BIO 612 **4 September 08**

Population Dynamics of Infectious Disease

Infection of Host by Microparasite

1. Contact with free-living pathogen stage

Bacillus anthrax spores in soil
Nuclear polyhedrosis virus on vegetation
Windborne, waterborne pathogen dispersal

2. Contact with biotic vector

Parasitism preceding pathogen transmission
Animal macroparasites: mosquito, tick ,fly
Parasitic plants: *Dodder* spp
Aphids: persistent and non-persistent viruses

3. Contact with previously infected host

S susceptible
E exposed (latent period)
I infectious
R removed (dead, recover with immunity)

Compartment models infection by direct-contact

Compare two transmission dynamics

Direct-contact transmission

Assumptions $(S, I) \rightarrow (S - 1, I + 1)$

Lotka-Volterra (density-dependent)

Mass action (frequency-dependent)

Local, probabilistic

SI Model with Immigration, Include Virulence

S_t **susceptible density, time** t

I_t **infectious-host density, time** t

Density-dependent dynamics:

$$dS/dt = \lambda - \mu S - \beta I S$$
$$dI/dt = \beta I S - (\mu + \gamma) I$$

λ **Birth/Immigration constant**

μ **Natural mortality, density-independent**

γ **Virulence (extra mortality from disease)**

β **Transmission coefficient**

Lotka-Volterra: Rate of contacts proportional to product of densities, IS

Pr [Transmit pathogen | contact] = β

Generalization $\beta I^p S^q$

Equilibrium Nodes

$$\left[S^*, I^*\right] = \left[\lambda/\mu, 0\right]$$ **Disease-free host population**

Invasion analysis: Can rare pathogen advance?

$$\left[S^*, I^*\right] = \left[\frac{\mu+\gamma}{\beta}, \frac{\lambda}{\mu+\gamma} - \frac{\mu}{\beta}\right]$$ **Endemic disease**

(Epidemic advance to final size, then fadeout)

Disease equilibrium (when stable)

$$\partial S^*/\partial\mu > 0\,;\, \partial I^*/\partial\mu < 0$$ **Background mortality**

$$\partial S^*/\partial\gamma > 0\,;\, \partial I^*/\partial\gamma < 0$$ **Virulence**

$$\partial S^*/\partial\lambda = 0\,;\, \partial I^*/\partial\lambda > 0$$ **Flow of susceptibles**

$$\partial S^*/\partial\beta < 0\,;\, \partial I^*/\partial\beta > 0$$ **Pathogen fitness (?)**

Stability of Disease-Free Equilibrium

Can pathogen invade host population?

Jacobian

How do densities affect dynamics?

Recall dynamics of *SI* model

$$dS/dt = \lambda - \mu S - \beta I S$$
$$dI/dt = \beta I S - (\mu + \gamma) I$$

$$\partial \dot{S} / \partial S = - \mu - \beta I$$

$$\partial \dot{S} / \partial I = - \beta S$$

$$\partial \dot{I} / \partial S = \beta I$$

$$\partial \dot{I} / \partial I = \beta S - \mu - \gamma$$

$$J = \begin{bmatrix} -\mu - \beta I & -\beta S \\ \beta I & \beta S - \mu - \gamma \end{bmatrix}$$

Jacobian Matrix

Evaluate J at disease-free equilibrium

$$J\left(\lambda/\mu, 0\right) = \begin{bmatrix} -\mu & -\beta\left(\lambda/\mu\right) \\ 0 & \beta\left(\lambda/\mu\right) - \mu - \gamma \end{bmatrix}$$

Characteristic equation: Det [J - ξ I] = 0

ξ **Eigenvalue; stability analysis**

Pathogen invades (Disease advances when rare):

$$\mu + \gamma - \beta\left(\lambda/\mu\right) < 0 \quad \Rightarrow \quad \mu + \gamma < \beta\left(\lambda/\mu\right) = \beta\, S^*$$

Pathogen invades *iff* $\quad S^* > (\mu + \gamma)/\beta$

Critical population size

Promote invasion: increase λ, β

Inhibit invasion: increase μ, γ

R_0 and Pathogen Invasion

Disease advances: $$\mu + \gamma - \beta\left(\lambda/\mu\right) < 0$$

$$\beta\left(\lambda/\mu\right) - \mu - \gamma > 0 \quad \Rightarrow \quad \frac{\beta\left(\lambda/\mu\right)}{\mu+\gamma} > 1 \Leftrightarrow \frac{\beta S^*}{\mu+\gamma} > 1$$

R_0 Criterion

Single infective

S^* contacts/unit time

Pr [Infect $|$ contact] = β

Duration infectious period = $(\mu + \gamma)^{-1}$

$$R_0 = \frac{\beta S^*}{\mu + \gamma}$$

Recover stability criterion

Criterion function of host disease-free density

Frequency-Dependent Transmission

Dynamics:

$$dS/dt = \lambda - \mu S - \beta I \left(S/N\right)$$
$$dI/dt = \beta I \left(S/N\right) - (\mu + \gamma) I$$

λ **,** μ **,** γ **as above**

N **=** ***S*** **+** ***I*** **total density**

I_t **infectives make** βI **total contacts/unit time**

Fraction contacts susceptible = S_t/N

Total transmission rate = $\beta I \left(S/N\right)$

N.B. : β **has different units in 2 models**

R_0 from Invasion Analysis

Disease-free equilibrium $$\left[S^*, I^*\right] = \left[\lambda/\mu, 0\right]$$

As above

Rare infective, total contact rate β

Susceptible frequency = S*/N = S*/S* = 1

Duration of infectious period = $(\mu + \gamma)^{-1}$

$$R_0 = \frac{\beta}{\mu + \gamma}$$

Independent of host density; no critical pop size

Equilibrate dynamics, endemic densities differ

Different assumptions make different predictions

Each has limited empirical support

Is there a critical population size?

Does R_0 depend on susceptible density?

Transients, endemic densities compare models?

In particular, effect of immigration on endemic S*

Do experiments address both density variation and number variation?

What are consequences for pathogen evolution via strain competition?