

Published 29 September 2008, Answers Due in Class Wednesday, 8 October 2008

1. In discrete time, a population with density-dependent dynamics grows according to:

$$x_{t+1} = 2 x_t (1 - x_t)$$

where  $x_t$  is proportional density. At dynamic equilibrium  $x^* = x_{t+1} = x_t$ . Find the equilibrium  $x^*$ .

2. At time  $t$  a population has size  $N_t$ , where  $0 \leq N_t \leq N_{\text{maximum}}$ . Let  $x_t = N_t/N_{\text{maximum}}$ , so that  $0 \leq x_t \leq 1$ . In discrete time  $x_t$  has dynamics:

$$x_{t+1} = 2.5 x_t (1 - x_t)$$

At positive equilibrium  $x_{t+1} = x_t > 0$ . (a) Find the positive equilibrium. (b) What will happen (briefly) if  $x_{t+1} = 0.5 x_t (1 - x_t)$ ?

3. Consider one form of the discrete time logistic:

$$N_{t+1} = N_t e^{r \left(1 - \frac{N_t}{K}\right)}$$

Show that  $N = K$  at positive equilibrium.

4. The following life table shows an initial cohort and the number of individuals left alive as age  $x$  advances.  $b_x$  is the average number of births per  $x$ -yr old. Estimate the intrinsic rate of increase. To do so, you must first find  $R_0$ , then find generation time, and then estimate the intrinsic rate of increase (under the assumption of density-independent growth).

Age	No. Alive	$b_x$
0	560	0.5
1	350	8/5
2	70	1.6
3	0	-

5. This problem has parts A and B.

(A) Briefly describe the hypothesis of character displacement between species.

(B) Consider a population with the following discrete-time dynamics

$$N_{t+1} = N_t \left[ \left(1 + r\right) - \frac{r}{K} N_t \right],$$

where  $r, K > 0$ . Show that if  $N_t = K$ , the population size has reached positive equilibrium.