Overstatement and Rational Market Expectation*

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Abstract

When an agent overstates his/her true performance, a rational market can simply discount the reported performance, and correctly guess the true performance. This paper shows, however, that such rational market discounting leads to less productive effort by the agent and less performance-pay by the principal. Therefore, a rational market and a profit-maximizing principal can exacerbate the lack of productive effort by the agent.

Keywords: overstatement; rational expectation; moral hazard

JEL: L20, D80, J30

1 Introduction

Overstatement of performance is a serious problem in many aspects of the economy. A CEO may overstate firm's earnings, and a president can overstate the merits of his or her policies. In response, however, the market has also learned to discount the reported performance rationally.

This paper considers the effect of such rational market expectation in a principal-agent model. We show that the rational market expectation reduces the agent's overstatement, but that it also reduces

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the agent's productive effort in the first place. Moreover, the rational market expectation reduces the principal's optimal performance pay, which exacerbates the lack of productive effort. Therefore, neither the rational market nor the profit-maximizing firms can resolve the fundamental problem of overstatement.

There has been an increasing number of theoretical and empirical studies on the relationship between overstatement (or earnings management) and performance-pay. (See, e.g., Bergstresser and Philippon 2006 and Crocker and Slemrod 2007) However, most of these studies do not consider the role of rational market expectation. The literature on strategic information transmission (e.g. Stein 1989) takes into account rational expectation, but largely ignores its effect on productive effort, which is the main focus of this paper.

2 The Model

There are three risk-neutral[1] players: a principal, a market, and an agent. The agent first exerts (productive) effort, a (≥ 0), at the cost of $\frac{1}{2}a^2$. The true performance (y) is realized as follows:

$$y = a + \epsilon \tag{1}$$

where ϵ $(\in [0, \bar{\epsilon}])$ is a random noise with mean μ_ϵ and pdf $f(\epsilon)$. For simplicity, we have normalized the true performance to be always positive. Neither the effort (a) nor the true performance (y) is observable to the principal or to the market.

After observing the realized (true) performance, the agent can overstate the performance by m (≥ 0) at the cost[2] of $\frac{k_m}{2}m^2$. The reported performance (denoted by $\widehat{y}$) is determined as follows:

$$\widehat{y} = y + m = a + \epsilon + m \tag{2}$$

Note that unlike in Holmstrom and Milgrom (1991), effort and overstatement are done sequentially, not simultaneously. Thus, we assume that each task has no effect on the marginal cost of the other.

The agent's overstatement (m) is not observable to the principal or to the market. However, the market can discount the agent's report by its expectation of the agent's overstatement, $m^e(\widehat{y})$, and determine the agent's market performance, $\widetilde{y}$, where

$$\widetilde{y} = \widehat{y} - m^e(\widehat{y}) \tag{3}$$

[1] Thus, our model differs from Lacker and Weinberg (1989) where their explanation of agent's overstatement relies on the agent's risk aversion.

[2] The cost of overstatement includes the manager's private cost of falsifying information and the expected penalty from possible audit. See Moran and Morgan (2003) for more details.

For example, the reported performance can be a CEO's earnings report, and the market performance can be the stock price of the firm.

We assume the principal also cares about the market performance (not the true performance). That is, shareholders may care about the stock price, not the fundamentals of a company.[3]

The contract for the agent depends on the market performance, $\widetilde{y}$, only.[4] For simplicity, we assume the following linear wage contract for the agent,

$$w = s + \beta \widetilde{y} \tag{4}$$

For example, β can be the CEO's share of stock holding. Even though we present the model in the context of a wage contract, one can also interpret β as a degree of competition that affects pay-performance sensitivity.

Benchmarks Let us first consider the agent's optimal effort and overstatement when they are observable to both the principal and the market, denoted by a^*_{FB} and m^*_{FB} respectively. Then, it is straightforward to show that $a^*_{FB} = 1$ and $m^*_{FB} = 0$.

In the opposite case, suppose that the agent's effort and overstatement are not observable, and that the market is naive and does not discount the reported performance (i.e. $m^e = 0$). Then, the agent's optimization problem for m is:

$$\max_{m} \; s + \beta \widetilde{y} - \frac{k_m}{2} m^2 = s + \beta(y + m) - \frac{k_m}{2} m^2 \tag{5}$$

From the first order condition, the agent's overstatement level (denoted by m^*_N) is $m^*_N = \frac{\beta}{k_m} > 0$.Then, the agent's optimization problem for the productive effort a is:

$$\max_{a} \; s + \beta(a + \mu_\epsilon + \frac{\beta}{k_m}) - \frac{1}{2}a^2 - \frac{k_m}{2}(\frac{\beta}{k_m})^2 \tag{6}$$

where $E(\epsilon) = \mu_\epsilon$.

From the first order condition, the agent's effort level (denoted by a^*_N) is $a^*_N = \beta > 0$.

Finally, the principal's optimization problem is:

$$\max_{m,a,s,\beta} \; E[\widetilde{y} - s - \beta \widetilde{y}] \tag{7}$$

[3] See Guthrie, Kwon, and Sokolowsky (2008) for related evidence.

[4] An implicit assumption is that the contract does not depend on the agent's reported performance or any other message. This is an arguably realistic assumption, though admittedly ad-hoc. For example, stock option contracts solely depend on the stock price, and not on the announced earnings.

subject to the participation constraint: $E[s+\beta(a+\epsilon+m)-\frac{1}{2}a^2-\frac{k_m}{2}m^2]\geq 0$, and the incentive constraints: $m=\frac{\beta}{k_m}$ and $a=\beta$.

Let us denote the optimal pay-performance sensitivity under naive market expectation by β_N^*.

Proposition 1 *When market expectation is naive,* $\beta_N^*=1$, $a_N^*=1$ $(=a_{FB}^*)$, *and* $m_N^*=\frac{1}{k_m}$ $(>m_{FB}^*=0)$.

Proof. Straightforward from (7), and omitted. ■

3 Rational Market Expectation and Optimal Contract

Now suppose that the agent can overstate true performance, and that the market can rationally expect it and discount the reported performance.

3.1 Rational Market Expectation and Overstatement

To solve backwards, given the true performance y, the agent's optimization problem for m is:

$$\max_m \; s+\beta\widetilde{y}-\frac{k_m}{2}m^2=s+\beta(y+m-m^e(y+m))-\frac{k_m}{2}m^2 \tag{8}$$

Note that the agent considers the market expectation as a given *function*.

From the first order condition, the agent's optimal overstatement, denoted by m^*, is:

$$m^*(y)=\frac{\beta}{k_m}(1-\frac{dm^e(\widehat{y})}{d\widehat{y}}). \tag{9}$$

The rational expectation hypothesis implies that, in equilibrium, the market's expectations should be correct:

$$m^e(\widehat{y})=m^*(y)=\frac{\beta}{k_m}(1-\frac{dm^e(\widehat{y})}{d\widehat{y}}). \tag{10}$$

Because $a\geq 0$, $m\geq 0$ and $\epsilon\geq 0$, if $\widehat{y}=a+\epsilon+m=0$, we must have $m=0$. For instance, when a professor reports a grade "F" for a student, there cannot be any overstatement. This provides a boundary condition, $m^e(0)=0$.

Then, by solving the differential equation (10), we can characterize the rational market expectations as follows[5]:

$$m^e(\widehat{y})=\frac{\beta}{k_m}[1-\exp(-\frac{k_m}{\beta}\widehat{y})]. \tag{11}$$

[5] There exists another rational expectation equilibrium where $m^e=m^*=\frac{\beta}{k_m}$. We ignore this equilibrium as the market expectation does not depend on the reported performance.

It is easy to see that the market expectation of overstatement increases in the reported performance (i.e. $\frac{dm^e(\hat{y})}{d\hat{y}} > 0$). Then, from (9), $m^*(y) < \frac{\beta}{k_m}(= m_N^*)$. Thus, compared to a naive market, a rational market reduces the level of overstatement without eliminating it. Also, note that the market expectation is concave. Thus, from (9), $m^*(y)$ must be increasing in y (i.e. $\frac{dm^*(y)}{dy} > 0$).

By substituting (11) into (9), the agent's optimal overstatement, $m^*(y)$, is determined by the following implicit function[6]:

$$m^*(y) = \frac{\beta}{k_m}[1 - \exp(-\frac{k_m}{\beta}(y + m^*(y))]. \tag{12}$$

To summarize,

Proposition 2 *(i) For a given* β, $0 < m^*(y) < m_N^*$ *for all* y.

(ii) The overstatement, $m^*(y)$*, increases in the true performance,* y.

(iii) $\frac{\partial m^*}{\partial k_m} < 0$.

Proof. Straightforward from (12), and omitted. ■

Interestingly, an agent's overstatement increases in his true performance. Intuitively, because the marginal cost of overstatement is increasing, when the reported performance is high, the market believes it cannot be all driven by large overstatement. Thus, the market expectation of overstatement increases in the reported performance at a decreasing rate. In response, the agent has more incentive to overstate when the true performance is large, because the market will discount relatively less at the margin. Not surprisingly, the overstatement decreases when the marginal cost of overstatement increases.

3.2 The Incentive for Productive Effort

Given the rational market expectation and the optimal overstatement $m^*(y)$, the agent's optimization problem for effort is:

$$\max_a \; s + \beta(a + \mu_\epsilon) - \frac{1}{2}a^2 - \frac{k_m}{2}\int m^*(a + \epsilon)^2 f(\epsilon)d\epsilon \tag{13}$$

From the first order condition,

$$a^* = \beta - k_m \int m^* \frac{\partial m^*}{\partial a} f(\epsilon)d\epsilon \tag{14}$$

Since $m^* > 0$ and $\frac{\partial m^*}{\partial a} = \frac{\partial m^*}{\partial y} > 0$ from Proposition 2, $a^* < \beta$. Recall that under naive market expectation, $a_N^* = \beta$. Therefore, compared with naive market expectation, when the market expectation is rational, the

[6] It is straightforward to show that the second order condition is satisfied given the market expectation (12).

agent exerts *less* productive effort for a given β. Also, note that, unlike the naive expectation case, the agent's optimal effort now depends on the cost of overstatement, reflected in k_m.

We summarize the features of the agent's optimal choice of effort in the following proposition.

Proposition 3 *When the market is rational,*

(i) for a given β, $a^* < a_N^*$;

(ii) $\frac{\partial a^*}{\partial k_m} > 0$;

(iii) $0 < \frac{\partial a^*}{\partial \beta} < 1$.

Proof. (i) From (14), $a^* < \beta$ because $\frac{dm^*}{da} = \frac{dm^*}{dy} > 0$ from Proposition 2. ■

(ii) From (14), $a^* = \beta - k_m \int m^*(\frac{\beta}{k_m m^*} - 1) f(\epsilon) d\epsilon = k_m \int m^*(a^* + \epsilon; k_m, \beta) f(\epsilon) d\epsilon$. Therefore,

$$\frac{\partial a^*}{\partial k_m} = \int m^* f(\epsilon) d\epsilon + k_m \int \left\{ \frac{\partial m^*}{\partial k_m} + \frac{\partial m^*}{\partial a} \frac{\partial a^*}{\partial k_m} \right\} f(\epsilon) d\epsilon$$

$$\Longleftrightarrow \frac{\partial a^*}{\partial k_m} = \frac{\int (m^* + k_m \frac{\partial m^*}{\partial k_m}) f(\epsilon) d\epsilon}{1 - k_m \int \frac{\partial m^*}{\partial a} f(\epsilon) d\epsilon}$$

$$= \int y \frac{e^{-\frac{k_m}{\beta}(y+m^*)}}{1 - e^{-\frac{k_m}{\beta}(y+m^*)}} f(\epsilon) d\epsilon / \int \frac{1 - e^{-\frac{k_m}{\beta}(y+m^*)} - k_m e^{-\frac{k_m}{\beta}(y+m^*)}}{1 - e^{-\frac{k_m}{\beta}(y+m^*)}} f(\epsilon) d\epsilon > 0$$

(iii) From (14),

$$\frac{\partial a^*}{\partial \beta} = \frac{1 - k_m \int \{ \frac{\partial m}{\partial \beta} \frac{\partial m^*}{\partial a} + m \frac{\partial^2 m^*}{\partial \beta \partial a} \} f(\epsilon) d\epsilon}{1 - k_m \int \{ (\frac{\partial m}{\partial a})^2 + m \frac{\partial^2 m}{\partial a^2} \} f(\epsilon) d\epsilon} = \frac{k_m \int \frac{\partial m}{\partial \beta} f(\epsilon) d\epsilon}{1 + k_m \int (\frac{\beta}{k_m m^*} - 1) f(\epsilon) d\epsilon}$$

Since $0 < \frac{\partial m}{\partial \beta} < \frac{1}{k_m}$ and $\frac{\beta}{k_m m^*} - 1 > 0$, we must have $0 < \frac{\partial a}{\partial \beta} < 1$.

Intuitively, the agent must take into account that choosing higher effort will require subsequently larger (and costly) overstatement, because the market expects him to do so. Thus, the agent's incentive for productive effort under rational market expectation is smaller than under naive market expectation.

Instead, when the marginal cost of overstatement increases (possibly due to antifraud legislation), it not only decreases the agent's incentive for overstatement, but also *increases* the agent's incentive for (productive) effort. Intuitively, since k_m is exogenous and common knowledge, an increase in k_m is a credible commitment to reduce market expectation of overstatement. Then, the agent will not be forced to make a more costly overstatement for higher true performance, which increases the agent's incentive for effort.

3.3 Optimal Contract

So far, we have treated β as exogenously given, for example, by market competition or exogenous contracts. In the example of CEO contracts, however, the principal (e.g. shareholders) can determine β optimally. Thus, we now characterize the optimal contract for the principal. Unlike the standard moral hazard model, however, now the principal must consider how a change in β will affect the market expectation.

The principal's optimization problem is[7]:

$$\max_{a,m,s,\beta} E\left[\widetilde{y} - s - \beta\widetilde{y}\right] \tag{15}$$

subject to the participation constraint,

$$E[s + \beta(a + \epsilon + m - m^e) - \frac{1}{2}a^2 - \frac{k_m}{2}m^2] \geq 0 \tag{16}$$

and the incentive constraints (12) and (14).

Since the participation constraint must be binding, the principal's optimization problem simplifies to

$$\max_{\beta} E[a^*(\beta) + \epsilon - \frac{1}{2}a^{*2}(\beta) - \frac{k_m}{2}m^*(\beta, a^*(\beta))^2] \tag{17}$$

Note that m^* depends on y $(= a+\epsilon)$. Thus, the optimization problem is not trivial because the principal must consider the indirect effect of β on m^* through a^* where m^* is defined by an implicit function (12). Let us denote the optimal β under rational market expectation by β^*. Then, we can characterize β^* as follows:

Proposition 4 $\beta^* = 1 - E[m^* \frac{\partial m^*}{\partial \beta}]/E[\frac{\partial m^*}{\partial \beta}] < \beta_N^*$.

Proof. From the first order condition,

$$\begin{aligned}
&\frac{\partial a^*}{\partial \beta} - a^* \frac{\partial a^*}{\partial \beta} - k_m E[m^*(\frac{\partial m^*}{\partial \beta} + \frac{\partial m^*}{\partial a}\frac{\partial a^*}{\partial \beta})] = 0 \\
&\Longleftrightarrow 1 - a^* - \beta - k_m E[m^* \frac{\partial m^*}{\partial \beta} / \frac{\partial a^*}{\partial \beta} - m^*] = 0 \\
&\Longleftrightarrow \beta^* = 1 - k_m E[m^* \frac{\partial m^*}{\partial \beta} / \frac{\partial a^*}{\partial \beta}] = 1 - E[m^* \frac{\partial m^*}{\partial \beta}]/E[\frac{\partial m^*}{\partial \beta}] < 1
\end{aligned}$$

The last inequality is due to $\frac{\partial m^*}{\partial \beta} > 0$ from (12). ■

[7] Recall that the principal in our model cares about the market performance of the agent, not the true performance. When the market expectation is rational, however, there is no difference between the true performance and the market performance.

Intuitively, under rational market expectation, the principal does not gain from the agent's overstatement, but still must pay for the agent's overstatement, due to the binding participation constraint. Since increasing β leads to more costly overstatement without additional gains, it is now optimal for the principal to reduce β.

3.4 Effects of Rational Expectation

We are now ready to state the full effect of rational market expectation, taking into account the endogenous choice of β and a.

Proposition 5 *Compared with naive market expectation, rational market expectation leads to less overstatement, less productive effort, and less pay-performance sensitivity.*

Proof. From proposition 2 and 3, compared with naive expectation, rational market expectation reduces the agent's effort and overstatement given β. Proposition 4 shows that rational expectation reduces β as well, which from proposition 2 and 3, will reduce the agent's effort and overstatement even further. ■

Our model shows that rational discount of overstated performance is not sufficient in dealing with the overstatement problem. Even though rational market expectation reduces the level of overstatement and prevents the agent (and the principal) from profiting from the overstatement, it reduces the agent's incentive for productive effort in the first place.

Furthermore, we can not rely on a principal, even if she exists, to improve the agent's incentive for productive effort, because the principal would reduce the pay-performance sensitivity, β, and reduce the agent's incentive for effort even further.

Therefore, in addition to rational 'discount of overstatement', Propositions 2 and 3 suggest that we must find a way to increase the marginal cost of overstatement, k_m, as it would decrease overstatement *and* increase productive effort at the same time. Recent antifraud legislation (e.g. Sarbanes-Oxley Act of 2002) provides a welcome change in this regard.

Note that even though most of this legislation has focused on reducing the level of overstatement, especially in firms' earnings reports, the benefits of this legislation are no longer clear if the market already knows how to discount overstated performance. Our paper shows that a more important benefit of this antifraud legislation can be the increased productive efforts by the agent.

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