Fiscal Risk in a Monetary Union

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Abstract

A country entering a monetary union gives up the right to determine its own monetary policy. Individual fiscal authorities promise passive fiscal policy, allowing the central monetary authority to use active monetary policy. Since a government, which can print its own money, can honor its nominal debt unconditionally, entrance into a monetary union raises new issues of potential fiscal insolvency. When there is an upper bound on the magnitude of the surplus and stochastic shocks to the surplus, a government can find itself in a position in which it cannot borrow to continue with its desired passive fiscal policy. This paper considers the risk of a fiscal financial crisis in a monetary union under alternative assumptions about how the fiscal authority would respond. The response affects the timing and probability of a crisis. We consider both outright default and policy switching, whereby the fiscal authority in crisis switches to active fiscal policy and the monetary authority switches to passive monetary policy. We apply the model to estimate fiscal risk in the European Monetary Union. Using panel estimates of the parameters in the surplus rule and initial values for government debt and the primary surplus, we simulate fiscal risk under the two alternative fiscal responses to a crisis. We find that countries with initial values bound by the Maastricht Treaty limits are safe, while countries like Italy and Greece, in which debt has strayed far above these limits, might not be.

Key Words: European Monetary Union, Fiscal Theory of the Price Level, Policy Switching, Default, Financial Crisis

1 Introduction

On January 1, 2002 twelve European countries gave up their individual currencies and adopted a common European currency, the Euro. This move has tremendous benefits in terms of the reduction in transactions costs that come with a larger market, but it also entails risks. Government budgets experience stochastic shocks, due to wars, natural disasters, business cycles, political decisions, etc. It is optimal for a government to smooth the effects of these shocks, and governments typically do this using nominal debt. A government which issues debt in terms of its own currency can honor its commitments unconditionally because it can always print money to repay its debt. Printing money can be costly in terms of inflation, but this seigniorage can be used to balance the government's budget (Sargent and Wallace 1981). Alternatively, the government can allow real returns on nominal government debt to be state-contingent even though nominal returns, as measured by nominal interest rates, are not (Chari, Christiano, and Kehoe 1991). This is achieved by surprise changes in the price level, which affect the real value of government debt, and is the mechanism in the "Fiscal Theory of the Price Level" (FTPL).(Sims 1994, Woodford 1994). In either case, fiscal solvency, as measured by intertemporal budget balance, is achieved through changes in the purchasing power of private agents.

When a country joins a monetary union, it looses seigniorage and debt devaluation through unexpected inflation as instruments for achieving intertemporal budget balance. The Maastricht rules were designed to assure that no country in the European Monetary Union (EMU) would ever exert pressure on the European Central Bank (ECB) to restore these instruments. The rules focused on fiscal soundness, requiring that each country's debt-GDP ratio remain well below the maximum any country could be expected to service, and that government budget deficits remain small. Violations of the rules were to be punished with fines. Governments with commitments to the Maastricht rules are planning to balance their intertemporal budget for any initial outstanding value of real debt. That is, they are committing to follow a passive fiscal policy as defined in the FTPL. Therefore, these rules and punishments can be viewed as a method of enforcing passive fiscal policy on member countries, leaving the ECB free to choose the price level with active monetary policy. In the absence of additional constraints, a government which adheres to a passive fiscal policy, will not face a fiscal financial crisis.[1]

However, every government faces limits on its ability to raise taxes, which implies an upper bound on the present value of the surplus and an upper bound on the value of debt which the surplus can service. Governments have violated the Maastricht rules, and future fiscal shocks could send a government's debt along a path expected to violate the upper bound on debt. The upper bounds, together with stochastic shocks to the surplus, imply fiscal risk. Specifically, stochastic shocks to the surplus could require very large values for future surpluses, values so large that they could be infeasible. If stochastic fiscal shocks place the system above the path leading to the maximum value for real debt, then there is no interest rate at which agents would agree to lend. A sudden stop of capital flows, which prevents the government from borrowing to continue its desired fiscal policy, defines a fiscal financial crisis.

[1] Other papers which analyze implications of the FTPL for the European Monetary Union include Bergin (2000), Sims (1997, 1999), Cochrane, (1998), Leith and Wren-Lewis (2006).

Crisis resolution requires policy action to restore equilibrium, and crisis dynamics are partially determined by expectations of the policy response to a crisis. We consider two types of policy response. The first is a policy response in which the crisis country reduces the magnitude of debt through default to restore fiscal solvency and continues passive fiscal policy. In the second, the fiscal authority in the crisis country switches to active fiscal policy with the union monetary authority accommodating by switching to passive monetary policy with a target for expected inflation. We show that default without fiscal reform leaves markets turbulent such that they continue to expect and experience default on the crisis country's debt. Markets are orderly after policy-switching with expected future inflation equal to its target value.

If the monetary union is willing to allow a member country to default, then fiscal policy has no consequences for the monetary authority's ability to control the price level. Therefore, crisis analysis with a policy response of default is a positive analysis of the consequences of allowing a member country to default. If default is unacceptable, as suggested by constraints EMU member countries have chosen to impose on fiscal policy, then the analysis with a response of policy switching characterizes the threat of a fiscal financial crisis to price stability.

Using panel estimates of the parameters in the surplus rule and initial values for government debt and the primary surplus for the EMU countries, we simulate fiscal risk under alternative fiscal responses to a crisis. We find that countries with initial values bound by the Maastricht Treaty limits are safe, while countries like Italy and Greece, in which debt has strayed far above these limits might not be.

This paper is organized as follows. The next section describes the behavior of monetary and fiscal policy in a monetary union, in which each country's primary surplus is subject to stochastic shocks, which the fiscal authority wants to smooth over time using debt. In the third section, we characterize an equilibrium, in which every country in the union initially follows a fiscal policy under which debt relative to output is expected to approach a long-run equilibrium value. Government debt has risk due to the upper bound on the primary surplus. The fourth section considers dynamics leading to a crisis under alternative responses to the fiscal crisis. The next section contains simulations of fiscal risk under alternative methods of response. The final section concludes.

2 Model

2.1 Overview

In this section, we set up a simple model of a monetary union which we can use to address fiscal risk. The model contains three key assumptions. First, international creditors lend to a government only when they expect to receive the market rate of return. Second, there is an upper bound on the present value of primary surpluses relative to output which a country can sustain. Third, fiscal policy implies risk on government debt, reflecting concern by founders of the EMU, as well as the reality, that a government's commitment to raise taxes to finance expenditures cannot be totally unconditional.

We fill out the model with enough structure to obtain an equation for the evolution of government debt relative to output. This requires specification of monetary and fiscal policy as well as government budget constraints. We assume that the monetary authority has a price level target and that the fiscal authority follows a rule relating the current primary surplus to past debt. The rule is subject to stochastic shocks, giving fiscal policy risk. The rule we choose is simple and does not require full specification of a general equilibrium model. However, any rule with fiscal risk could be used to complete the model. The government's intertemporal budget constraint, combined with the upper bound on the present value of primary surpluses, imply an upper bound on the value of debt.

2.2 Goods and Asset Markets

We assume that the monetary union consists of N countries. The $j = 1, 2, ..., N$ countries are small enough that they cannot affect the world price level or world interest rate. There is a single good in the world, implying that equilibrium in goods markets requires the law of one price. Normalizing the world price level at unity and assuming no world inflation implies that the equilibrium price level in the monetary union is the exchange rate.

The **first key assumption** is that international creditors are willing to buy and sell country j's government bonds as long as its interest rate, i_{jt}, satisfies interest rate parity. Interest rate parity can be derived as the Euler equation for a representative world agent when the covariance of the country j interest rate with world-agent consumption is zero, or when the world agent is risk neutral. Under the additional assumptions that the world interest rate (i) is constant, interest rate parity can be expressed as

$$\frac{1}{1+i_{jt}} = \left(\frac{1}{1+i}\right) E_t \left[\frac{P_t}{P_{t+1}}\delta_{jt+1}\right], \qquad j = 1, 2, ...N \tag{1}$$

where E_t denotes the expectation conditional on time t information, P_t denotes the price level in the monetary union, and δ_{jt+1} is the fraction of the value of the j country's bond that will be repaid in period $t + 1$.

Interest rates in the monetary union countries can rise above the world interest rate when there is some possibility of a crisis which will be resolved with either default ($\delta_{jt+1} < 1$) or inflation $\left(\frac{P_t}{P_{t+1}} < 1\right)$. If default is used to resolve fiscal crises, then a country with a positive probability of default in the next period, such that $E_t\delta_{jt+1} < 1$, would have an interest rate which is higher than the rate in other member countries for which there is no probability of default. If default is ruled out as a policy response to a crisis, then $\delta_{jt+1} \equiv 1 \; \forall j, \; t$, and all N member-country interest rates are equal. They can be higher than the world rate when there is some probability that debt devaluation through a price level surprise will be used to resolve a crisis.

2.3 Monetary Policy

We assume that with the creation of the monetary union, all N countries in the union agree to follow a strongly passive fiscal policy, which we define below, leaving the common monetary authority free to determine the price level with an active monetary policy. Monetary policy is assumed to have a fixed price level target, implying an inflation target of zero. When fiscal policy for every country in the union is strongly passive and there is no probability of default in the next period for any of the N countries, the price level is fixed at its target and interest rate parity from equation (1) implies that the nominal interest rates for all countries are equal at $i_{jt} = i$.[2]

[2] This policy could be implemented with a Taylor Ruler, whereby the monetary authority has a credible threat to raise the nominal interest rate substantially in the event that price rises.

2.4 Fiscal Policy

2.4.1 Government Flow Budget Constraint

We assume that each government issues bonds denominated in the common currency. The $j'th$ country's bonds are held by the public $\left(B_{jt}^{p}\right)$ and by the monetary authority $\left(B_{jt}^{M}\right)$,

$$B_{jt} = B_{jt}^{p} + B_{jt}^{M}.$$

The supply of the common currency in the union (M_t) is given by the sum of each country's government bonds held by the monetary authority,

$$M_t = \sum_{j=1}^{N} B_{jt}^{M}.$$

Assuming that the monetary authority returns the interest on bonds to the governments issuing those bonds, and letting η_j be the fixed fraction of the union monetary base provided by country j, then seigniorage revenues for country j are given by $i_{jt}B_{jt}^{M} = i_{jt}\eta_j M_t$. Allowing for the possibility of default and simplifying notation by dropping the subscript j, a government's nominal flow budget constraint is given by

$$B_t^p + \eta M_t = \delta_t \left[(1 + i_{t-1}) B_{t-1}^p + \eta M_{t-1}\right] + G_t - \tau_t P_t Y_t,$$

where G_t is nominal government expenditures, P_t is the price level, Y_t is real output and τ_t is the tax rate on nominal output. Letting small letters denote values relative to output, the values of debt relative to output and the primary surplus relative to output can be expressed respectively as

$$b_t = \frac{1}{P_t Y_t}\left(B_t^p + \frac{1}{1+i_t}\eta M_t\right),$$

$$s_t = \frac{1}{P_t Y_t}\left(\tau_t P_t Y_t - G_t + \left(\frac{i_t}{1+i_t}\right)\eta M_t\right).$$

Allowing for inflation and default, either of which could be created by a fiscal financial crisis, the government's flow budget constraint can be expressed in terms of debt and primary surplus relative to output as[3]

$$b_t = \left(\frac{\delta_t}{1+\pi_t}\right)\left(\frac{1+i_{t-1}}{1+g}\right) b_{t-1} - s_t, \tag{2}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate, and $g = \frac{Y_t}{Y_{t-1}} - 1$ is the output growth rate.[4] Imposing interest rate parity from equation (1), and defining γ_t as capital loss on the outstanding stock of debt, such that

$$\gamma_t = \left(1 - \frac{\delta_t}{1+\pi_t}\right)\left(\frac{1+i_{t-1}}{1+g}\right) b_{t-1},$$

the equation for the evolution of debt relative to output can be expressed as

$$b_t = (1+r)\, b_{t-1} - s_t - (\gamma_t - E_{t-1}\gamma_t)\,. \tag{3}$$

The growth-adjusted interest rate is denoted by $r = \left(\frac{i-g}{1+g}\right)$, and $(\gamma_t - E_{t-1}\gamma_t)$ is the unexpected capital loss on government debt. Capital loss on debt can occur due to either unexpected inflation or default. Debt accumulates in response to expectations of capital loss which are not realized. Expectations of capital loss raise the interest rate, and when the capital loss does not occur, debt accumulates in response to the higher interest rate.

Optimization by the representative agent, together with the assumption that governments do not allow their debt to become negative in the limit,[5] implies a government's

[3] This ignores the effect of capital gains or losses on seigniorage revenue $\left(\frac{i_t}{1+i_t}\frac{\eta M_t}{P_t Y_t}\right)$ under the assumption that the fiscal authority can adjust the surplus to offset these.
[4] We assume growth is non-stochastic to simplify the analysis. We could analyze the implications of stochastic growth using a linearized model, but we reserve this for future work.
[5] Sims (1997), Woodford (1997) and Daniel (2001) argue that no country, acting to maximize utility of its own agents, would allow the present-value of its debt to become negative in the limit.

intertemporal budget constraint given by[6]

$$\lim_{T\to\infty} E_t b_{t+T}\left(\frac{1}{1+r}\right)^T = (1+r)\,b_{t-1} - (\gamma_t - E_{t-1}\gamma_t) - E_t\sum_{k=0}^{\infty} s_{t+k}\left(\frac{1}{1+r}\right)^k = 0. \quad (4)$$

Note that unexpected capital loss on debt, created either by default or by a price level jump and represented by $\gamma_t - E_{t-1}\gamma_t > 0$, creates revenue.

2.4.2 Upper Bound

The **second key assumption** is that there is an upper bound on the present value of the primary surplus relative to output that a government can sustain. We motivate this with the realization that taxes are distortionary such that there will be an upper bound on the fraction of income that a government can collect in taxes. Using the government's intertemporal budget constraint, equation (4), this implies an upper bound on the current value of debt relative to output given by

$$b_t \leq \frac{\bar{\varphi}}{r}, \quad (5)$$

where $\bar{\varphi}$ is the value of the upper bound on the primary surplus relative to output.

The upper bound on debt relative to output rules out an equilibrium in which debt relative to output is explosive. This differs from original presentations of the FTPL, in which debt relative to output can increase forever in equilibrium, as long as its growth rate is less than the growth-adjusted rate of interest.[7]

6 Woodford (1994) derives of the constraint as an equilibrium condition for a closed economy.
7 Canzoneri, Cumby, and Diba (2001) base their empirical test determining whether monetary policy in the US is active or passive on the intertemporal budget constraint, as in early presentations of the FTPL. Sims (1997) argues that governments instead should be concerned with stabilizing debt relative to GDP, as in the current paper. Cochrane (1998) explains the difference in the two perspectives.

2.4.3 Fiscal Policy Rule

We assume that the fiscal authority is able to commit to a surplus rule[8], in which the primary surplus responds to its own lag and a linear combination of the target value of the long-run primary surplus and debt service at the growth-adjusted interest rate. The surplus rule for a particular country is given by

$$\begin{aligned} s_t &= (1-\alpha)\, s_{t-1} + \alpha \left[(1-\lambda)\,\varphi + \lambda r b_{t-1}\right] + \nu_t, \qquad (6) \\ 0 &< \alpha < 1, \qquad 0 \leq \lambda, \qquad 0 < \varphi \leq \bar{\varphi}, \end{aligned}$$

where φ is the value of the target for the long-run primary surplus relative to output, $(1-a)$ measures persistence in the primary surplus, λ represents the responsiveness of the surplus to the value for debt service relative to its long-run target value, and ν_t is a bounded, zero-mean, stochastic disturbance representing fiscal shocks. Stochastic shocks represent both truly unanticipated fiscal shocks, as with a war or natural disaster, as well as fiscal policy responses to the state of the economy. The lagged value of the primary surplus relative to output reflects the desire to smooth the effect of shocks over time and is consistent with empirical evidence showing persistence in the primary surplus. Since the model is specified in terms of debt and the primary surplus relative to output, we refer to these variables simply as debt and the surplus when there is no confusion.

Equations (3) and (6) imply dynamic equations for the surplus and debt;

$$\begin{aligned} s_t &= (1-\alpha)\, s_{t-1} + \alpha\,(1-\lambda)\,\varphi + \alpha\lambda r b_{t-1} + \nu_t \qquad (7) \\ b_t &= (1+r-\alpha\lambda r)\, b_{t-1} - (1-\alpha)\, s_{t-1} - \alpha\,(1-\lambda)\,\varphi - \nu_t - \gamma_t + E_{t-1}\gamma_t \qquad (8) \end{aligned}$$

[8] The rule gives the government credibility, limiting the effect of negative fiscal shocks on the expected present value of future surpluses.

The **third key assumption** is that fiscal policy entails risk. In our specification, risk is due to the upper bound and stochastic shocks. Governments understand this risk, and the parameters they choose reflect their risk tolerance, determined in part, by the cost of a crisis. For the simulation exercises later in the paper, we let the data reveal the parameter values the authorities chose in solving their optimization problem. Empirically, countries do choose rules with risk, and the Maastricht limits on debt and deficits reflect policy-maker concerns that at least some EMU countries might choose risky rules.

2.4.4 Stability and Dynamics

The time paths for each country's surplus and debt can be determined by solving equations (7) and (8). Letting θ represent eigenvalues, which are assumed to be real and distinct, the characteristic equation for each country is given by

$$(1-\alpha)(1+r) - \theta\left[1 + r(1-\alpha\lambda) + 1 - \alpha\right] + \theta^2 = 0. \tag{9}$$

No Upper Bounds To understand the behavior of the model without crises, consider the dynamic stability of the model for different values of λ when there is no upper bound on the value of the surplus. Equilibrium in the absence of upper bounds is defined as

Definition 1 *Given constant values for the world interest rate and world price level, a monetary-policy price-level target, and a surplus rule (equation 7) for each country, an equilibrium is a set of time series processes for each country's primary surplus, debt, and capital loss on debt,* $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$, *such that each government's flow and intertemporal budget constraints (equations 8 and 4) hold, expectations are rational, and world agents expect to receive the return on assets determined by interest rate parity, (equation 1).*

For $\lambda \geq 1$ both eigenvalues are on or inside the unit circle, and the model is globally stable. Debt and the surplus are each expected to reach a long-run equilibrium values for any initial values of the variables, including γ_t. Therefore, the expected present value of debt relative to output goes to zero in the limit, implying that equation (4) is satisfied for any stochastic process for γ_t. A fiscal policy for which the present value of debt relative to output is zero in the limit for any initial value of the price level is defined as passive fiscal policy. When fiscal policy is passive, the monetary authority is free to follow an active policy and choose $\gamma_t = 0$ in equilibrium, consistent with its fixed price-level target.

Alternatively when $0 \leq \lambda < 1$, one eigenvalue of the characteristic equation is inside the unit circle and the other is outside, implying that the model is saddlepath stable. The system reaches a long-run equilibrium only if it begins on the saddlepath. Otherwise, debt can be on a path where it is expected to grow faster than output. For $0 < \lambda < 1$, debt relative to output is always expected to grow more slowly than the growth-adjusted interest rate. This implies that the expected present value of debt relative to output in the limit is zero, or equivalently, that equation (4) is satisfied for any stochastic process for γ_t. Fiscal policy is passive, and in the absence of any upper bounds, $\lambda > 0$ is sufficient for the monetary authority to freely choose $\gamma_t = 0$, maintaining its fixed-price target.

For $\lambda = 0$, the expected present value of debt relative to output is no longer zero in the limit unless the system is on the saddlepath. Therefore, there is no equilibrium unless there is a jumping variable, offsetting shocks, to keep the system on the saddlepath. The only candidate is the real value of debt through γ_t. Therefore, γ_t must be free to experience positive and negative jumps to keep the system on the saddlepath, as in the FTPL. This represents an active fiscal policy because the government's intertemporal budget is balanced only for the value of real debt along the saddlepath, not for all values. When some fiscal policies in the union are active, then the active fiscal policies together determine γ_t. Therefore, a fiscal rule with $\lambda = 0$ implies that the monetary authority does not have the freedom to determine the price level.

Upper Bounds The upper bound on debt has different implications for the constraints on monetary policy when λ takes on values between zero and unity. For $0 < \lambda < 1$, debt relative to output is always expected to grow more slowly than the growth-adjusted interest rate, but debt can grow faster than output. Paths along which debt grows faster than output violate the upper bound and cannot be equilibrium paths. Since such paths are inconsistent with equilibrium, there must be a jumping variable to move the system away from these paths onto the saddlepath. In equilibrium, the value of γ_t jumps to keep the system on the saddlepath, implying that the monetary authority loses its ability to control the price level.

Effectively, with an upper bound given by (5), monetary freedom to control the price level in equilibrium requires that each fiscal authority follow a rule with $\lambda \geq 1$. We refer to such policy as "strongly passive" because it rules out explosive debt relative to output. The standard definition of passive fiscal policy without an upper bound restricts debt relative to output to grow more slowly than the growth-adjusted interest rate in the limit, requiring only $\lambda > 0$. However, this definition does not rule out growth of debt relative to output, which would eventually violate any upper bound.

In summary, consideration of upper bounds implies that a necessary condition for the monetary authority to be able to choose the price level, that is to choose γ_t, is that the surplus rule restrict $\lambda \geq 1$. This restriction assures that the long-run values for debt and the surplus are not expected to violate their upper bounds. However, upper bounds can imply crises even under a surplus rule with $\lambda \geq 1$. This can occur when the adjustment path toward long-run equilibrium values requires a value for debt which exceeds its upper bound. We turn to this below.

3 Dynamics under Strongly Passive Fiscal Policy

Consider the dynamic behavior of debt and the surplus in a newly-formed monetary union in which each country is committed to strongly passive fiscal policy. Equations (7) and (8) with $\lambda \geq 1$ can be solved to express the time paths for the surplus and debt relative to output in each country yielding

$$\begin{aligned} s_t &= \varphi + \frac{(\theta_2 - 1 + \alpha)\,\theta_1^t}{(1-\alpha)(\theta_1 - \theta_2)} \Big\{ (\alpha - 1)(s_0 - \varphi) + (\theta_1 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \\ &\qquad + \sum_{k=1}^{t} \theta_1^{-k} \left[-\theta_1 \nu_k - (\theta_1 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \Big\} \\ &\quad + \frac{(\theta_1 - 1 + \alpha)\,\theta_2^t}{(1-\alpha)(\theta_1 - \theta_2)} \Big\{ (1-\alpha)(s_0 - \varphi) - (\theta_2 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \\ &\qquad + \sum_{k=1}^{t} \theta_2^{-k} \left[\theta_2 \nu_k + (\theta_2 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \Big\} \end{aligned} \tag{10}$$

$$\begin{aligned} b_t &= \frac{\varphi}{r} + \frac{\theta_1^t}{\theta_1 - \theta_2} \Big\{ (\alpha - 1)(s_0 - \varphi) + (\theta_1 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \\ &\qquad + \sum_{k=1}^{t} \theta_1^{-k} \left[-\theta_1 \nu_k - (\theta_1 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \Big\} \\ &\quad + \frac{\theta_2^t}{\theta_1 - \theta_2} \Big\{ (1-\alpha)(s_0 - \varphi) - (\theta_2 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \\ &\qquad + \sum_{k=1}^{t} \theta_2^{-k} \left[\theta_2 \nu_k + (\theta_2 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \Big\} \end{aligned} \tag{11}$$

where $\theta_1 \leq 1$ and $\theta_2 < 1$ are the eigenvalues of the characteristic equation (9). The time paths depend on initial values, fiscal shocks, capital losses and their expectations. When the country is far from a crisis, $\gamma_t = E_{t-1}\gamma_t = 0$. The values for γ_t and its expectations in the neighborhood of a crisis are endogenized below.

3.1 No Upper Bound

When there is no upper bound on debt and $\lambda \geq 1$, equation (11) can be used to show that the government's intertemporal budget constraint is satisfied for any stochastic process for γ_t. With institutions strong enough to prevent default the fiscal authority can borrow freely and the monetary authority is free to set $\gamma_t = E_{t-1}\gamma_t = 0$, achieving its price level target. In equilibrium, with no possibility of either default or inflation, agents are always willing to lend at the world interest rate (equation 1).

To facilitate understanding, it is useful to represent the dynamics of the debt-surplus system using country phase diagrams. We can construct the phase diagram for each country by subtracting lagged values of the surplus from equation (7) and lagged values of debt from equation (8) to yield:

$$\Delta s_t = s_t - s_{t-1} = -\alpha s_{t-1} + \alpha(1-\lambda)\varphi + \alpha\lambda r b_{t-1} + \nu_t, \tag{12}$$

$$\Delta b_t = b_t - b_{t-1} = (1-\alpha\lambda) r b_{t-1} - (1-\alpha) s_{t-1} - \alpha(1-\lambda)\varphi - \nu_t - \gamma_t + E_{t-1}\gamma_t. \tag{13}$$

The phase diagram under passive fiscal policy with $\lambda > 1$ and with $\nu_t = \gamma_t - E_{t-1}\gamma_t = 0$ is given in Figure 1. Debt service (rb) is on the vertical axis and the surplus is on the horizontal axis. The $\Delta s = 0$ and $\Delta b = 0$ schedules intersect at point P with $s_t = \varphi = rb_t$, representing a long-run equilibrium. The system is globally stable around its long-run equilibrium target values. If initial debt and the surplus are at point A, then the system is expected to travel along AP, eventually reaching the long-run equilibrium point P. Equations (12) and (13) can be used to show that with $\nu_t = \gamma_t - E_{t-1}\gamma_t = 0$, the relationship between debt and surplus along any passive adjustment path is given by

$$\frac{r\left(E_{t-1}b_t - b_{t-1}\right)}{E_{t-1}s_t - s_{t-1}} = r\left[\frac{rb_{t-1} - s_{t-1}}{\alpha\left(\lambda r b_{t-1} - s_{t-1} + (1-\lambda)\varphi\right)} - 1\right]. \tag{14}$$

Over time, fiscal shocks (ν_t) could move the system away from its initial passive adjustment path, labelled AP, possibly to an adjustment path like HP. Along HP, the debt and the surplus are expected to overshoot their long-run equilibrium values. However, in the absence of upper bounds, the government's intertemporal budget constraint is satisfied along any adjustment path. Since the monetary authority's choice of price affects the initial position and any initial position is consistent with equilibrium, global stability allows the monetary authority to adhere to its fixed price level target, setting $\gamma_t = E_{t-1}\gamma_t = 0$.

3.2 Upper Bounds

Consider the implications of an upper bound for the viability of passive fiscal policy, using Figure 1. Assume that the initial adjustment path is AP. A fiscal shock moves the system in either a northwest or southeast direction from the initial path. Consider a sequence of shocks which eventually moves the system above the adjustment path AP, to point H. In the absence of an upper bound, the adjustment path HP is an equilibrium path. However, when debt has an upper bound given by equation (5), adjustment along the HP path requires values for debt greater than its upper bound. This path violates the government's intertemporal budget constraint because it requires that debt be expected to pass through a point where it exceeds the maximum present value of future surpluses. Since the fiscal authority could never service or repay such a large debt, agents could not expect to receive the market rate of return on debt along the path HP, implying that HP cannot be an equilibrium path.

As a country nears a crisis, which could require $\gamma_t > 0$, agents begin to anticipate the capital loss. The expectation affects the evolution of debt and surpluses. Once shocks together with expectations send the system onto a path like HP, agents refuse to lend. This sudden stop of capital flows requires a fiscal response since the government cannot continue its policy of smoothing fiscal shocks using government debt. The timing of the sudden stop itself and the actual dynamics depend on how the fiscal authority is expected to react to the crisis. We consider two possible policy responses to the crisis, default to reduce the magnitude of the debt, and policy reform with fiscal policy switching to active and monetary policy switching to passive.[9]

4 Fiscal Financial Crises

Consider the equilibrium dynamics leading to a fiscal financial crisis under alternative assumptions about the government's response to the financial crisis. We assume that agents know the fiscal response to the crisis. Crises are most likely to occur in the region in which both debt and the surplus are rising. Below, we restrict attention to this region. Equilibrium in the presence of upper bounds is defined below.

Definition 2 *Given constant values for the world interest rate and world price level, a monetary price-level target, a surplus rule (equation 7) and an upper bound on debt (equation 5) for each of the j countries, and a policy-response in the event of a fiscal crisis, an equilibrium is a set of time series processes for each country's primary surplus, debt, and capital loss on debt, $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$, such that each government's flow and intertemporal budget constraints (equations 8 and 4) hold, expectations are rational, the debt for each country does not exceed its upper bound, and world agents expect to receive the return on assets determined by interest rate parity (equation 1).*

[9] Cooper, Kempf, and Peled (2008) show how alternative policy responses can represent multiple equilibria based on agents' beliefs about the policy response.

4.1 Default

Consider the case in which the country responds to a sudden stop of capital by reducing the magnitude of debt through default. With this crisis response, the fiscal authority remains committed to the strongly passive fiscal policy rule, given by equation (12). When the monetary union is willing to allow a member country to default, the possibility of a fiscal financial crisis poses no threat to the monetary authority's ability to control the price level. As agents anticipate default in country h, $E_t\delta_{ht+1} < 1$, and the monetary authority upholds its price level target by keeping $i_{jt} = i$ for all $j \neq h$, allowing i_{ht} to rise to satisfy equation (1) for the crisis country. Although a default policy response poses no threat to price stability, its economic consequences could be judged so detrimental that the union could choose to rule out default. Therefore, crisis analysis with a policy response of default should be viewed as a positive analysis of the characteristics of such a crisis.

To determine the probability of a crisis next period and expectations of one-period-ahead capital loss on government debt, it is useful to compare the current value of debt, whose evolution is given by equation (8), with the value of debt along the adjustment path to the upper bound on debt $\left(\frac{\bar{\varphi}}{r}\right)$, denoted as $\bar{b}_t$. We cannot obtain a closed-form expression for $\bar{b}_t$ as a function of s_t. However, we can take a piecewise linear approximation of this path about s_{t-1} and $\bar{b}_{t-1}$, using equation (14) to yield

$$\bar{b}_t = \bar{b}_{t-1} + \left(\beta_{t-1} - 1\right)\left(s_t - s_{t-1}\right), \tag{15}$$

where

$$\beta_{t-1} = \frac{r\bar{b}_{t-1} - s_{t-1}}{\alpha\left(\lambda r\bar{b}_{t-1} - s_{t-1} + (1-\lambda)\varphi\right)}, \tag{16}$$

and $s_t - s_{t-1}$ is given by equation (12).

The vertical distance between debt along the path to the upper bound and the current value of debt is given by

$$x_{t-1} = \bar{b}_{t-1} - b_{t-1}.$$

Equations (8), (12), (15), and (16) can be used to show that the distance evolves as

$$\bar{b}_t - b_t = x_t = \mu_{t-1} x_{t-1} + \beta_{t-1}\nu_t + \gamma_t - E_{t-1}\gamma_t, \tag{17}$$

where

$$\mu_{t-1} = 1 + \frac{\alpha r (1-\lambda)(\varphi - s_{t-1})}{\alpha \left(\lambda r \bar{b}_{t-1} - s_{t-1} + (1-\lambda)\varphi\right)}.$$

Assume that, when faced with a crisis in which it cannot borrow the desired amount, the government reduces the magnitude of debt through a default to assure that debt is not expected to travel above $\frac{\hat{\varphi}}{r} \leq \frac{\bar{\varphi}}{r}$. Note that we are allowing the government to choose a default magnitude larger than necessary to restore solvency, but we are assuming that agents know this choice. This requires that the government reduce the magnitude of current debt to the value of debt along the path that is expected to reach a maximum at $\frac{\hat{\varphi}}{r}$, given by $\hat{b}_t \leq \bar{b}_t$. Therefore, if unable to borrow, the government is expected to use default to set the distance between $\hat{b}_t$ and b_t equal to zero. This assures that debt is not expected to travel above the government's desired maximum value given by $\frac{\hat{\varphi}}{r}$. Letting $\psi_{t-1} = \left(\hat{b}_{t-1} - \bar{b}_{t-1}\right) \leq 0$ and redefining the β_{t-1} and μ_{t-1} to be a function of $\hat{b}_{t-1}$ instead of $\bar{b}_{t-1}$,[10] the distance between $\hat{b}_t$ and b_t is given by

$$\Omega_t = \hat{b}_t - b_t = x_t + \psi_t = \mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right) + \beta_{t-1}\nu_t + \gamma_t - E_{t-1}\gamma_t. \tag{18}$$

[10] We are approximating the adjustment path which reaches a maximum value of debt given by $\frac{\hat{\varphi}}{r}$ about values along that path, given by $\hat{b}_{t-1}$.

Note that the magnitude of default necessary to set $\Omega_t = 0$ is determined by the desired maximum value for debt $\left(\frac{\hat{\varphi}}{r}\right)$ and not by debt's possibly larger upper bound $\left(\frac{\bar{\varphi}}{r}\right)$. The state variable determining a crisis becomes $x_{t-1} + \psi_{t-1} = \hat{b}_{t-1} - b_{t-1}$.

Definition 3 *Conditional on the expectation that a lending crisis will be resolved with default to keep expected values for future debt from rising above* $\frac{\hat{\varphi}}{r} \leq \frac{\bar{\varphi}}{r}$, *a* ***boundary locus*** *for debt service* (rb) *is defined as the piecewise continuous path, given by the expected path for debt service passing through the maximum value for* rb *of* $\hat{\varphi}$ *for* $s \leq s^*$ *and by* $rb = \hat{\varphi}$ *for* $s \geq s^*$, *where* $s^* = \frac{\hat{\varphi}(1-\alpha\lambda)-\alpha(1-\lambda)\varphi}{1-\alpha}$ *is the value of* s *along the expected adjustment path at the point with* $rb = \hat{\varphi}$.

Figure 1 shows the boundary locus for debt as BLM. Note that the boundary locus is defined with respect to the government's desired maximum value of debt, not by its upper bound. For $\nu_t = \gamma_t = E_{t-1}\gamma_t = 0$, a positive value for $x_{t-1} + \psi_{t-1}$ implies that b_{t-1} and b_t are below the boundary locus. However, fiscal shocks (ν_t), expectations of default $(E_{t-1}\gamma_t)$, and default (γ_t) can all affect the position of b_t relative to the boundary locus.

We define a shadow value of default, analogous to the shadow value of the exchange rate in generation one currency crisis models (Flood and Garber 1984). Conditional on a crisis in which agents refuse to lend, the shadow value of default represents the reduction in the value of debt needed for the economy to reach the boundary locus. The shadow value can be positive or negative.

Definition 4 *The* ***shadow value*** *of capital loss on debt due to default at time* t, $\tilde{\gamma}_t$, *is defined as the value of* γ_t *for which* $\Omega_t = 0$.

Setting Ω_t to zero in equation (18) implies

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - \mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right) - \beta_{t-1}\nu_t. \tag{19}$$

Equations (18) and (19) imply that the distance between $\hat{b}_t$ and b_t can be expressed as

$$\Omega_t = \gamma_t - \tilde{\gamma}_t. \tag{20}$$

Assume that agents believe that the fiscal borrowing constraint will bind, creating default, iff $\tilde{\gamma}_t > 0$. We prove that this assumption is consistent with a rational expectations equilibrium below in Proposition 2. Under this assumption, the actual value of the capital loss due to default is given by

$$\gamma_t = \max\left\{\tilde{\gamma}_t, 0\right\}. \tag{21}$$

To determine the probability of a crisis and expectations of default, define ν_t^* as a critical value for ν_t such that $\gamma_t > 0$ for $\nu_t < \nu_t^*$, and $\gamma_t = 0$ for $\nu_t \geq \nu_t^*$. Letting $f(\nu_t)$ be a bounded, symmetric, mean-zero distribution for ν_t, with bounds given by $\pm\,\bar{\nu}$, the expectation for γ_t can be expressed as

$$E_{t-1}\gamma_t = \int_{-\bar{\nu}}^{\nu_t^*} \gamma_t f(\nu_t)\, d\nu_t = \int_{-\bar{\nu}}^{\nu_t^*} \left[E_{t-1}\gamma_t - \mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right) - \beta_{t-1}\nu_t\right] f(\nu_t)\, d\nu_t.$$

Defining $F(\nu_t^*)$ as the cumulative at ν_t^* and collecting terms on the expectation yields

$$\left[1 - F(\nu_t^*)\right] E_{t-1}\gamma_t = -\mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right) F(\nu_t^*) - \beta_{t-1}\int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t. \tag{22}$$

Substituting into equation (21) yields an implicit expression for γ_t as

$$\left[1 - F(\nu_t^*)\right]\gamma_t = \max\left\{0, -\left[\mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right) + \beta_{t-1}\int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + \beta_{t-1}\left[1 - F(\nu_t^*)\right]\nu_t\right]\right\}, \tag{23}$$

where $F(\nu_t^*)$ has the interpretation as the probability of default. Define $\chi_t = \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + \left[1 - F(\nu_t^*)\right]\nu_t^*$. A solution for ν_t^* exists iff there exists a value for ν_t^*, satisfying $-\bar{\nu} \leq \nu_t^* \leq \bar{\nu}$, which sets $\mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right) + \beta_{t-1}\chi_t = 0$ such that $\gamma_t = 0$ in equation (23).

Lemma 1 *There is no equilibrium solution for ν_t^* when $x_{t-1} + \psi_{t-1} < 0$, that is, when debt is above the boundary locus at time $t-1$.*

Proof. Given that $\beta_{t-1} > 0$ and $\mu_{t-1} > 0$, the proof must show that $\chi_t \leq 0$. To prove that $\chi_t \leq 0$, let ν_t^* take on its smallest possible value of $-\bar{\nu}$. Then $\chi_t = -\bar{\nu} < 0$. The derivative of χ_t with respect to ν_t^* is given by $1 - F(\nu_t^*)$. For $\nu_t^* < \bar{\nu}$, the derivative is positive. Therefore, as ν_t^* rises, χ_t rises monotonically. Once ν_t^* takes on its largest possible value, given by $\bar{\nu}$, $1 - F(\bar{\nu}) = 0$, and χ_t takes on its maximum value of zero. Therefore, $\chi_t \leq 0$ for all feasible values of ν_t^*. This implies that when $x_{t-1} + \psi_{t-1} < 0$, there is no feasible value for ν_t^* which sets $\mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right) + \beta_{t-1}\chi_t = 0$ such that $\gamma_t = 0$ in equation (23).

■

Lemma 2 *When $x_{t-1} + \psi_{t-1} > 0$, the probability of a crisis in period t is less than one, and when $x_{t-1} + \psi_{t-1} = 0$, the probability of a crisis in period t is one.*

Proof. When $x_{t-1} + \psi_{t-1} > 0$, Lemma 1 implies that $\chi_t < 0$, requiring $\nu_t^* < \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\nu_t^*)$, is less than one. When $x_{t-1} + \psi_{t-1} = 0$, ν_t^* must set $\chi_t = 0$, implying that $\nu_t^* = \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\bar{\nu})$, is one. ■

Intuitively, when the debt is below the boundary locus at time $t-1$, the probability that a monetary union country following a strongly passive fiscal policy will encounter a fiscal crisis in the next period is less than one. Even though expectations of default raise the interest rate and raise debt, sending it toward the boundary locus, it is possible to receive a large positive fiscal shock and still be safe. However, once the debt reaches the boundary locus, a fiscal crisis and default occur almost surely with any fiscal shock. The only time that default does not occur is when the government receives the largest positive fiscal shock, and the probability of receiving the largest positive shock is zero.

Lemma 3 *A solution for $E_{t-1}\gamma_t$ exists if and only if $x_{t-1} + \psi_{t-1} \geq 0$.*

Proof. When $x_{t-1}+\psi_{t-1} > 0$, Lemma 2 implies that the probability for default is positive. Expectations of default are given by the solution to equation (22).

When $x_{t-1} + \psi_{t-1} = 0$, Lemma 2 implies $\nu_t^* = \bar{\nu}$. With the critical value equal to its upper bound, any value of the fiscal shock ν_t requires $\gamma_t \geq 0$. Together $x_{t-1}+\psi_{t-1} = 0$ and equation (19) imply that $\gamma_t = \tilde{\gamma}_t = E_{t-1}\gamma_t - \beta_{t-1}\nu_t \geq 0$ for any realization of ν_t, including its upper bound, $\bar{\nu}$. Therefore, expectations of capital loss on debt due to default must satisfy

$$E_{t-1}\gamma_t \geq \beta_{t-1}\bar{\nu}.$$

When b_{t-1} is along the boundary locus, expectations of default are subject to a lower bound and can be arbitrarily large.

When $x_{t-1} + \psi_{t-1} < 0$, the shadow value of default is positive for any value for ν_t, implying an unitary probability of default. Taking expectations of equation (21), using equation (19) when the probability of default is unity yields

$$E_{t-1}\gamma_t = E_{t-1}\gamma_t - \mu_{t-1}\left(x_{t-1} + \psi_{t-1}\right).$$

This equation has a solution for the expectation only if $x_{t-1}+\psi_{t-1} = 0$. When $x_{t-1}+\psi_{t-1} < 0$, there can be no value for default such that it equals its expectation minus a negative gap. ■

Intuitively, if the debt will be above its boundary locus at time t with probability one, then there will be a crisis at time $t-1$. Creditors stop lending at time $t-1$ because there is no interest rate which can compensate them for expectations of default at time t. Only when $x_{t-1} + \psi_{t-1} \geq 0$, can creditors be compensated for expectations of default, keeping borrowing constraints from binding.

Proposition 1 *For positions of* b_{t-1} *on or below the boundary locus* $\left(x_{t-1}+\psi_{t-1}\geq 0\right)$*, the equilibrium interest rate in period* $t-1$ *increases to adjust for rational expectations of default* $\left(E_{t-1}\gamma_t>0\right)$*, allowing the government to borrow at its desired level in period* $t-1$*. However, for positions of* b_{t-1} *above the boundary locus* $\left(x_{t-1}+\psi_{t-1}<0\right)$*, there is no interest rate which can compensate agents for expectations of default, implying that such positions cannot represent an equilibrium.*

Proof. For positions for debt on or below the boundary locus, Lemma 2 shows that the probability of a crisis is one or less than one, respectively. Equations (22) and (1) can be used to solve for the values of expected default and the interest rate. Lemma (3) shows that for positions above the boundary locus, there is no solution for the expected value of default. Therefore, there is no value of the interest rate which can compensate agents for lending, implying that these positions cannot satisfy the definition of equilibrium. ■

It is now necessary to prove that whenever $\tilde{\gamma}_t>0$, agents refuse to lend, thereby eliciting a financial crisis with $\gamma_t=\tilde{\gamma}_t>0$. Assume that b_{t-1} is in a position below the boundary locus BLM, such that $x_{t-1}+\psi_{t-1}>0$. Additionally, assume that the position is near enough to the boundary locus that $E_{t-1}\gamma_t>0$. From this initial position, the economy receives a fiscal shock, given by ν_t.

Proposition 2 *There is no equilibrium without default in period* t *if* $\tilde{\gamma}_t>0$*. Default, given by* $\gamma_t=\tilde{\gamma}_t$ *restores equilibrium.*

Proof. Equilibrium in period t requires $x_t+\psi_t\geq 0$. This is because Lemma 3 shows that there can be no equilibrium rational expectations value for $E_t\gamma_{t+1}$ when $x_t+\psi_t<0$. Therefore, if $x_t+\psi_t<0$, then there is no equilibrium unless the country defaults. Using equation (17), yields

$$x_t+\psi_t=\mu_{t-1}\left(x_{t-1}+\psi_{t-1}\right)+\beta_{t-1}\nu_t-E_{t-1}\gamma_t+\gamma_t=\gamma_t-\tilde{\gamma}_t.$$

Therefore, when $x_t + \psi_t < 0$, $\tilde{\gamma}_t > 0$. A positive shadow rate triggers default. Default, with $\gamma_t = \tilde{\gamma}_t$, sets $x_t + \psi_t = 0$, restoring equilibrium by Lemma 1. ■

Therefore, Proposition 2 validates agents' assumption that the government will default whenever $\tilde{\gamma}_t > 0$. Intuitively, in the event of a sudden stop, the country promises default in magnitude sufficient to restore fiscal solvency. The sudden stop occurs when $\tilde{\gamma}_t > 0$, and the government responds as promised.

Corollary 1 *A government which wants to sustain current fiscal policy as long as possible chooses $\hat{\varphi} = \bar{\varphi}$, implying that $\psi_t = 0$ for all t.*

Proof. The position of the boundary locus is determined by $\hat{\varphi}$, and the boundary locus is higher the larger is $\hat{\varphi}$. This is because, by Propositions 1 and 2, the state variable determining a crisis becomes $x_{t-1} + \psi_{t-1} = \hat{b}_{t-1} - b_{t-1}$, and is independent of the upper bound. ■

We can use the phase diagram in Figure 1 to illustrate crisis dynamics. When the system is far from its boundary locus BLM, such that no fiscal shock could send it over, expectations of default are zero, and the system is governed by the arrows of motion sending it to its long-run equilibrium target values. Once the system reaches the neighborhood of the boundary locus, agents begin to expect default, and the associated risk premium on debt causes debt to increase more quickly than shown along illustrated adjustment paths. Once a shock, combined with equilibrium expectations of default, sends the system above the boundary locus, default is necessary to bring the system back to the boundary locus.

Proposition 3 *In the absence of fiscal reform, equilibrium after default requires additional default each period until debt falls below the boundary locus on its approach to the long-run equilibrium value.*

Proof. A default in period t, which brings the system to the boundary locus, implies that $x_t + \psi_t = 0$. By Lemma 2, the probability of a crisis in period $t+1$ is unity and by Lemma 3, $E_t \gamma_{t+1} \geq \beta_t \bar{\nu}$. Given a realization for $\nu_{t+1,}$ default occurs in the magnitude to set $x_{t+1} + \psi_{t+1} = 0$. The pattern persists until the dynamics imply that debt travels along a path like the solid path LP, which is below the boundary locus BLM. ■

Post-crisis equilibrium is characterized by repeated default which can be arbitrarily large in magnitude. Expectations of default must be large enough that default occurs for any fiscal shock. This is because of the one-sided nature of default, whereby default always reduces the value of debt. Expectations of default must be correct on average, implying that expectations of default must be the average value of default. Therefore, following the crisis, markets remain turbulent for some time. Agents expect additional default, interest rates are high, and additional default is necessary. This pattern does eventually end once the dynamics move the economy toward the long-run equilibrium below the boundary locus.

4.2 Monetary and Fiscal Policy Switching

The second possibility we consider is that a government facing a sudden stop reneges on its commitment to strongly passive fiscal policy. With this fiscal response, existence of an equilibrium requires the cooperation of the monetary authority. The monetary authority could prefer to cooperate over allowing default with its post-crisis turbulence.[11] Under policy switching, expectations of post-crisis capital loss on debt are zero, in contrast to high expectations of additional capital loss after default. We consider a switch in fiscal policy from strongly passive to active, accompanied by a monetary policy switch from

[11] An alternative, but analytically equivalent possibility, is that the crisis country could withdraw from the monetary union, reissue its own currency, and follow passive monetary policy.

active to passive. With this response, the monetary authority looses control of the price level, reflecting concern by the EMU founders regarding fiscal restraint.

When a crisis is anticipated, the monetary authority increases the interest rate, to accommodate $E_t \frac{P_t}{P_{t+1}} < 1$, while keeping the current price level fixed. After the switch to passive monetary policy, the monetary authority replaces its price level target with an inflation target with $E_t \frac{P_t}{P_{t+1}} = 1$. Therefore, after the switch, the monetary authority retains control of expected inflation, but not of actual inflation.

Before analyzing the switching model, it is useful to understand equilibrium in a monetary union with one active fiscal policy country, $N-1$ strongly passive fiscal policy countries, and a passive monetary authority.

4.2.1 Active Fiscal Policy in the N'th Country and Strongly Passive in the Others

Consider a monetary union in which fiscal policy is active in the $N\prime th$ country and strongly passive in all others. Active fiscal policy is modeled as $\lambda = 0$. The active-fiscal-policy system we solve analytically is comprised of equations (7) and (8) with $\lambda = 0$, in which the eigenvalues of the characteristic equation (9) are $1 + r$ and $1 - \alpha$. We assume that with fiscal reform, the government can choose a different value for φ subject to $\varphi \leq \bar{\varphi}$. Under active fiscal policy, the intertemporal budget constraint holds only for a unique initial real value of debt and hence for a unique initial price level. With default ruled out, monetary policy must be passive allowing the value for γ_t to set the coefficient on the explosive root to zero. This is the policy combination analyzed in the FTPL.

The time paths for the surplus and debt in the active-fiscal-policy country with $\varphi = \bar{\varphi}$

are given by[12]

$$s_t = \bar{\varphi} + (1-\alpha)^t \left[s_0 - \bar{\varphi} + \sum_{k=1}^{t} (1-\alpha)^{-k} \nu_k \right], \tag{24}$$

$$b_t = \frac{\bar{\varphi}}{r} + (1-\alpha)^t \left(\frac{1-\alpha}{r+\alpha} \right) \left[s_0 - \bar{\varphi} + \sum_{k=1}^{t} (1-\alpha)^{-k} \nu_k \right]. \tag{25}$$

These equations can be used to express the saddlepath relationship between debt and the surplus with $\varphi = \bar{\varphi}$ as

$$b_t = \left(\frac{1-\alpha}{\alpha+r} \right) s_t + \frac{\bar{\varphi}\alpha (1+r)}{r(\alpha+r)}. \tag{26}$$

When there are stochastic shocks to the surplus, the real value of debt must jump to keep the system on the saddlepath, as in the FTPL. Since all other fiscal policies are passive, there is only one unstable root in the system of N countries.

The equations for changes in the surplus and debt with $\lambda = 0$ and $\varphi = \bar{\varphi}$ can be computed from equations (7) and (8) to yield

$$\Delta s_t = s_t - s_{t-1} = -\alpha s_{t-1} + \alpha \bar{\varphi} + \nu_t, \tag{27}$$

$$\Delta b_t = b_t - b_{t-1} = r b_{t-1} - (1-\alpha) s_{t-1} - \alpha \bar{\varphi} - \gamma_t + E_{t-1}\gamma_t - \nu_t. \tag{28}$$

The phase diagram under active fiscal policy and with $\nu_t = \gamma_t - E_{t-1}\gamma_t = 0$ is given in Figure 2. The saddlepath has a slight positive slope. The saddlepath with $\varphi = \bar{\varphi}$, which is the saddlepath determining the largest possible value of debt under active fiscal policy, is labeled as SP1.

Since the system does not reach an equilibrium for arbitrary starting values, this is an active fiscal rule. Fiscal shocks, ν_t, move the system away from the saddlepath. To assure

[12] The requirement that the coefficient on the explosive root be zero implies: $b_0 - \left(\frac{1-\alpha}{\alpha+r}\right) s_0 + \sum_{k=1}^{t} (1+r)^{-k} \left[E_{k-1}\gamma_k - \gamma_k - \frac{1+r}{\alpha+r}\nu_k \right] = 0.$

that debt does not violate its upper bound, there must be one jumping variable to assure that the system is on the saddlepath. Price level jumps create jumps in γ_t. From equation (8), b_t jumps with each jump in γ_t, allowing the system to remain on the saddlepath. For an equilibrium to exist, monetary policy must be passive, as assumed, allowing γ_t to jump. Capital gains and losses on government debt are symmetric, implying that expectations of gains and losses are zero in the active-fiscal-policy, passive-monetary-policy regime. The upper bound poses no constraints other than the the fact that it sets an upper bound on the value for the φ.[13] If the government chooses $\varphi = \hat{\varphi} < \bar{\varphi}$, then in Figure 2, shifts $\Delta s = 0$ left and $\Delta b = 0$ right (not shown) shifting the saddlepath down to SP2.

4.2.2 Active Fiscal Policy in Two Countries

Consider a monetary union with active fiscal policy in 2 countries and strongly passive fiscal policy in $N-2$. Although there are N values for γ_t, there is only a single independent one. The value for γ_t is determined such that the present-value of total monetary union debt equals the present-value of total monetary union surpluses. Debt in each strongly-passive-fiscal-policy country must equal the expected present-value of surpluses. Therefore, the value for γ_t must equate the sum of the expected present-value of surpluses for the two active-fiscal-policy countries with the sum of their initial debt.

When there are two countries with active fiscal policy, the equilibrium jump in γ_t, which places the sum of the two countries' debt on a saddlepath, would land one country's debt above its saddlepath and the other country's debt below its saddlepath. Therefore, one country would expect rising debt and the other falling debt. The country with falling

[13] There are no constraints in the region in which debt and the surplus are both rising. The upper bound on debt does imply that positions on the saddlepath beyond $b_t = \frac{\bar{\varphi}}{r}$ are not feasible.

debt would be transferring resources to the other over time. This suggests that an equilibrium in which two fiscal policies are active is unlikely to persist. The country with falling debt would optimally choose to switch back to strongly passive fiscal policy and reduce its taxes in accordance with its lower debt to avoid a resource transfer away from its citizens, leaving a single country with active fiscal policy.

4.2.3 Fiscal Crisis Resolved with Fiscal Policy Switching

Consider crisis dynamics under the assumption that the monetary union has agreed to respond to a fiscal financial crisis in one country by allowing the crisis country to switch to active fiscal policy with accommodation by the monetary authority. We assume that all countries initially follow a strongly passive fiscal rule and maintain this policy for as long as possible. Figure 3 superimposes the saddlepaths for an active policy system on the passive policy system for a particular country.

After the policy switch, the system must be on the saddlepath in order to reach an equilibrium. Therefore, the largest possible value for debt after policy reform is given by the saddlepath leading to $\bar{\varphi}$, labeled SP1. Redefine the state variable at time t, as the distance between the largest possible post-crisis value of debt, given by SP1, and the current value of debt under passive fiscal policy. Using equations (26) and (8), the state variable is given by

$$x_{t-1} = \frac{(1-\alpha)}{\alpha} s_{t-1} - \frac{(r+\alpha-\alpha\lambda r)}{\alpha} b_{t-1} + \frac{\bar{\varphi}}{r} + (1-\lambda)\,\varphi. \tag{29}$$

Note that, as in the default case, the state variable determining the time t distance receives a $t-1$ subscript since its value is known at time $t-1$. Using equations (7) and (8), the state variable evolves as

$$x_t = \frac{(r+\alpha)}{\alpha}(\gamma_t - E_{t-1}\gamma_t) + (1+r)\left(x_{t-1} + \frac{\nu_t}{\alpha}\right) - (\bar{\varphi} - \varphi) - \lambda(\varphi - rb_t). \tag{30}$$

Assume that, when faced with a crisis in which it cannot borrow the desired amount, the fiscal authority institutes fiscal reform. It switches to an active fiscal policy with $\lambda = 0$, and raises the target surplus from φ to $\hat{\varphi} \leq \bar{\varphi}$, assuring that debt is not expected to travel above $\frac{\hat{\varphi}}{r} \leq \frac{\bar{\varphi}}{r}$, as in the default case. In Figure 3, the saddlepath to $\hat{\varphi}$ is labeled SP2. Under policy-switching, the system must begin on SP2, implying that the distance between the saddlepath value of debt and the current value of debt must be zero. Using equations (7), (8), (26) and (29), this distance can be expressed as

$$\Omega_t = \frac{\alpha(1+r)}{\alpha+r}\left(x_{t-1} + \psi + \frac{\nu_t}{\alpha}\right) + \gamma_t - E_{t-1}\gamma_t. \tag{31}$$

where $\psi = \left(\frac{\hat{\varphi}-\bar{\varphi}}{r}\right)$. Note that the distance between the post-reform value of debt along the saddlepath and its current value is determined by the desired maximum value for debt $\left(\frac{\hat{\varphi}}{r}\right)$ and not by debt's possibly larger upper bound since $x_{t-1} + \psi$ does not contain the term $\frac{\bar{\varphi}}{r}$. Additionally, the evolution of $x_t + \psi$ depends on its own lag and on $\hat{\varphi}$, not $\bar{\varphi}$.

Definition 5 *Conditional on the expectation that a lending crisis will be resolved with policy switching, accompanied by a new target surplus of $\hat{\varphi} \leq \bar{\varphi}$, a boundary locus for debt service (rb) is defined as the piecewise continuous path, given by the saddlepath leading to $\hat{\varphi}$ for $s \leq \hat{\varphi}$ and by $rb = \hat{\varphi}$ for $s \geq \hat{\varphi}$.*

Figure 3 shows the boundary locus for debt as CKM. Note that the boundary locus is defined with respect to the government's desired maximum debt, not by its upper bound. For $\nu_t = \gamma_t = E_{t-1}\gamma_t = 0$, a positive value for $x_{t-1} + \psi$ implies that b_{t-1} and b_t are below the boundary locus. However, fiscal shocks (ν_t), expectations of inflation $(E_{t-1}\gamma_t)$, and inflation (γ_t) can all affect the position of b_t relative to the boundary locus.

We define a shadow value of capital loss on government debt due to inflation. The shadow value of capital loss represents the reduction in the value of debt needed for the economy to reach the boundary locus. The shadow value can be positive or negative.

Definition 6 *The shadow value of capital loss on debt due to inflation at time t,* $\tilde{\gamma}_t$*, is defined as the value of* γ_t *which sets* $\Omega_t = 0$.

Setting $\Omega_t = 0$ and solving yields

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - \frac{\alpha(1+r)}{\alpha+r}\left(x_{t-1} + \psi + \frac{\nu_t}{\alpha}\right). \tag{32}$$

We assume that in the event of a crisis the fiscal authority never raises the value of debt to reach the saddlepath. In the event of a lending crisis with debt below the boundary locus, the fiscal authority reduces the long-run target value of the surplus such that the current value of debt without inflation is on the saddlepath to lower long-run values for the surplus and debt. However, if a fiscal shock sends the system above the boundary locus, then inflation is necessary because post-reform equilibrium requires $\Omega_t = 0$. Using equations (31) and (32), the distance between the saddlepath value of debt and the current value of debt can be expressed as $\Omega_t = \gamma_t - \tilde{\gamma}_t$.

Assume that agents believe that the fiscal borrowing constraint will bind, creating policy switching with $\gamma_t = \tilde{\gamma}_t$ if $\tilde{\gamma}_t > 0$. We prove that this assumption is consistent with a rational expectations equilibrium below.[14] This implies that the value for inflation in the crisis period is given by equation (21), where we redefine $\tilde{\gamma}_t$ using equation (32). If we redefine $\mu_{t-1} = \mu = \frac{\alpha(1+r)}{\alpha+r}$ and $\beta_{t-1} = \beta = \frac{(1+r)}{\alpha+r}$, then Lemmas 1, 2, and 3, and Proposition 1 apply directly to the switching case.

[14]In contrast to the default case, under switching, a crisis could occur with $\tilde{\gamma}_t < 0$, as we show below. Therefore, the statement is expressed as an *if* statement, not as an *iff* statement.

Consider how a crisis arises, when it will be resolved with policy-switching. Assume that b_{t-1} is in a position below the boundary locus SP2, such that $x_{t-1} + \psi > 0$. Additionally, assume that the position is near enough to the boundary locus that $E_{t-1}\gamma_t > 0$. From this initial position, the economy receives a fiscal shock, given by ν_t.

Proposition 4 *Given initial policy and expectations about policy-switching, a crisis occurs in period t if $x_t + \psi < 0$. Policy switching restores equilibrium.*

Proof. Lemma 3 shows that there is no equilibrium rational expectations value for $E_t\gamma_{t+1}$ when $x_t + \psi < 0$. There is no interest rate at which agents would lend under the original strongly passive fiscal policy, triggering a crisis and policy switching. Therefore, if $x_t + \psi < 0$, then there is no equilibrium in the absence of policy switching

Policy switching restores equilibrium by setting $\Omega_t = 0$. There are two ways in which this can happen, depending on the value for $\tilde{\gamma}_t$. When $\tilde{\gamma}_t > 0$ $(\Omega_t < 0)$, a price level jump setting $\gamma_t = \tilde{\gamma}_t$, assures $\Omega_t = 0$, placing the system on the saddlepath.

However, it is possible for $x_t + \psi < 0$, when $\tilde{\gamma}_t \leq 0$ $(\Omega_t \geq 0)$. This is because equations (30) and (32) can be used to show that the state variable evolves as

$$x_t + \psi = \frac{r+\alpha}{\alpha}(\gamma_t - \tilde{\gamma}_t) - (\hat{\varphi} - \varphi) - \lambda(\varphi - rb_t).$$

In this event, we assume that there is no deflation. Instead, policy switching entails choosing a target surplus lower than $\hat{\varphi}$, in order to place the system on a lower saddlepath without a price level change. The lower target surplus reduces the distance between debt along the new lower saddlepath and its current value to zero, reducing Ω_t to 0. ■

A crisis occurs when the government can no longer borrow to continue with the strongly passive fiscal rule. Assume that debt at time $t-1$, is at point H along path HP in Figure

3. Along the path HP, the distance between the debt along the boundary locus and the current value of debt becomes negative. Since this is inconsistent with equilibrium, HP cannot be an equilibrium path. However, the expectation of a regime switch in the future makes point H feasible because the expectation raises the expected present-value surplus to equal the value of outstanding debt.

In the neighborhood of the boundary locus, the market begins to anticipate inflation. This anticipation forces the interest rate to increase to incorporate the increase in expected inflation. The monetary authority accommodates to allow an equilibrium with regime switching. Once agents anticipate inflation, the system approaches the boundary locus SP2 at a faster rate than implied by the adjustment path HP, as shown in Figure 3 by the arrow from point H.

A crisis occurs when agents refuse to lend, and there are two ways in which this can happen. As the passive-fiscal system approaches the saddlepath, a negative fiscal shock could send it over such that $x_t + \psi < 0$ and $\tilde{\gamma}_t > 0$. The government's response is to promise fiscal reform. This implies a regime switch with a price level jump to bring the system to the saddlepath. After the policy switch, the system travels along the saddlepath SP2.[15]

Alternatively, the dynamics of the surplus and debt under passive policy could imply that debt next period, in the absence of regime switch, would travel above the saddlepath such that $x_t + \psi < 0$, but $\tilde{\gamma}_t < 0$.[16] Agents would not lend into this position since no rationally-expected value for future inflation could place the system on the saddlepath.

[15]Since the probability of devaluation is less than one, when a shock occurs requiring devaluation, its magnitude is greater than expected allowing b to jump downwards.

[16]This could occur since the passive fiscal policy adjustment path can be steeper than the saddlepath SP.

Regime-switch with no change in the price level allows debt and the surplus to move along a saddlepath below SP2, implying a long-run surplus below $\hat{\varphi}$.

Proposition 5 *Equilibrium after policy switching is characterized by the FTPL. The price level jumps following fiscal shocks to keep the system on the saddlepath. On average the jumps are zero, implying that expected inflation and* $E_{t-1}\gamma_t$ *are both zero.*

Proof. Expected inflation is determined by the monetary authority's price level target, implying an inflation target of zero. Since the mean of fiscal shocks is zero, the mean of price level shocks is zero. ■

Equilibrium after policy switching entails both positive and negative shocks to the price level, offsetting fiscal shocks, but expected inflation remains at the monetary authority's target of zero.

4.3 Summary of Crisis Characteristics

It is useful to summarize the characteristics of a fiscal financial crisis. First, a crisis generally occurs when debt is below its upper bound. There are two reasons for this. One is the upward sloping boundary locus, which implies that the upper bound on debt is lower for values of the surplus below the long-run equilibrium value. Second, a government might not be willing to let debt travel as high as its absolute maximum, effectively lowering the boundary locus and the value of debt which elicits a crisis.

Crises are imperfectly predictable. Once a crisis becomes possible, the interest rate rises, reflecting the expected capital loss on debt. The increase in the interest rate causes debt to accumulate more quickly, increasing the probability of a crisis. The more rapid growth in debt, due to the higher interest rate, implies that a crisis can occur even when the economy receives a favorable shock. This is possible when the favorable shock is small relative to the expected capital loss. However, if a country receives large enough favorable shocks, then it can escape the crisis.

Crises develop suddenly. For a country whose debt is substantially below the boundary locus, the probability of ever having a crisis is very low. However, once its debt is close enough to the boundary locus to elicit expectations of one-period-ahead capital loss, then rising interest rates increase the rate of growth of debt. This implies that to avoid a crisis, the country must on average receive favorable fiscal shocks. Therefore, as soon as interest rates begin to rise, the probability of a crisis sometime in the future jumps from something very low to something greater than fifty percent.

Crises are preceded by rising interest rates, but the sudden stop of capital flows is indeed sudden. A country can be conducting its desired fiscal policy and borrowing at risk-adjusted interest rates until the combination of expected capital loss and a fiscal shock sends the country above the boundary locus. This creates a sudden stop of capital flows because there is no interest rate which could compensate lenders for expected capital loss. Agents lend again only after the fiscal response has restored fiscal solvency.

Finally, it is also useful to compare the two policy responses to a crisis, debt reduction through default and policy switching, possibly with debt reduction through inflation. The relative probability of a crisis under alternative policy responses depends on the relative slopes of the boundary loci. We address this issue in simulations later and find that crisis probability is very similar under the two responses. The effects of the alternative policy responses are most different in their post-crisis equilibria. Since default is one-sided (no revaluations of debt), the post-crisis equilibrium after default is characterized by high expectations of additional default and by additional default until debt has traveled along its adjustment path below the boundary locus BLM. In contrast, the post-crisis equilibrium after policy-switching is characterized by both positive and negative price level shocks, which offset fiscal shocks, but whose expected value is zero.

4.4 Other Possible Policy Responses

Reduction of the magnitude of debt through default and policy switching are not the only possible policy responses to a crisis. Other possible responses are briefly considered here, but full analysis of them is left to future research.

Once a crisis becomes anticipated with some positive probability, the government could implement fiscal reform with the objective of reducing the probability of a crisis. However, given that the probability of a crisis becomes positive following negative fiscal shocks, the promise of larger near-term surpluses, in the presence of economic circumstances that reduce surpluses, is unlikely to be credible. And, even after fiscal reform, fiscal policy still has risk unless the government can eliminate the source of stochastic shocks.

Another possible response would be a promise of fiscal transfers from member countries to the crisis country. Countries might not be willing to make this promise given the obvious moral hazard problem. And, even if countries do promise to use fiscal transfers, then fiscal risk applies to aggregate member debt instead of to individual country debt. Once the aggregated fiscal authority faces a crisis, there must be some alternative response since there can be no fiscal transfers in the aggregate.

Alternatively, the union's monetary authority could resort to an increase in traditional seigniorage to provide additional revenue to the crisis country, effectively increasing the upper bound on debt. However, acceptable magnitudes are likely to be small. And implementing this policy without fiscal transfers requires increasing seigniorage for all member countries, not just for the crisis country. Additionally, the increase in seigniorage is likely to require an increase in both crisis and average post-crisis inflation, as in Sargent and Wallace (1981). This sustained increase in inflation after the crisis is likely to be more objectionable than stochastic inflation around its target, as implied by a response of policy-switching.

Finally, a country could withdraw from the monetary union and reissue its own currency, as suggested by Sims (1999). If the country also institutes policy reform, switching fiscal policy to active and its own new monetary policy to passive, then the analysis would be much as in the switching model presented here. Alternatively, the new monetary authority could be pressured to provide additional seigniorage, as in Sargent and Wallace (1981), yielding larger seigniorage revenues and a larger value for the upper bound on debt.

5 Simulations of Crisis Risk

In this section, we use simulations to consider the fiscal risk faced by different countries in a monetary union under the two possible fiscal responses to a crisis, default (D) and policy switching (SW). Given parameter values for the N fiscal rules, the distribution of ν_t, and the method of crisis resolution, the system can be solved numerically and simulated to generate the risk of one country in the N-country monetary union encountering a crisis over a given period of time.[17]

[17]The methodology is similar to that in Garcia and Rigobon (2004) who use simulation to determine the probability that Brazilian debt will reach a fixed upper bound within a particular time frame. In our model, the upper bound is not fixed, but is given by the boundary locus.

Using annual data over the 1970-2006 period, Daniel and Shiamptanis (2008) provide group mean estimates of parameters for the surplus rule using cointegration and error-correction models for a panel of ten EMU countries. The baseline parameters we use for the simulations are reported in Table 1.

Table 1: Baseline Parameters

	i	φ	α	λ	g
parameters	0.0422	0.0388	0.4987	1.3003	0.0262
standard errors	0.0061	0.0076	0.0717	0.0901	0.0027

We use these estimates together with the group mean panel estimate of the long-run value of output growth g, to specify values for the real interest rate, i, the target value of the long-run primary surplus, φ, the growth-adjusted real interest rate, r, the persistence, $1 - \alpha$, and the responsiveness of the surplus to the value for debt service relative to its long-run target value, λ.[18] Under the assumption that fiscal shocks have a normal distribution with mean zero, the panel estimate of their standard error is 1.42% of GDP. We let the upper bound on the fiscal shocks, $\bar{\nu}$, be 2.84% of GDP, which corresponds to two standard deviations. We set the desired maximum value of the surplus, $\hat{\varphi}$, at 4% of GDP which is larger than ninety percent of the actual primary surplus ratios achieved in our sample.[19]

We consider risk faced by an individual country. We use 1,000 replications of a ten-year simulation, under the two fiscal responses to a crisis, to estimate the probability of a fiscal

[18]The variables in the paper of Daniel and Shiamptanis (2008) are in levels, whereas the variables in this paper are expressed as percentages of output. This implies that the α in this paper is $\alpha = \frac{\hat{\alpha}}{1+g}$, where $\hat{\alpha} = 0.5118$, and the growth-adjusted real interest rate is $r = \frac{i-g}{1+g}$.

[19]Out of the 370 primary surplus/GDP observations, there are only 35 observations in which their values are larger than 4% of GDP.

crisis and the average time to a crisis. In each simulation, initial values of debt/GDP, b_{t-1}, and the primary surplus/GDP, s_{t-1}, are used to set the initial value for the distance, $x_{t-1}+\psi_{t-1}$. For the default case, we use a numerical approximation of the boundary locus to obtain a value for $\hat{b}_{t-1}$. The dynamic system then receives a fiscal shock, ν_t, from the truncated normal distribution. Based on $x_{t-1}+\psi_{t-1}$ and ν_t, the critical value for the shock, ν_t^*, the expectation for capital loss, $E_{t-1}\gamma_t$, and the value for capital loss, γ_t, are calculated. If $\gamma_t = 0$, then next period's surplus and debt are updated using equations (7) and (8), which are then used to update $x_t+\psi_t$. The process is repeated for ten years. If during the ten-year simulation we have a value of $\gamma_t > 0$ or $x_t+\psi_t < 0$ then there is a crisis and the simulation ends. We repeat the ten-year simulation 1000 times. The probability of a crisis over ten-years is the number of crises divided by 1000, the number of replications.

To determine the safety of a country which adheres to the Maastricht rules, we simulated the model with values for initial debt and the primary surplus under the Maastricht rules. We set the actual surplus (not primary) at -3% of GDP and debt at 60% of GDP. Under the baseline parameter values, fiscal policy is very safe with no crises over ten years in the 1,000 replications. We considered several sensitivity analysis scenarios to raise the risk. These include changing parameter values one at a time by two standard deviations in the risky direction and simultaneously increasing the nominal interest rate and reducing the growth rate, both by two standard deviations.[20] Under each of the simulations to increase risk, a country under Maastricht rules is perfectly safe over the ten

[20]Experiments included raising i to 0.0544, raising λ to 1.4805, reducing α to 0.3553, reducing g to 0.0208, reducing φ and $\hat{\varphi}$ to 0.0236, and increasing the upper bound on $\bar{\nu}$ to 4.26% of GDP (which corresponds to a three standard deviation increase). Note that an increase in i to 0.0544 and a decrease in g to 0.0208, increase r to 0.0329.

year horizon. We also find that countries like France, Germany, and Belgium, with small deviations from the Maastricht rules are perfectly safe under the risk-increasing scenarios. This implies that the limits in the Maastricht Treaty might be too strict.

Next we consider whether high-debt countries like Italy and Greece, which have violated the Maastricht rules, face any risk over the next ten years. For Italy, the 2007 value of debt/GDP was 116.14%, and the primary surplus/GDP was 1.95%. For Greece, the 2007 value of debt/GDP was 103.82% and the primary surplus/GDP was 1.01%.[21] Under the baseline parameter values and sensitivity analyses, whereby we move parameters one at a time in the risky direction by two standard deviations, both Italy and Greece are perfectly safe over the next ten years. However, as we increase the real growth-adjusted interest rate from this point, possibly due to a simultaneous increase in the nominal interest rate and a decrease in the growth rate, crisis probability becomes positive and rises at an increasing rate, as shown in Figure 4. Since debt/GDP is higher for Italy than for Greece, the crisis probability becomes positive at a lower value for the growth-adjusted real interest rate for Italy. We also considered how crisis probability changes as debt increases with parameter values set at their baseline values. Since the estimated value for φ is large, debt can become quite large under the baseline parameters before the ten-year probability of a crisis becomes positive. Once the probability becomes positive, crisis probability increases at an increasing rate in debt, as shown in Figure 5. The expected number of years to a crisis falls as its probability rises.

Crisis probability rises at an increasing rate, either as parameter values change in the risky direction or as debt increases, because expectations of debt devaluation rise as the

[21]Source: OECD

economy approaches the boundary locus. Higher expected debt devaluation increases the interest rate and increases the rate at which debt accumulates. As a country approaches the boundary locus, a slight change in parameters or in debt can create a dramatic change in crisis probability. This illustrates forcefully that a country receiving favorable shocks can substantially reduce and/or eliminate the probability of crisis without fiscal reform. It also illustrates the reverse. A country can substantially increase its crisis probability with small changes in debt which push it critically toward the boundary locus.

These simulations of fiscal risk are conditional on fiscal policy following the rule estimated for the panel over the period 1970-2006. They ignore any policy differences among countries. We are also ignoring the likely correlation of fiscal shocks across countries combined with the fact that a fiscal crisis in one country can affect the interest premium in another under policy-switching. This implies that risk is actually higher, and future research is needed to address this.

6 Conclusions

Countries in the EMU no longer feel compelled to follow the rules set out in the Maastricht Treaty. Some economists argue that there is no need for any coordinated fiscal restraint. Yet, others are concerned that unrestrained fiscal policy could pose problems for the monetary authority's ability to control inflation

We present a model to analyze how a country following a strongly passive fiscal policy, subject to stochastic shocks and an upper bound on the present value of surpluses, could experience a fiscal financial crisis in which agents refuse to lend. When faced with a sudden stop in capital flows, such that a government is unable to continue with its desired fiscal policy, some fiscal response is needed. We consider two responses; maintenance of the strongly passive fiscal policy combined with default to reduce the magnitude of outstanding debt, and policy switching. If the monetary authority is willing to allow a member country to experience default and the associated post-crisis market turbulence, then a fiscal financial crisis in one country need not impair the monetary authority's ability to control inflation. However, if the monetary authority prefers policy switching to allowing member default, then a fiscal financial crisis in one country can impair the monetary authority's ability to control inflation.

The paper makes two primary contributions. The first is theoretical. We use insights from the Fiscal Theory of the Price Level to determine when an agent would refuse to lend to the government in equilibrium. This allows us to model the dynamics of a fiscal financial crisis, demonstrating how an upper bound on the value of debt relative to output, combined with stochastic shocks to fiscal policy, could give government debt risk, even when policy is governed by a strongly passive fiscal rule.

Second, it provides simulations using estimated parameter values and initial conditions from EMU countries to determine the probability of a fiscal financial crisis in the next ten years, under alternative assumptions about the fiscal response to a crisis. We find that a country operating at the upper bound of the Maastricht Treaty is perfectly safe over a horizon of ten years. Additionally, countries like Belgium, France and Germany with small violations, are also perfectly safe. However, countries like Italy and Greece with high debt, are safe under the baseline parameter values, but there are reasonable variations which increase risk over a relatively short time horizon.

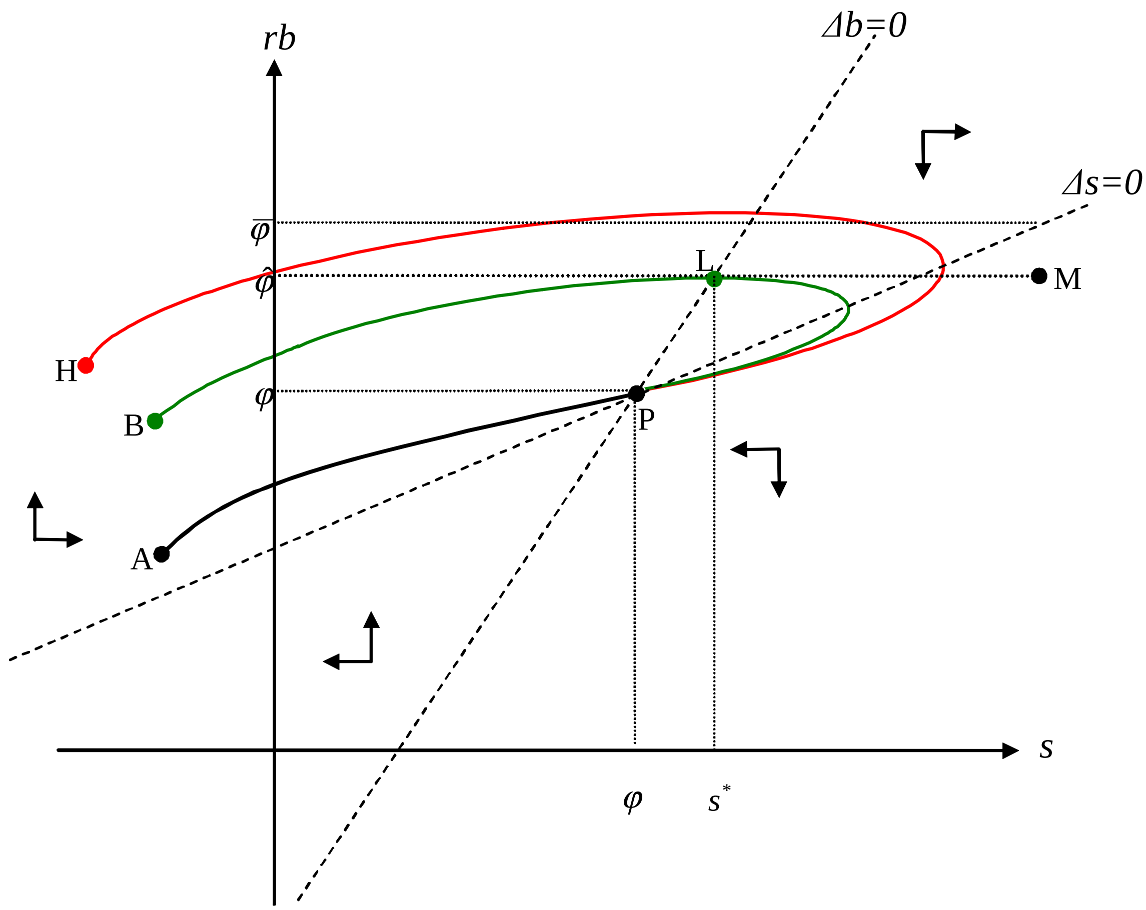


Figure 1: Passive Fiscal Policy

Note: $s^{*} = \frac{\hat{\varphi}(1-\alpha\lambda)-\alpha(1-\lambda)\varphi}{1-\alpha}$ is the value of s along the adjustment path BP at the point L with $rb = \hat{\varphi}$.

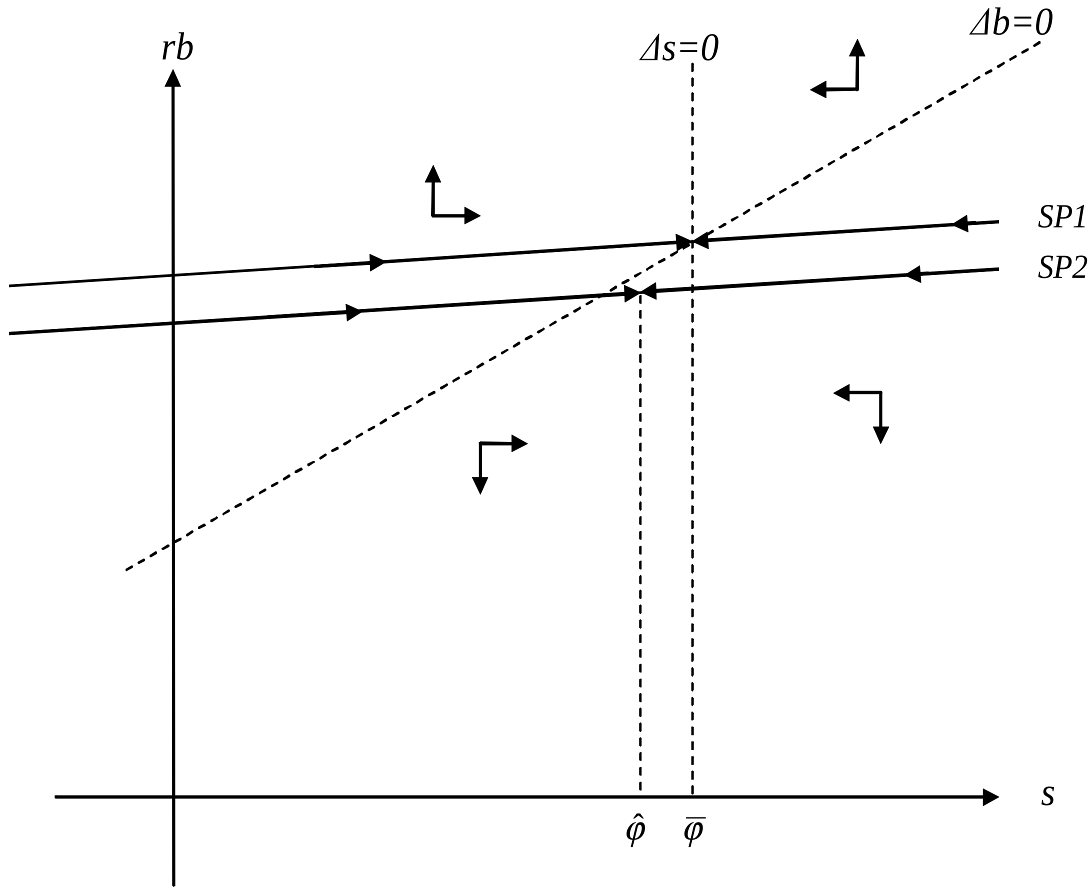


Figure 2: Active Fiscal Policy

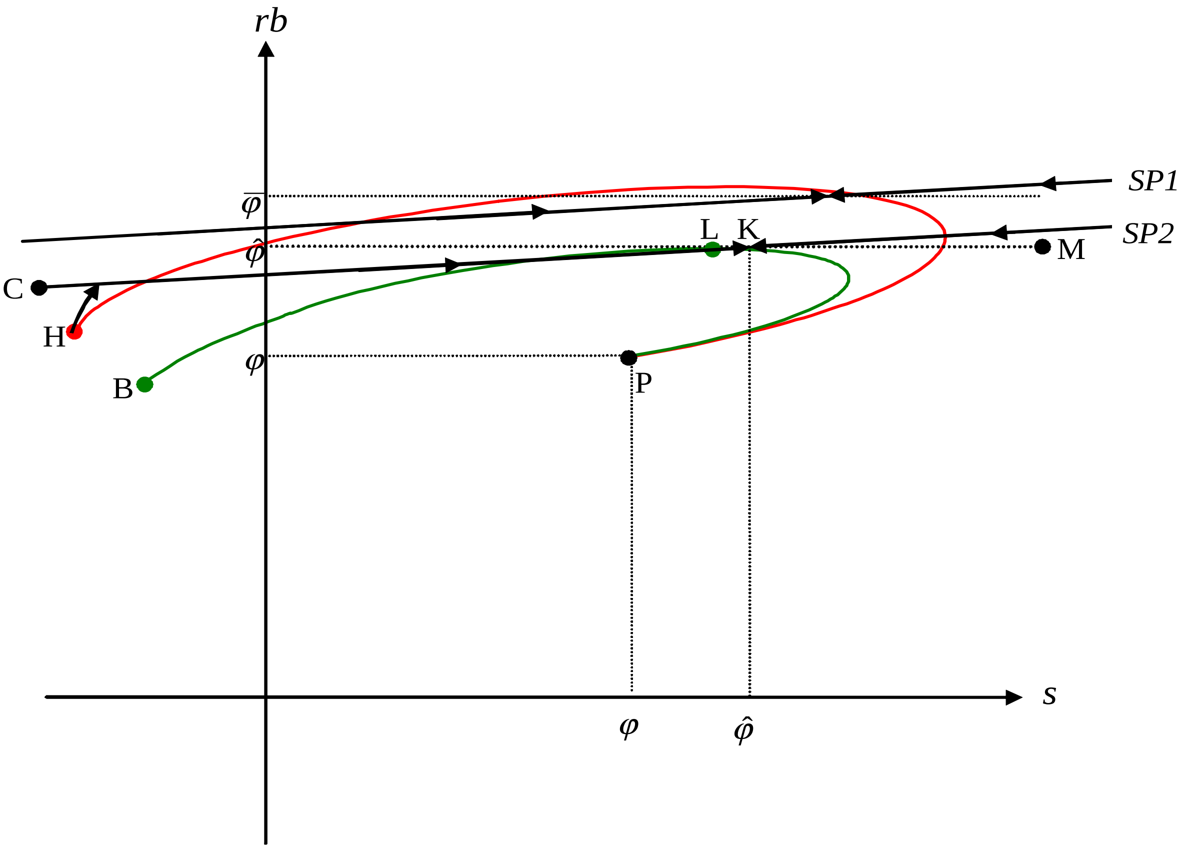


Figure 3: Switching

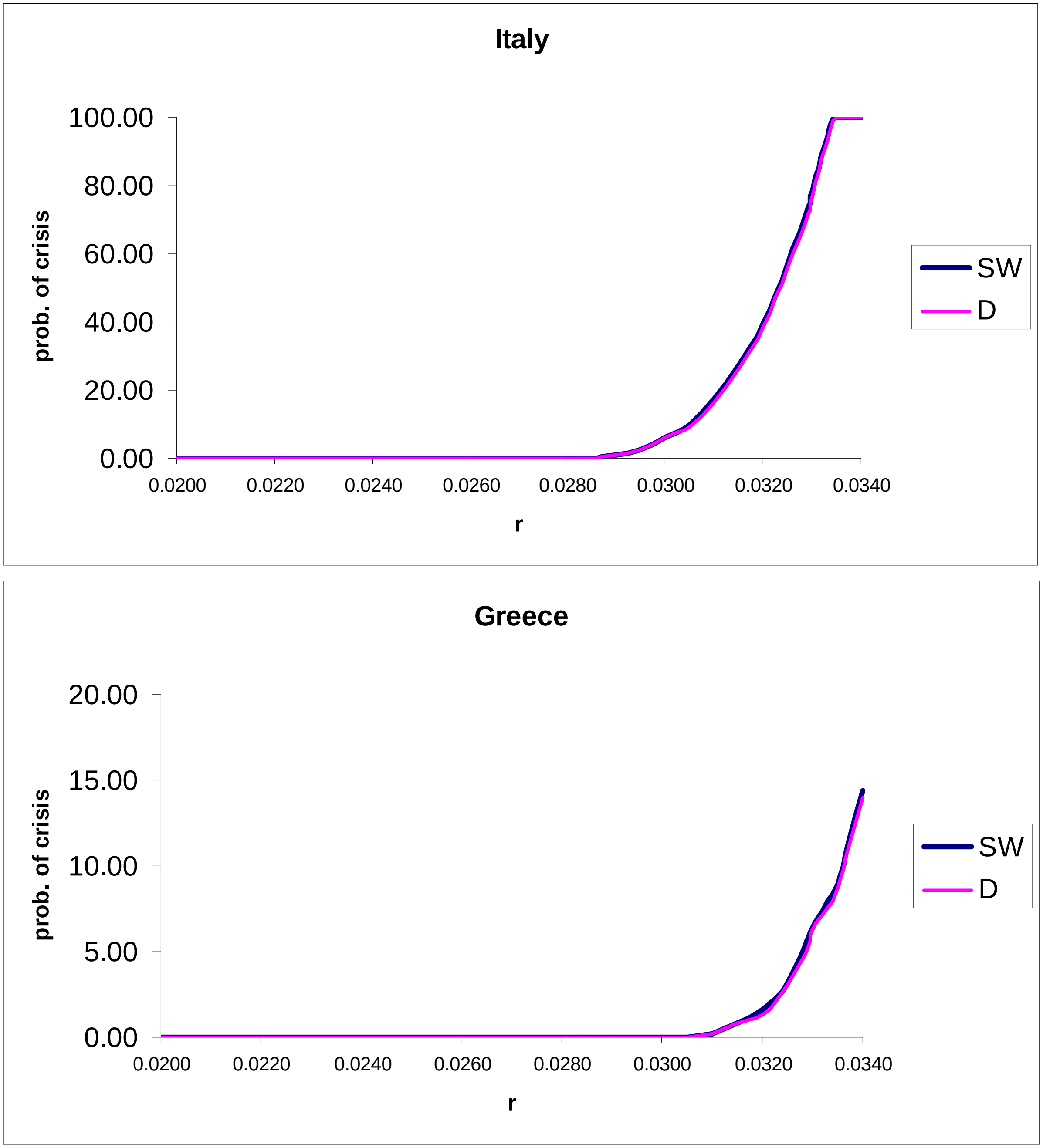
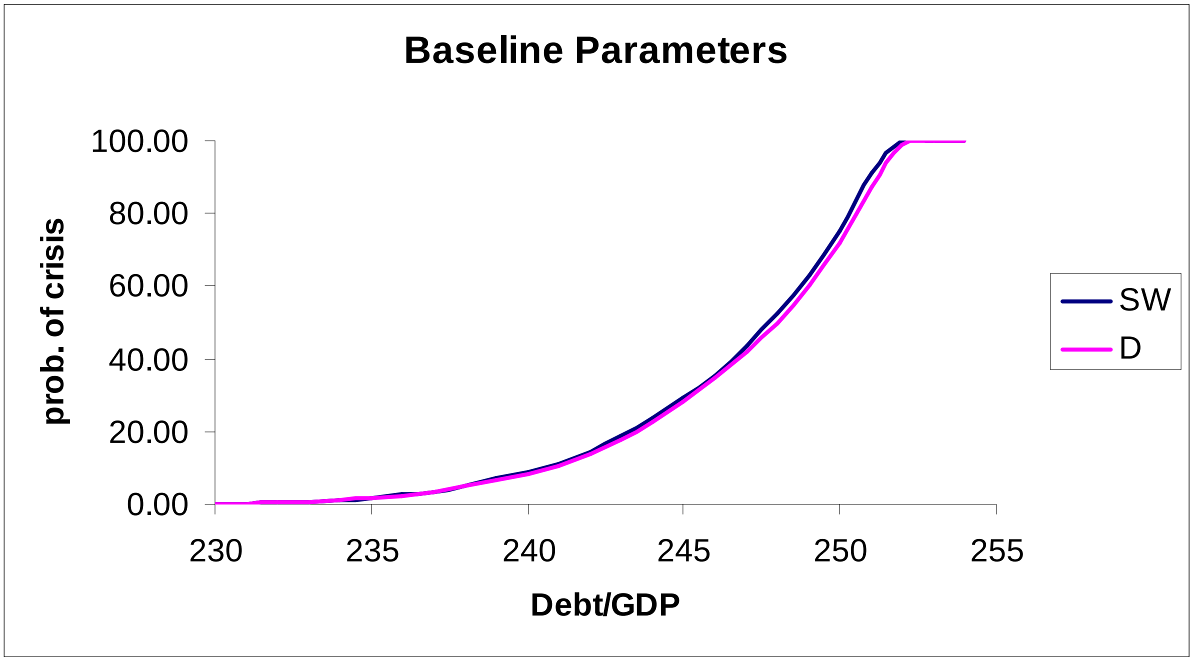


Figure 4: Italy and Greece


Baseline Parameters
prob. of crisis
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80.00
60.00
40.00
20.00
0.00
230
235
240
245
250
255
Debt/GDP
SW
D

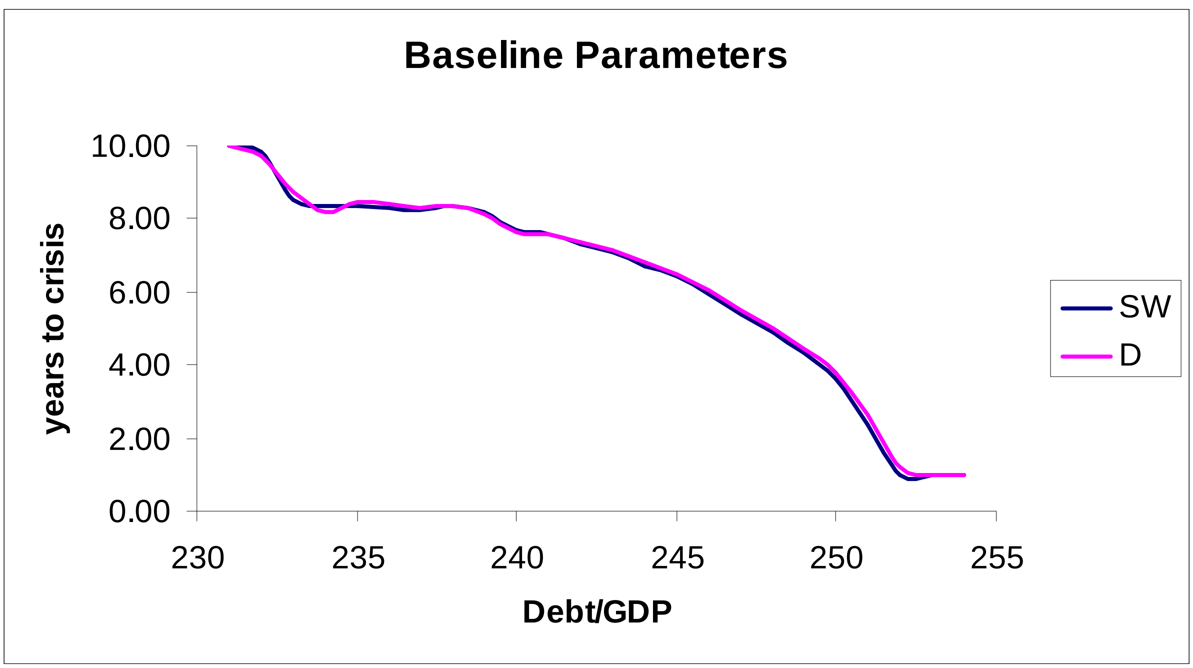
Baseline Parameters
years to crisis
10.00
8.00
6.00
4.00
2.00
0.00
230
235
240
245
250
255
Debt/GDP
SW
D

Figure 5

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