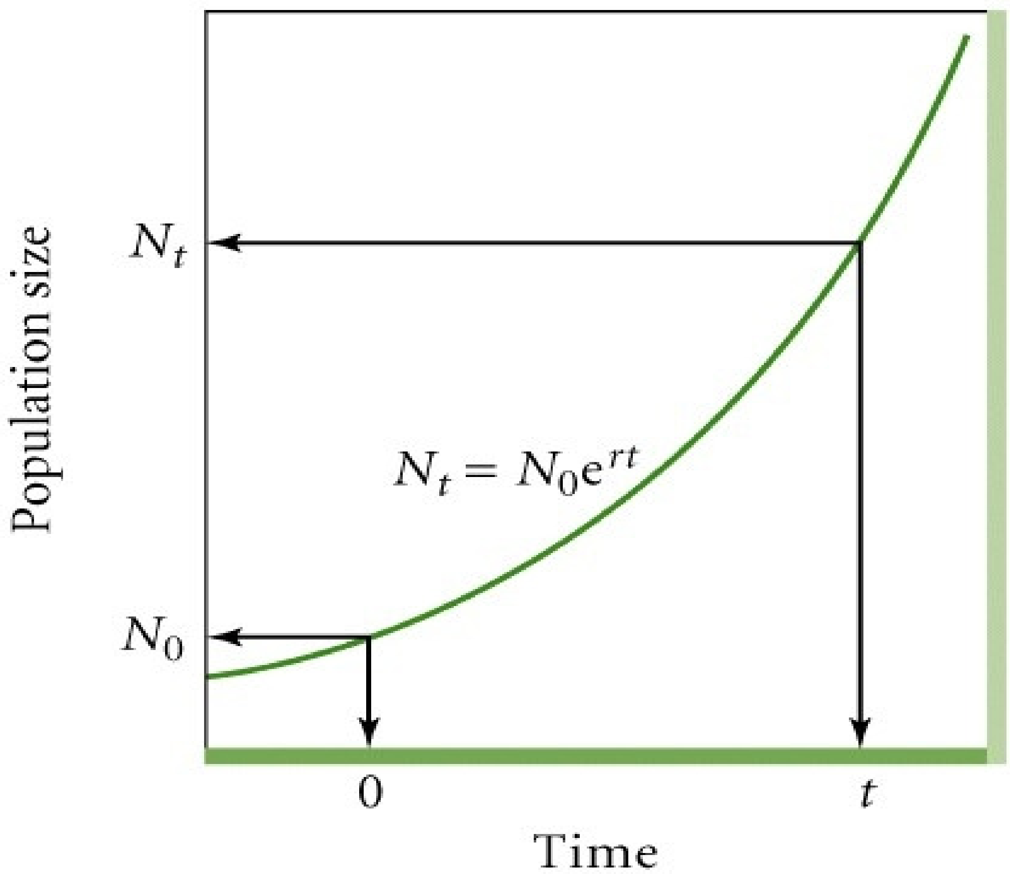
Know $N_t = N_0 \, e^{rt}$

Then $N_5 = N_{2.5} \, e^{rt}$ with $t = 2.5$

$180 = N_{2.5} \, e^{0.8\,(2.5)} \;\Rightarrow\; N_{2.5} = 180 \, e^{-2}$

$\boldsymbol{N_{2.5} = 24.36 \approx 24}$

Exponential Growth



$$\frac{dN}{dt} = rN$$

Population dynamics

Density-independent

$$r = b - d$$

Individual level, constant

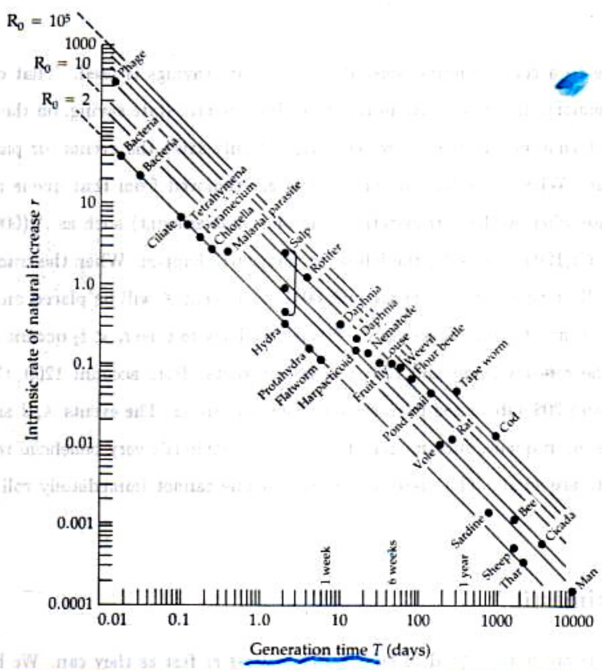


***Figure 9.10** The relationship of the intrinsic rate of natural increase, r, and generation time, with diagonal lines representing values of R_0 from 2 to 10^5, for a variety of organisms.* (Redrawn from Heron 1972.)

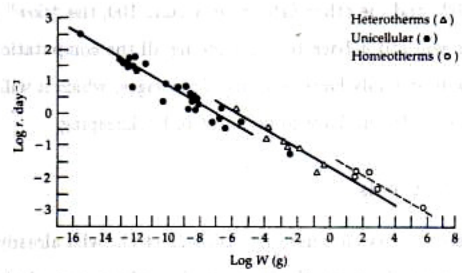


***Figure 9.11** The relationships of the intrinsic rate of natural increase to weight for various animals.* (Redrawn from Fenchel 1974.)

Large *r*: Short generation, small mass

Large *r*: Long generation, large mass

Geometric Growth

Discrete time: t, (t + 1), (t + 2), …

N_t Population density at time t

"Annual" organisms

Assume:

$$N_{t+1} / N_t = \lambda$$

Growth rate taken as constant

Density-independent and time-independent

Discrete-time parallel to exponential growth

$$\Delta N_t = N_{t+1} - N_t$$

$$= (\lambda N_t) - N_t = N_t(\lambda - 1)$$

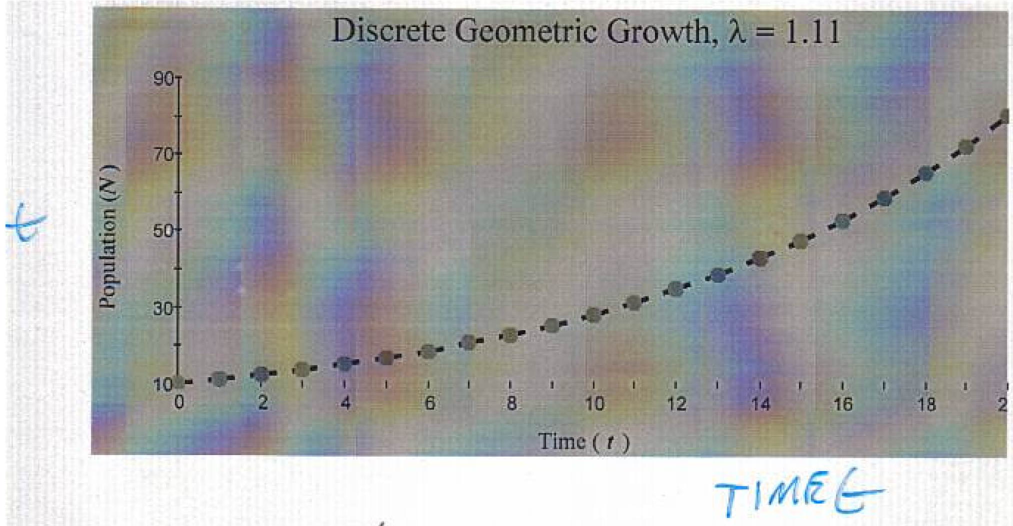
$\lambda > 1 \iff \Delta N_t > 0$ Unbounded growth

$\lambda < 1 \iff \Delta N_t < 0$ Decline to extinction

Geometric growth

Constant individuals/individual

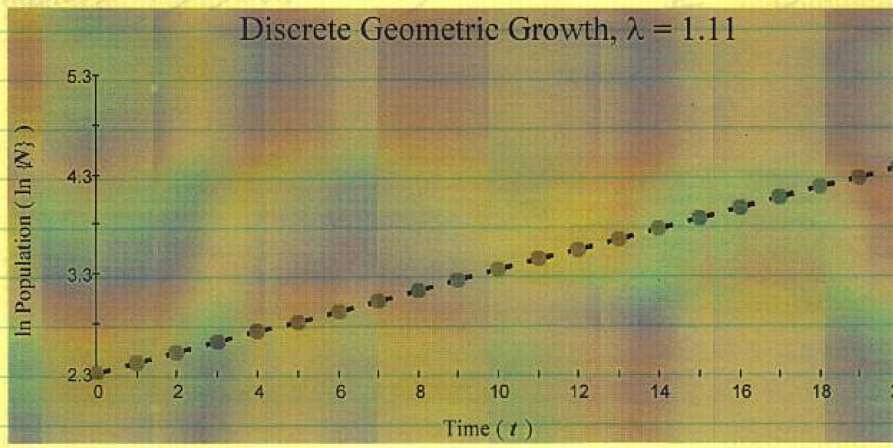
$$\boxed{N_t = \lambda^t N_0}$$



$N_t = \lambda^t N_0$

GEOMETRIC GROWTH

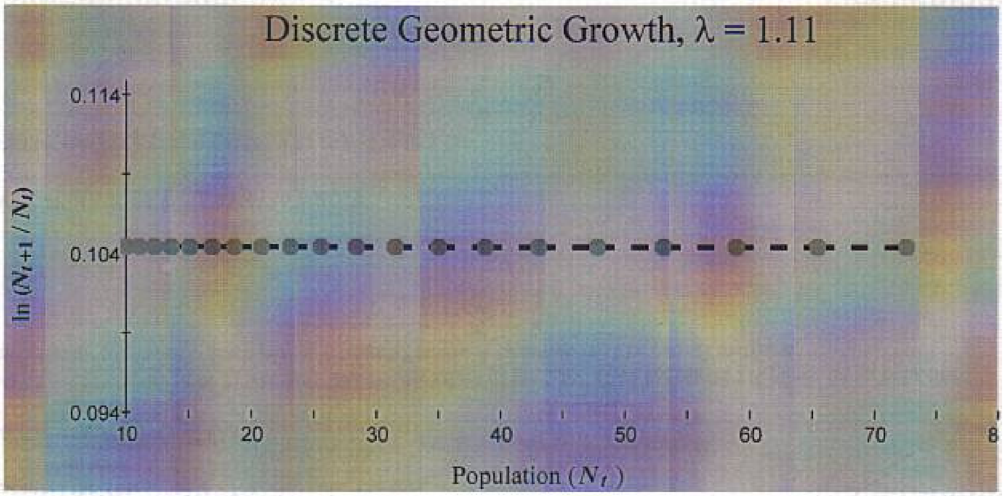
∈ DENSITY-INDEPENDENT GROWTH



LOG N_t

TIME t

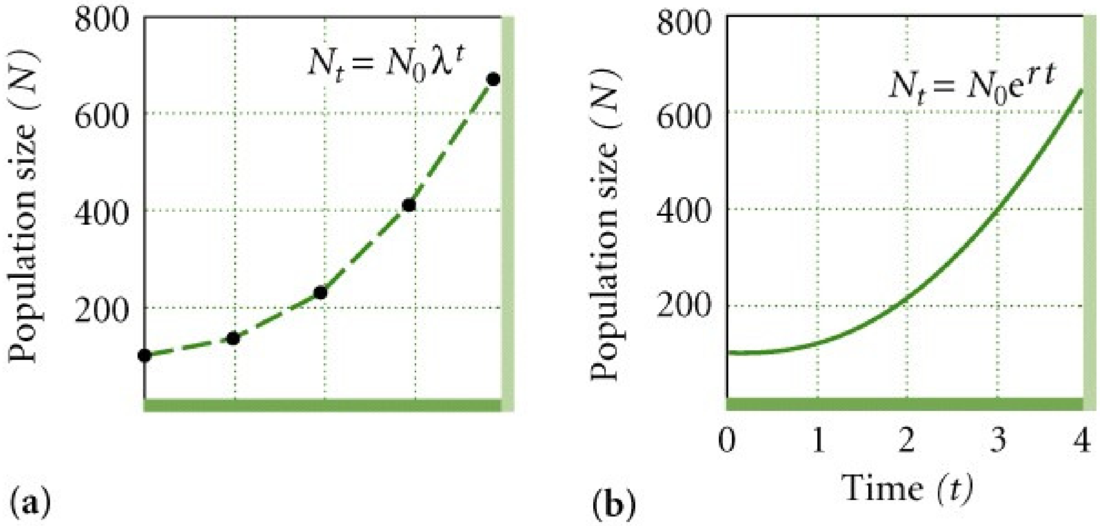
LINEAR



$LOG\ (N_{t+1} / N_t) = LOG\ \lambda$

N_t POPULATION SIZE

DENSITY - INDEPENDENCE



From Ricklefs and Miller (1999)

Exponential Growth

Overlapping Generations

Birth, Death Continuous Processes

$$t \geq 0$$

Geometric Growth

Non-Overlapping Generations

Birth, Death Seasonal: "Annuals"

$$t = 0, 1, 2, \ldots$$

1. For any continuous-time population dynamics, there is a faithful discrete-time dynamics.

2. For *some* discrete-time population dynamics, there is *no* faithful continuous-time model.

Good to know something about discrete-time growth.