

1.  $V(t)$  and  $E(t)$  are, respectively, prey density and predator density at time  $t$ . The densities have dynamics:

$$dV(t)/dt = V - 0.01 VE$$

$$dE(t)/dt = 0.002 VE - E$$

Find the positive equilibrium densities; recall that you first set both rates to zero.

2. Graphically analyze the “paradox of enrichment.”
3. Let  $S_t$ ,  $I_t$  and  $R_t$  represent, respectively, the number of susceptible, infective, and removed individuals in a general epidemic.  $S_0$  is the initial number of susceptibles, and  $I_0 = 1$ . Consider the dynamics:

$$dS/dt = -0.001 S_t I_t$$

$$dI/dt = 0.001 S_t I_t - 0.5 I_t$$

$$dR/dt = 0.5 I_t$$

Note that the “infection rate” appears in the first two equations, and that the removal rate appears in the second and third equations. Find the relative removal rate. If the initial population of susceptibles is  $S_0 = 501$  individuals, do we expect the infection to advance to an epidemic?

4. Consider the predator-prey system with dynamics

$$dH / dt = rH - \alpha HP - cH^2$$

$$dP / dt = \alpha HP - mP$$

where  $H$  is the prey density, and  $P$  is the predator density. How, biologically, does this interaction differ from the introductory Lotka-Volterra model of predator and prey? What happens in the absence of predation? What is the equilibrium where predator and prey coexist?