Directed Search without Wage Commitment and the Role of Labor Market Institutions

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Abstract

An urn-ball matching model of directed search is analyzed in which the usual assumption of commitment to posted wages is dropped. One-on-one matches lead to a Nash bargained wage but when multiple applicants arrive competition drives the workers down to their continuation value.

A minimum wage can act as a commitment device when (as in the USA) willful underpayment carries a stiffer penalty than "inadvertent" underpayment. The theory sheds new light on why firms appear to voluntarily bind themselves into paying higher wages than they would otherwise pay. Robustness to various sources of heterogeneity is considered

1 Introduction

This paper explores the idea that firms use labor market institutions such as the minimum wage or labor unions as commitment devices to avoid paying low wages. Essentially, if firms cannot commit to a particular wage, competition among applicants drives the wage down. In such a market, a firm that can credibly commit to a higher wage will attract more applicants. More applicants mean vacancies can be filled faster and with better workers, the benefit of which can outweigh the increased cost of labor.

Non-compliance with the minimum wage in the USA is significant and persistent. Ashenfelter and Smith [1979] was the first serious attempt to measure the extent of noncompliance. More recently, Eckstein *et al* [2005] estimate a structural search-based model of the labor market to back out a measure of non-compliance. While they are 25 years apart and based on different data sets, both studies reveal that between 30 and 40 % of those workers who should receive the minimum wage are underpaid. Yet another different data source was used by Holtzer *et al* [1991] who look at application rates at jobs paying below, at and above the minimum wage. The rate of noncompliance in their sample (after removing workers in exempt industries) is 25%.

Despite this evidence, these studies provide no real discussion as to the cause of non-compliance. Actually, the question raised here is why do any firms comply? The regulation stipulates that workers can be awarded a *maximum* of twice the backpay for up to 2 years if they lodge a successful complaint against their employer. Only when there is "willful" or repeated disregard of the law is there any criminal penalty incurred by the firm.[1]

[1] See http://www.dol.gov/asp/programs/guide/minwage.htm

While the minimum wage provides a simple example for analytical purposes, perhaps the clearest instances of the kind of behavior highlighted here are of voluntary recognition of unions. While unions provide many other services to workers it is well established that they also provide a wage premium (see Booth [1995]). Some evidence on voluntary recognition of unions in the USA comes from the Federal Mediation and Conciliation Service (FMCS) [2004] p. 18. It reports that of the 1,311 initial contract cases assigned to federal mediators in FY 2004, 258 were assigned from certification sources other than the National Labor Relations Board (NLRB) "such as voluntary recognitions". How many of these are truly voluntary is not reported but these figures do understate the proportion of non-NLRB certifications. This is because only NLRB certifications are necessarily referred to the FMCS. For the UK, Central Arbitration Committee [2004] does report voluntary recognitions. At any stage in the formal proceedings, employers can choose to voluntarily recognize unions. Between 2000 and 2004, of 361 applications for recognition, 85 were accepted by employers without a ballot.

As directed search explicitly incorporates the application decision of workers, it provides a natural environment to address the issue of why firms might associate themselves with institutions that lead to paying higher wages than they would otherwise pay.[2] Essentially, if there is a large number of workers and vacancies in a market and, in any period, workers apply to a subset of the vacancies, then the number of applicants at any one vacancy can be described by a discrete probability distribution which puts positive probability on no applicants at all. The usual approach assumes that firms can commit to posted wages and that workers can direct their applications accordingly.

[2]See Rogerson *et al* [2005] for more background to this approach to modelling the labor market.

In the absence of any coordination mechanism, it is assumed that homogeneous workers follow symmetric strategies. In equilibrium, workers will be indifferent across applying to each firm and attach a probability of applying to each which depends on the characteristics (including the wage) of the vacancy.

The central theoretical deviation from the literature of this work is that firms cannot commit to posted wages. Knowing this workers duly ignore them. Julien *et al* [2005a] assert that firms rarely post wages. Here, the issue is moot, when the set of applicants is realized, wage formation will occur without regard to whatever wage was posted. In the baseline model with homogeneous firms and workers, I assume that when there is only one applicant the wage is negotiated. Otherwise, one worker is hired at random from the pool of applicants with a wage equal to the workers' continuation value. In general, it is well known that such a mechanism for wage formation may be suboptimal from the perspective of the firm. The point here is to look into when firms might use labor market institutions such as the minimum wage as a commitment device to prevent themselves from paying wages that are too low.

Wages can be too low from the firms' perspective if the implied increase in the application rate from raising wages outweighs the increased cost of labor. It is shown that as long as firms retain some bargaining power in the one-on-one matches, there is always a binding minimum wage to which firms would like to commit. It should be clear how recognition of a union could represent a commitment to a particular wage structure. It is less obvious however, that offering the minimum wage carries any more commitment than offering any other wage. The assertion here is that (at least in the USA) if firms declare themselves to be minimum wage payers, subsequent violation of the regulation would be deemed *willful*. In that case the employer, as mentioned above, will be subject to criminal prosecution.

Further support for this model of minimum wage compliance is provided by Holtzer *et al* [1991]. They find that the number of applicants for jobs paying the minimum wage was higher than for jobs paying either just above or just below the minimum wage. The data set they used only reports the realized wage. In the model, the whole point of offering the minimum wage is to increase the size of the pool of applicants. A prediction therefore, is that (on average) firms offering the minimum wage will have more applicants than similar firms which do not. Also, as the only reason to pay more than the minimum wage would be that only one applicant showed up, the number of applicants for jobs paying more than the minimum wage is necessarily small.

In reality, the number of applicants to the high wage jobs was not precisely one. What Holtzer *et al* [1991] did find was that those jobs were typically occupied by better qualified workers - presumably, those whose continuation values exceeded the minimum wage. Some discussion of how to adapt the environment to incorporate heterogeneity and how this might change the results is provided in Section 3 of the paper.

When firms can commit to wages, the allocation under directed search is efficient (see Moen [1997], Sattinger [1990]). If the prevailing wage were something other than the efficient wage as identified by Hosios (1990), firms could always make a market at a wage closer to it in which both workers and firms are better off. The freedom to make markets causes the externalities that exist in the matching framework to be internalized by the firms so that what is good for them is good for the economy. In the baseline model of this paper, efficiency pertains when workers get all the surplus in one-on-one matches. As mentioned above, this is precisely when the there is no strictly binding minimum wage to which firms would like to commit. An implication of this is that when workers do not have all of the bargaining power minimum wage policy can improve welfare even under voluntary adoption.

Related theoretical work is provided by Julien *et al* [2005a,b]. The first of these papers looks at what happens when wages are determined after firms and workers meet. Firms can contact at most one worker and workers auction their services among those firms that contact them. They show that this arrangement causes efficient vacancy creation. The second paper shows that more generally, any mechanism that assigns the match surplus to the contacting agent in any encounter leads to a socially optimal allocation. If the contacting agent is the worker, this means that when there is a one-on-one meeting, the worker gets all the surplus. When more than one worker applies to the same job, there is no surplus - workers get their continuation value. This coincides with the special case of the wage formation in this paper in which the worker has all the bargaining power in one-on-one meetings. Coles and Eeckhout [2003] also look at the possibility of firms choosing from a larger class of mechanisms. They find that simple wage posting as in Moen [1997] can be supported as an equilibrium outcome. The main difference between all of these papers and the baseline model presented here is that they all impose some degree of commitment to a wage formation mechanism by individuals on one side of the market. The idea here is that the bargaining protocol is simply a cultural norm.

Most of the prior work on directed search has been focussed on the theoretical development of the framework (see Rogerson *et al* [2004]). An exception to this has been Acemoglu and Shimer [1999] who show that with risk-averse workers, unemployment insurance (UI) can increase output. Essentially, they show that the investment decisions of firms is influenced by the search decisions of workers. Workers will look for more productive jobs if they are insured against long periods of unemployment. Consequently, firms create fewer but more productive jobs in the presence of a UI system than they would in the absence of UI. The net effect for moderate UI coverage is an increase in economic output.

While in principle, the argument put forward in this paper applies to any institution that permits firms to commit to paying higher wages than it otherwise would, for simplicity the analysis is restricted to the example of the minimum wage. The paper proceeds as follows. The next section lays out the baseline model with homogeneous workers and firms. The model is analyzed in 3 versions: without minimum wages, with compulsory minimum wages and with voluntary adoption of the minimum wage. Section 3 considers the robustness of the results to various sources of heterogeneity. Section 4 concludes.

2 Model

2.1 Basic Environment

The discrete time infinite horizon economy comprises a continuum of *ex ante* homogeneous infinite lived workers and firms. Workers who get jobs are replaced by new entrants to the market so that the mass of unemployed workers is fixed; normalized to 1. Both workers and firms are risk neutral and discount the future at a rate r per period. Workers experience utility from leisure at the rate b per period.

Firms can create as many atomistic vacancies as they like but have to pay an advertising cost a per period that the vacancy is held open. The mass of vacancies, v, is controlled by a zero-profit condition. If they so wish, a firm can assign a wage or range of wages to a particular vacancy. The wage so assigned becomes common knowledge to all market participants. When a firm hires a worker to a vacancy, the match produces $p > b$ units of the perfectly divisible (perishable) consumption good per period. Consumption of one unit of the good provides one unit of utility to firms or workers.

Within any time period, firms post vacancies and then workers simultaneously apply to whichever job they like but they are restricted to one application per period.[3] The main informational restriction is that, as workers apply simultaneously they do not know precisely how many others have applied for any particular vacancy. Following Burdett *et al* [2001], I assume that the number of applicants for any particular vacancy in any period is a random variable with a Poisson distribution. (This emerges as the limiting distribution of applicants as the economy grows large and workers follow a Nash equilibrium in mixed strategies.) The appropriate parameter for the Poisson distribution is q, the expected queue length or number of applicants per vacancy. If vacancies are completely indistinguishable, $q = 1/v$. Specifically, for a vacancy with expected queue length, q, the implied probability that it will receive exactly n applications is $q^n e^{-q}/n!$, for $n = 0, 1, 2...$When vacancies differ, the expected queue lengths adjust so that workers are indifferent across vacancy types supporting their propensity to randomize.

So far, the environment I have described is the large market version of the model of Burdett *et al* [2001] adapted to a labor market context (see Rogerson *et al* [2005]). The point of departure from standard directed search is

[3]This restriction can be relaxed. Albrecht *et al* [2005] study a directed search environment with multiple applications. As long as workers do not apply to every opening in every period there is a non-degerate distribution of applicants across vacancies.

that I do not assume that firms can commit to posted wages. Instead here, wage formation depends on the realized match configuration. When 2 or more workers apply to the same vacancy, the firm hires one worker, chosen at random, at a wage equal to the workers' (flow) continuation value. The workers are clearly indifferent between employment and continued unemployment at this wage and I assume the worker takes the job. One can think of this wage as emerging from (unmodelled) rounds of Bertrand competition between workers. Where meetings are one-on-one, the firm and worker use generalized Nash bargaining in which the parameter $\beta \in [0, 1]$ represents the bargaining power of the worker. In terms of the allocation, the wage that any firm might post is immaterial. Wage posting does not, therefore, feature in the description of equilibrium.[4]

For the workers, the probability that they get to bargain their wage is equal to the number of vacancies multiplied by the probability that any vacancy gets exactly one applicant divided by the number of unemployed workers:

$$qe^{-q}\left(\frac{v}{1}\right) = e^{-q}$$

Let the asset value to unemployment be V. By assumption, the value to meeting a firm with more than one applicant is V whether the worker gets the job or not. Thus,

$$rV = b + e^{-q}\left(\frac{\hat{w}}{r} - V\right) \tag{1}$$

[4]There is an issue as to whether the firm is simply committing to the Nash bargaining outcome instead of the posted wage. The view put forward here is that when there is a surplus to be divided the Nash bargaining outcome represents a cultural norm. (When there are multiple homogeneous applicants, competition means that there is nothing to bargain over.)

where, $\hat{w}$ is the bargained wage and the non-bargained wage is rV.

The asset value to holding open a vacancy, V_f, is obtained from:

$$rV_f = -a + qe^{-q}\left(\frac{p-\hat{w}}{r} - V_f\right) + \left[1 - e^{-q} - qe^{-q}\right]\left(\frac{p - rV}{r} - V_f\right) \quad (2)$$

The (flow) match surplus for one-on-one meetings is $p - rV - rV_f$. Nash bargaining leads to the workers getting a share β of this in addition to their continuation value, rV. As long as the surplus is positive

$$\hat{w} = \beta\left(p - rV_f\right) + (1-\beta)rV \quad (3)$$

If the match surplus is strictly negative there is no match.

A *zero-profit equilibrium* is a mass of vacancies, v^*, such that $q = q^* \equiv (1/v^*)$ solves (2) with $V_f = 0$ where $\hat{w}$ and V are obtained from (1) and (3).

Solving (1) and (3) for V and $\hat{w}$ indicates that for any q,

$$rV = \frac{\beta e^{-q}p + rb}{\beta e^{-q} + r}, \quad \hat{w} = \frac{\beta p(e^{-q} + r) + (1-\beta)rb}{\beta e^{-q} + r} \quad (4)$$

As $p > b$, one-on-one match surplus is always positive. This also shows that for any given value of q, $\beta = 1$ means $\hat{w} = p$ and $\beta = 0$ means $\hat{w} = rV = b$. Otherwise, $p > \hat{w} > rV > b$. Substituting for V and $\hat{w}$ into (2) and setting $V_f = 0$ yields the following implicit expression for the equilibrium queue length, q^*:

$$a = (p-b)\left[\frac{1 - e^{-q^*}\left(1 + \beta q^*\right)}{\beta e^{-q^*} + r}\right] \quad (5)$$

As q varies from 0 to ∞, the expression

$$\left[\frac{1 - e^{-q}\left(1 + \beta q\right)}{\beta e^{-q} + r}\right]$$

increases strictly monotonically from 0 to $1/r$. Existence of equilibrium therefore requires that $ra < p - b$. This is because no matter how tight the market, the firms have to incur the advertising cost for at least one period. Strict monotonicity ensures that whenever the equilibrium exists it is unique.

Clearly, an increase in a or b or a decrease in p causes the equilibrium queue length to increase as firms produce less vacancies. The parameter r here is inversely related to the "thickness" of the market. In thicker markets, the meeting rate is higher so that the extent of discounting between possible meetings is lower. As firms expect to fill their openings more quickly, vacancies become effectively cheaper to create which leads to a decrease in the expected number of workers per vacancy.

2.2 Minimum wage with full enforcement

Let the value to unemployment when all firms comply with a minimum wage, $\bar{w}$, be $\bar{V}$. The minimum wage binds when it exceeds the workers' flow continuation value, $r\bar{V}$. When it does not bind, the market is identical to that without a minimum wage and $\bar{V} = V$ as derived above. The analysis therefore only considers the case in which $\bar{w} > rV$.

There is some question as to how Nash bargaining should be applied in this circumstance. As long as the workers threatpoint is $r\bar{V}$, the Independence of Irrelevant Alternatives axiom means that while $\bar{w}$ lies between $r\bar{V}$ and $\hat{w}$ the minimum wage will not directly influence the outcome of the bargaining.[5] The question is really whether the worker's threatpoint should be $r\bar{V}$ or $\bar{w}$. A minimum wage paying firm has no obligation to hire a worker. Rather, the obligation is that if the worker is hired, the wage to be paid must be at least $\bar{w}$. Because of this, the relevant threatpoint should be $r\bar{V}$ as this is all the worker can base his negotiations on. The worker cannot demand

[5] See Osborne and Rubinstein [1990] for a complete exposition of Nash bargaining.

that as a last resort he be hired at $\bar{w}$. Consequently, if $\bar{V}_f$ represents the value to holding open a minimum wage vacancy and $\bar{w} < \beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}$, then $\hat{w} = \beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}$. It is possible, however, that the minimum wage is so high that $\bar{w} > \beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}$. In this case, the minimum wage becomes a relevant alternative as the parties cannot agree (by law) to match at a wage below $\bar{w}$. In general, we have

$$\hat{w} = \max\left\{\beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}, \bar{w}\right\} \tag{6}$$

When $\hat{w} = \bar{w}$, the minimum wage is *completely* binding otherwise it is termed *partially* binding.[6] The circumstances under which the minimum wage can bind completely are discussed below.

Given an expected queue length, $\bar{q}$, the number of firms who end up with 2 or more applicants in a given time period is

$$\left(1 - e^{-\bar{q}} - \bar{q}e^{-\bar{q}}\right)\bar{v}$$

where $\bar{v}$ is the mass of vacancies. As $\bar{v} = 1/\bar{q}$, and the number of workers hired by firms with more than one applicant equals the number of firms that get more than one applicant, we have

$$r\bar{V} = b + e^{-\bar{q}}\left(\frac{\hat{w} - r\bar{V}}{r}\right) + \left(\frac{1 - (1+\bar{q})e^{-\bar{q}}}{\bar{q}}\right)\left(\frac{\bar{w} - r\bar{V}}{r}\right) \tag{7}$$

For firms,

$$r\bar{V}_f = -a + \bar{q}e^{-\bar{q}}\left(\frac{p - \hat{w} - r\bar{V}_f}{r}\right) + \left[1 - (1+\bar{q})e^{-\bar{q}}\right]\left(\frac{p - \bar{w} - r\bar{V}_f}{r}\right) \tag{8}$$

A zero-profit equilibrium here is a mass of vacancies, $\bar{v}^*$, such that $\bar{q} = \bar{q}^* \equiv 1/\bar{v}^*$ solves (8) with $\bar{V}_f = 0$.

[6] Notice that had I used $\bar{w}$ as the worker's threatpoint during negotiations, the minimum wage would never completely bind and the environment would be analytically simpler.

Two types of equilibria are possible: equilibria with a partially binding minimum wage and equilibria with a completely binding minimum wage. Straightforward algebra reveals that for partially binding minimum wages, $\bar{q}^*$ solves

$$ra = \frac{(p-b)(1-\beta)r\bar{q}^2 e^{-\bar{q}} + (p-\bar{w})\left[1-(1+\bar{q})e^{-\bar{q}}\right]\left[r\bar{q}+1-e^{-\bar{q}}\right]}{\bar{q}(\beta e^{-\bar{q}}+r)+1-(1+\bar{q})e^{-\bar{q}}} \tag{9}$$

For completely binding minimum wages, $\bar{q}^*$ solves

$$ra = (p-\bar{w})\left(1-e^{-\bar{q}}\right). \tag{10}$$

Neither of these equilibria can exist when $p - \bar{w} < ra$. Straight forward algebra shows that multiple equilibria of either type cannot occur.

Uniqueness of either equilibrium, however, does not rule out coexistence. Imposing

$$\beta p + (1-\beta)r\bar{V} = \bar{w} \tag{11}$$

along with the set of equations that characterize equilibrium with partially binding minimum wage yields the upper bound on the values of $\bar{w}$ for which that type of equilibrium exists. Imposing the same equality on the set of equations that characterize equilibrium with completely binding minium wages will similarly yield the lower bound on the set of values of $\bar{w}$ for which that equilibrium type exists. Continuity of $\hat{w}$ in $\bar{w}$ (from equation (6)) ensures that the resulting critical values for the existence of either type of equilibrium are the same. These equilibrium types, therefore, do not coexist which means that equilibrium is unique.

To ascertain which equilibrium type is relevant for any given parameter configuration, let w_T be the threshold value of the minimum wage which just completely binds. That is $w_T = \bar{w}$ such that equations (7), (10), and (11) all hold. Eliminating $r\bar{V}$ yields

$$\bar{w} = \frac{\beta p \left[r\bar{q} + 1 - e^{-\bar{q}}\right] + (1-\beta) r\bar{q}b}{\left[r\bar{q} + \beta\left(1 - e^{-\bar{q}}\right)\right]} \tag{12}$$

$$ra = (p - \bar{w})\left(1 - e^{-\bar{q}}\right) \tag{13}$$

As $\bar{q}$ increases, equation (12) generates a monotonically decreasing value of $\bar{w}$ which approaches $\bar{w} = \beta p + (1-\beta)b$ as $\bar{q}$ gets large. Meanwhile, equation (13) generates a monotonically increasing value of $\bar{w}$ which approaches $p - ra$ as $\bar{q}$ gets large. So, as long as

$$\beta p + (1-\beta)b < p - ra \qquad \text{or equivalently} \qquad p - b > \frac{ra}{(1-\beta)} \tag{14}$$

w_T exists and it is unique. A minimum wage larger than w_T, that is consistent with equilibrium (i.e. $p - \bar{w} > ra$), will be fully binding. A minimum wage below w_T will only partially bind.

When condition (14) does not hold, there is no value of the minimum wage such that it completely binds in equilibrium. Thus, given $p - b > ra$, for sufficiently large values of β only partially binding equilibria are possible. When $\beta = 0$, $w_T = b$ and a minimum wage that binds at all binds completely. Beyond that, it is straightforward to show that, while it continues to exist, w_T strictly increases with β. Moreover, from the definition of $\hat{w}$, it should be clear that for $\beta > 0$, $w_T > rV$ so there is always some range of minimum wages which will only partially bind.

2.3 Voluntary adoption of the minimum wage

The issue considered here is whether a firm might prefer to adopt the minimum wage if the associated legal framework imbues sufficient credibility. It is therefore assumed that when a firm declares itself a minimum wage payer, violation of the law is considered willful. And, the penalty for willful violation is sufficiently punitive that no firm adopting the minimum wage will ever violate the law. Firms can, however, completely ignore the law. The penalty for underpayment in that case is assumed to be insignificant. Throughout this analysis, the value of the minimum wage, $\bar{w}$, remains exogenous to the firms. Firms simply choose whether to adopt the minimum wage or not.

Let ϕ represent the propensity with which an individual firm adopts the minimum wage. If Φ represents the propensity with which all other firms adopt the minimum wage, an equilibrium in this extended environment is a $\phi^* \in \{0, 1\}$ such that $\phi^* = \Phi$ is each individual firm's optimal adoption choice. Equilibrium is therefore restricted to pure strategy, symmetric Nash. (The possibility of firms using mixed strategy equilibria is considered below.) Under this restriction, two types of equilibrium are possible, $\phi^* = 0$ and $\phi^* = 1$. Clearly, the values to being in equilibrium with $\phi^* = 0$ are precisely those that pertain in the equilibrium in the basic environment described above. Similarly, the values to being in equilibrium with $\phi^* = 1$ are precisely those that pertain in equilibrium when the firms are fully compliant. The issue here, then, is for what values of $\bar{w}$ is either outcome described in the preceding subsections an equilibrium of the extended environment with optional compliance?

Let $\tilde{V}_f$ be the value to creating a minimum wage vacancy ($\phi = 1$) when all other vacancies are non-minimum wage ($\Phi = 0$). Then,

$$r\tilde{V}_f = -a + \tilde{q}e^{-\tilde{q}} \left(\frac{p - \tilde{w}}{r} - \tilde{V}_f \right) + \left(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}} \right) \left(\frac{p - \bar{w}}{r} - \tilde{V}_f \right) \quad (15)$$

where

$$\tilde{w} = \max \left\{ \beta \left(p - r\tilde{V}_f \right) + (1 - \beta) rV, \bar{w} \right\} \quad (16)$$

and $\tilde{q}$ is the expected number of applicants at the firm offering the minimum wage, $\bar{w}$. As workers are fully aware of the characteristics of all vacancies, they apply to the deviant firm in such numbers that makes them indifferent between applying to the minimum wage vacancy and all the other vacancies. The value of $\tilde{q}$ is therefore obtained from

$$rV = b + e^{-\tilde{q}} \left(\frac{\tilde{w} - rV}{r} \right) + \left(\frac{1 - (1 + \tilde{q})e^{-\tilde{q}}}{\tilde{q}} \right) \left(\frac{\bar{w} - rV}{r} \right) \tag{17}$$

where V has the same value that emerged in the basic model without minimum wages. Noncompliance, $\phi = \Phi = 0$, is an equilibrium if and only if $\tilde{V}_f \leq V_f = 0$.

For full-compliance $\phi = \Phi = 1$ to be an equilibrium, firms should not prefer deviation to noncompliance. Let $\tilde{\tilde{V}}_f$ be the value to noncompliance ($\phi = 0$) when all other vacancies comply ($\Phi = 1$) with the minimum wage, $\bar{w}$. Then,

$$r\tilde{\tilde{V}}_f = -a + \tilde{\tilde{q}} e^{-\tilde{\tilde{q}}} \left(\frac{p - \tilde{\tilde{w}}}{r} - \tilde{\tilde{V}}_f \right) + \left[1 - e^{-\tilde{\tilde{q}}} - q e^{-\tilde{\tilde{q}}} \right] \left(\frac{p - r\bar{V}}{r} - \tilde{\tilde{V}}_f \right) \tag{18}$$

where

$$\tilde{\tilde{w}} = \beta \left(p - r\tilde{\tilde{V}}_f \right) + (1 - \beta) r\bar{V} \tag{19}$$

is calculated using $r\bar{V}$ as the worker's threat point and $\tilde{\tilde{q}}$ is the expected number of applicants at the noncompliant firm. Workers apply to the deviant firm in such numbers that they are indifferent across all vacancies. The value of $\tilde{\tilde{q}}$ is therefore obtained from

$$r\bar{V} = b + e^{-\tilde{\tilde{q}}} \left(\frac{\tilde{\tilde{w}} - r\bar{V}}{r} \right) \tag{20}$$

Compliance, $\phi = \Phi = 1$, is an equilibrium if and only if $\tilde{\tilde{V}}_f \leq \bar{V}_f = 0$.

Claim 1 *Under the parameter restrictions required for existence of equilibria in the basic environment and under minimum wage with full enforcement,* *either $\phi^* = 0$ or $\phi^* = 1$ type equilibria exist under voluntary compliance. These equilibrium types do not generically coexist.*

Proof. The boundary to the set of parameter values for which $\phi^* = 0$ is an equilibrium is defined by $\tilde{V}_f = V_f = 0$. The boundary to the set of parameter values for which $\phi^* = 1$ is an equilibrium is defined by $\tilde{\bar{V}}_f = \bar{V}_f = 0$. After substituting these values into the appropriate equations above (respectively (1), (2), (3), (15), (16), (17) and (7), (8), (9), (18), (19), (20)), simple inspection reveals that the two boundaries are identical. On the common boundary these equilibria coexist. Smoothness of the functional forms ensures that the boundary is non-generic (zero measure) in the permissible parameter space. ■

In particular this means that a minimum wage that any firms voluntarily adopt will be adopted by all firms. Furthermore, any value of the minimum wage at which firms are indifferent between adoption and non-adoption, the proportion of firms choosing to adopt does not affect the workers' continuation value, i.e. $V = \bar{V}$. This is because for firms to be indifferent between adoption and non-adoption $V_f = \bar{V}_f = 0$ and the associated queue length from offering the minimum wage when every other firm does, is the same as when no other firm offers it.

Claim 1 also explains why possible mixed strategy equilibria were ignored. By definition, in any mixed strategy equilibrium firms have to be indifferent between adoption and non-adoption of the minimum wage. This means that mixed strategy equilibria only occur at those critical parameter values for which both pure strategy equilibria also exist. At those parameter values there is a continuum of mixed strategy equilibria indexed by the proportion of firms adopting the minimum wage.

The foregoing does not prove that the $\phi^* = 1$ type equilibrium ever exists. Claim 2 addresses this question.

Claim 2 *For every $\beta < 1$, there exists a minimum wage, $\bar{w}$, sufficiently close to rV such that the unique equilibrium under voluntary adoption is $\phi^* = 1$*

Proof. From Claim 1, we simply have to show that for low enough $\bar{w}$, $\phi^* = 0$ is not an equilibrium when $\beta < 1$. This requires that individual firms would find it profitable to adopt the minimum wage when all other firms do not. A deviant firm will adopt some $\bar{w} > rV$ as long as

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} > 0.$$

Restricting attention to partially binding minimum wages, substituting for $\tilde{w}$ from (16) into (15) and (17) yields the following pair of equations in $\tilde{V}_f$ and $\tilde{q}$.

$$G^1(\tilde{V}_f, \tilde{q}; \bar{w}) \equiv r^2\tilde{V}_f + ra - \tilde{q}e^{-\tilde{q}}(1-\beta)\left(p - rV - r\tilde{V}_f\right) - (1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})\left(p - \bar{w} - r\tilde{V}_f\right) = 0$$

$$G^2(\tilde{V}_f, \tilde{q}; \bar{w}) \equiv r^2\tilde{q}V - r\tilde{q}b - \tilde{q}e^{-\tilde{q}}\beta\left(p - r\tilde{V}_f - rV\right) - (1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})(\bar{w} - rV) = 0$$

Using implicit differentiation and Cramer's rule,

$$\frac{d\tilde{V}_f}{d\bar{w}} = \frac{-\begin{vmatrix} G_3^1 & G_2^1 \\ G_3^2 & G_2^2 \end{vmatrix}}{\begin{vmatrix} G_1^1 & G_2^1 \\ G_1^2 & G_2^2 \end{vmatrix}}$$

where G^i_j represents the partial derivative of the ith component of G with respect to the jth argument. Obtaining each of the partial derivatives is straightforward. Once they have been obtained, we can impose $\bar{w} = rV$, which also means $\tilde{V}_f = V_f = 0$, and $\tilde{q} = q^*$. After substituting for rV from (4),

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{(1-\beta)(1-e^{-q^*}-q^*e^{-q^*})}{r\beta q^*(r+1-\beta e^{-q^*})}$$

which is strictly positive while $\beta < 1$ and q^* is finite.

As the preceding analysis was carried out for partially binding minimum wages, it is only valid for $\beta > 0$.When $\beta = 0$, $\hat{w} = rV = b$ and a minimum wage that binds at all binds completely. In that case substituting for $\tilde{w} = \bar{w}$ into (15) and (17) yields

$$\begin{aligned} r\tilde{V}_f &= -a + \frac{(1-e^{-\tilde{q}})}{r}\left(p - \bar{w} - r\tilde{V}_f\right) \\ rV &= b + \left(\frac{1-e^{-\tilde{q}}}{\tilde{q}}\right)\left(\frac{\bar{w}-rV}{r}\right) \end{aligned}$$

which imply

$$r\left(r+1-e^{-\tilde{q}}\right)\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} = e^{-\tilde{q}}\left(p-\bar{w}\right)\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} - \left(1-e^{-\tilde{q}}\right)$$

where

$$\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{1-e^{-\tilde{q}}}{rV-b}$$

As $rV = b$, deviation to a minimum wage that just binds generates an unbounded queue length of applicants and any firm would choose to adopt the minimum wage. ■

The gist of the proof is that, if the value to posting the minimum wage when no one else does is increasing at the point where it just begins to bind, then for some range of values of $\bar{w}$ sufficiently close to rV, $\phi^* = 0$ cannot be an equilibrium. From Claim 1, this implies that over that range, the equilibrium is of type $\phi^* = 1$.

The intuition is clearest in the $\beta = 0$ case. Because workers get at least b whether they apply to the minimum wage job or not, every unemployed worker might as well apply. By offering the minimum wage, the deviant firm will fill its job with probability 1 while incurring an infinitesimal increase in the wage. When $\beta > 0$, minimum wage jobs will still attract more workers but to a lesser extent than occurs under $\beta = 0$. This is because, while $\hat{w} > \bar{w}$ workers experience some opportunity cost from applying to minimum wage jobs. They have to trade off the improved outcome when there are multiple applicants with the reduced probability of getting to negotiate their wage. While β is small, $\hat{w}$ is close to rV and the former effect dominates so that queue length increases rapidly with $\bar{w}$. The impact of adoption on the expected queue length continues to make adoption of low enough but binding minimum wages worthwhile to firms as long as $\beta < 1$. Ultimately when $\beta = 1$, the deviant prefers not to implement any binding minimum wage. Here, the increased probability of multiple applicants exactly offsets the increased cost of the wage bill.

2.4 Welfare

As both workers and firms are risk neutral, welfare in the model amounts to output minus costs. However, in order to focus on match formation, there are no separations in this model which means that total output is always growing. Welfare comparisons amount to comparing different economies and the welfare measure is the value to being born in any economy.

Regardless of the method of wage formation or the implied distribution, free-entry of vacancies means that any payments firms receive simply compensate them for vacancy creation costs. Welfare is therefore the utility contribution of a birth minus the associated vacancy cost.

If V_b is the present value of utility generated by a birth then

$$rV_b = b + \frac{(1 - e^{-q})}{q}\left(\frac{p}{r} - V_b\right)$$

For each worker there are $v = 1/q$ vacancies which cost a/q to maintain over the expected duration of the workers job search period. If C_b represents the present value of the costs then

$$rC_b = \frac{a}{q} - \frac{(1 - e^{-q})}{q} C_b$$

the flow value of welfare is then $W \equiv rV_b + rC_b$. That is

$$W = \frac{(1 - e^{-q})(p - b) - ra}{rq + 1 - e^{-q}} + b$$

It is simple to show that in the absence of a minimum wage, $W = rV$.[7] In the minimum wage with full enforcement, $W = r\bar{V}$.[8] Because of free entry, the firms wash out of the welfare calculation.

The first order condition from maximization of W with respect to q implies that any first best queue length, q_p solves

$$(p - b)\left[1 - e^{-q} - qe^{-q}\right] = a(r + e^{-q}) \tag{21}$$

As $W''(q_p)$ is negative, $W(.)$ is quasi-concave meaning that the unique solution to (21) is a global optimum. Under the maintained assumption that $p - b > ra$ (required for existence of equilibrium), q_p always exists.

Comparison of equations (21) and (5). shows that if $\beta = 1$, then $q_p = q^*$. Of course, β is a deep parameter of the model and not directly controlled

[7] Set $V_f = 0$ in equation (2) and add it to equation (1)

[8] Set $V_f = 0$ in equation (8) and add it to equation (7)

by policy. A second question, therefore, is whether the minimum wage can achieve optimality for a given value of β. This amounts to asking whether minimum wage policy can implement $\bar{q} = q_p$. Recall that from equations (10) and (9) $\bar{q}$ is continuous and strictly increasing in $\bar{w}$. When $\bar{w} = rV$ $\bar{q} = q^* < q_p$. As $\bar{w}$ approaches $p - ra$, $\bar{q}$ approaches infinity. So, there exists a unique value of $\bar{w}$ at which $\bar{q} = q_p$.

Of greater interest for the purpose of this paper is whether firms will voluntarily adopt a Pareto improving minimum wage.

Lemma 3 *Full enforcement of a minimum wage that just binds is welfare improving*

Proof. *We need to show that*

$$\left.\frac{dW}{d\bar{w}}\right|_{\bar{w}=rV} = \left.\frac{dW}{d\bar{q}}\right|_{\bar{w}=rV} \frac{d\bar{q}}{d\bar{w}} > 0$$

That $\frac{d\bar{q}}{d\bar{w}} > 0$ *for all* $\bar{w}$ *was established earlier. The sign of* $\frac{dW}{dq}$ *is the same as that of* Ω *where*

$$\Omega \equiv a(r + e^{-q}) - (p - b)\left[1 - e^{-q} - qe^{-q}\right]$$

Evaluation of Ω *at* $q = q^*$ *implies*

$$\Omega = (1 - \beta)e^{-q^*}\left[1 - e^{-q^*} - qe^{-q^*}\right] > 0$$

as long as $\beta < 1$ ■

As Claim 2 establishes that minimum wages sufficiently close to rV will voluntarily be adopted it follows from Lemma 1 that voluntarily adopted minimum wages can lead to efficiency gains.

3 Heterogeneity

The preceding analysis provides an example of how firms might adopt the minimum wage as a commitment to not paying extremely low wages. This section considers how the model could be extended to incorporate various sources of heterogeneity in order to examine the extent to which this idea can be generalized. Introducing heterogeneity of any form in such models vastly complicates the analysis. Considered here are: *ex ante* differences across firms, *ex ante* differences across workers (either in productivity or value of leisure) and *ex post* (i.e. match specific) heterogeneity. Whatever the source, heterogeneity raises further questions about who knows what and when. And, for each set of assumptions as to the nature of private information, there may be many ways of modelling the determination of the terms-of-trade that are consistent with the model described above. Consequently, it goes beyond the scope of this paper to provide a complete analysis of each alternative.

3.1 Ex ante heterogeneity across firms

A drawback of the model described so far is that either all firms adopt the minimum wage or none of them do. One way to address this clearly counter-factual outcome is the introduction of vacancies for jobs that incorporate different technologies. In that way, the same worker may produce different amounts of output in different jobs. The simplest way to model this is to incorporate an initial job-creation cost and for the productivity to be realized only after the cost has been incurred. This would be the same approach used by Moen [1997]. In his model, firms could commit to posted wages so whether the worker knew the true productivity of the firm was not an issue. Here, because the terms of trade are determined after the workers and firms meet, assumptions as to what the worker (or even the firm) know about the productivity of the job have to be made.

The simplest way forward is to assume that the productivity of the job is common knowledge and is used by workers as a basis for directing their search. In equilibrium the expected queue length at each vacancy type will adjust so that workers are indifferent between searching all active vacancy types.[9] In the absence of a minimum wage, one-on-one matches result in a bargained wage which will reflect the productivity of the job. If multiple applicants show up, workers get pushed down to their common outside option.

In this model the firms' choices are essentially the same as before. We know from the previous analysis that firms will voluntarily adopt a binding minimum wage as long as it is not too high. Of course, "too high" is relative to the productivity of the job. Sufficiently high productivity jobs will adopt the minimum wage while low productivity ones will not.

Perhaps more interesting, but left for future work, is the possibility that firm productivity is not observable (at least prior to the meeting). Firms in that case might offer the minimum wage as a signal of their productivity. Then, even low productivity firms may have to offer the minimum wage in order to attract enough workers to make the job viable.

[9] There is an implicit assumption here (as used by Moen [1997]) that vacancies with sufficiently low productivity that they will never match, can be freely disposed of.

3.2 Ex ante heterogeneity across workers

Holtzer et al [1991] provide evidence on the number of applicants for jobs that hire workers at wages close to the minimum wage. They find that:[10]

(i) queue lengths are longer for jobs paying the minimum wage than those paying just below the minimum wage

(ii) queue lengths are longer for jobs paying the minimum wage than those paying just above the minimum wage

(iii) those jobs that pay more than the minimum wage have longer queues than those paying below.

The baseline model of this paper is consistent with the first and second observations. The first follows because workers have to be indifferent between applying to minimum and non-minimum wage firms. The second occurs because workers only get more than the minimum wage when they are the only worker to show up at that particular firm. The third observation is problematic. Firms that could commit to paying higher than minimum wages would (on average) get more applicants than even the minimum wage firms. But without the ability to commit, they will only pay high wages if there is only one applicant.

A clue as to how the model can be reconciled with the third observation is provided by Holtzer et al [1991] when they examine the nature of the workers. They find that,

> ...workers hired to minimum wages jobs are on average less educated, younger, less experienced and more likely to be female than workers who are hired into low-paying jobs that pay

[10] The data they use comes from the Employment Opportunity Pilot Project Survey. This survey reports realized wages only. (There are no data collected on what the firm expected to pay.)

more than the minimum wage, while workers with starting wages less then the minimum wage have similar personal characteristics and training to workers whose starting wage equals the minimum wage.

Incorporating workers that have outside options that exceed the minimum wage because, say, they have higher expected productivity as evidenced by their qualifications could clearly lead to the kind of outcome required here.

Actually extending the model to include multiple worker types involves some non-trivial modelling choices. Again the simplest informational arrangement is that worker productivity is common knowledge.[11] Even in that case, wage formation requires further assumptions. What should be clear is that with complete information matching should be efficient. Whenever a higher productivity worker and a lower productivity worker apply for the same job, the high productivity worker will get hired. Also, regardless of how wages are determined, higher productivity workers will attract higher wages than their lower productivity counterparts. These facts combined will mean that high productivity workers have higher outside options. If the minimum wage is chosen so that it partially binds for low productivity workers but does not bind at all for high productivity workers, firms potentially paying below the minimum wage can have shorter expected queue lengths than those who pay more. Even still, in this model with heterogeneous workers and homogeneous

[11]If worker productivity is private information the high productivity workers would need some way of separating themselves from the low productivity workers. If this is not possible, the outside options of the workers would be the same and any binding minimum wage would bind on everyone. But, as Holtzer *et al* [1991] were able to identify the workers as coming from different expected productivity groups, it seems reasonable to suppose that prospective employers can too.

firms, firms offering the minimum wage would not coexist with those who do not offer it.

For all the facts identified by Holtzer *et al* [1991] to be realized in the same equilibrium requires both worker and firm heterogeneity. With assortative matching, as found in the data, the market is essentially segmented and the queue lengths at high paid jobs could well be lower than those for low paid jobs.

3.3 Match-specific heterogeneity

The model of the previous section provides a simple example of how firms can benefit from minimum wage adoption. The basic idea is that as long as firms are able to commit to the minimum-wage, the implied improvement in application rate by workers makes adoption worthwhile. The increased queue length increases the possibility for the firms of filling the vacancy more quickly and at a lower wage. Another possible benefit from a higher application rate is a better match. This sub-section investigates this possibility by incorporating match-specific heterogeneity.[12]

I assume that any encounter between a worker and a firm generates a draw of the match productivity, p, from a continuous distribution F with support between $\underline{p}$ and $\bar{p}$. If the variation across matches is attributed to subjective assessments by the firm as to how the worker would fit within the organization, then the realized value of p should be private information to the firm.[13]

[12] Moen [2003] provides a model of directed search with match-specific heterogeneity. In his model, however, firms always meet with a continuum of workers so that the size of the applicant pool does not affect the realized match productivity.

[13] The heterogeneity could also emerge from non-pecuniary aspects of the job over which

There are many wage formation mechanisms that are consistent with the homogeneous worker/firm model. For concreteness, one example is analyzed here. To explore the extent to which this extension provides an additional incentive for firms to voluntarily adopt the minimum wage, I will focus on the case where workers have all the bargaining power in one-on-one matches. That is, they get to make a take-it-or-leave-it wage offer to the firm. When the realized queue length at any vacancy exceeds one, I assume the firm gets to hire the most productive worker at the workers' common outside-option value.[14]

Let $G(.|q)$ represent the distribution function of the highest productivity among the workers conditional on 2 or more of them showing up. It is helpful to derive G and some of its properties before continuing with the general analysis of this example. For a given realized queue length, n, the probability that every realized productivity is below p is $F^n(p)$. For given q, n has a Poisson distribution so that contingent on $n \geq 2$, the probability that every realized productivity is below p is

$$G(p|q) = \sum_{n=2}^{\infty} \frac{q^n e^{-q} F^n(p)}{n!(1 - e^{-q} - qe^{-q})}$$

Clearly, as $F^n(p) < F(p)$, $G(p|q) < F(p)$ for all q.

The second important property of $G(.|.)$ is that of first-order stochastic

workers have preferences. In that case, the natural assumption is that the worker has the private information. Such an arrangement, with random matching and wage posting, is considered in Masters [1998].

[14]Perhaps a more internally consistent model of wage formation would be to have the workers make take-it-or-leave-it offers which depend only on the number of other workers in the realized queue. In that case, however, the wage distribution is difficult to characterize making the effect of a minimum wage hard to assess.

dominance with respect to q. That is

$$\frac{\partial G(p|q)}{\partial q} < 0. \text{ for } p \in (0,1)$$

To see why this is true, notice that we can also write

$$G(p|q) = \frac{e^{qF(p)} - 1 - qF(p)}{e^q - 1 - q}$$

so that the sign of $\partial G(p|q)/\partial q$, after suppressing the argument in F is the same as the sign of

$$\Gamma(q,F) \equiv \frac{F(e^{qF} - 1)}{e^{qF} - 1 - qF} - \frac{e^q - 1}{e^q - 1 - q}$$

As $\Gamma(q,1) = 0$, if for $F < 1$, $\partial\Gamma(q,F)/\partial F > 0$,then $\Gamma(q,F) < 0$. Now,

$$\frac{\partial\Gamma(q,F)}{\partial F} = \frac{(e^{qF} - 1)^2 - q^2F^2e^{qF}}{(e^{qF} - 1 - qF)^2} = \frac{(e^{qF} - 1 + qFe^{\frac{qF}{2}})(e^{qF} - 1 - qFe^{\frac{qF}{2}})}{(e^{qF} - 1 - qF)^2}$$

the sign of which is the same as the sign of

$$\Phi(q,F) \equiv (e^{qF} - 1 - qFe^{\frac{qF}{2}})$$

Clearly, $\lim_{F\to 0}\Phi(q,F) = 0$ for all q and for $F > 0$,

$$\frac{\partial\Phi}{\partial F} = qe^{\frac{qF}{2}}\left(e^{\frac{qF}{2}} - 1 - \frac{qF}{2}\right) > 0$$

So, $\Phi(q,F) > 0$ for $F > 0$ which means for $F < 1$, $\partial\Gamma(q,F)/\partial F > 0$ and $\Gamma(q,F) < 0$.

With the essential properties of G established, I move to the analysis of the model. First, consider the workers' choice. They have to pick a wage offer to make in the case of a single match but are otherwise indifferent between remaining unemployed and getting a job when there are multiple applicants.

If $V(w)$ is the present discounted expected value to offering wage w, the relevant asset value equation is

$$rV(w) = b + \frac{e^{-q}}{r}(1 - F(w))(w - rV) \tag{22}$$

where

$$V \equiv \max_{w} V(w)$$

Let $\hat{w}$ indicate the wage in single matches (when they occur). Because of the recursive nature of equation (22) it should be clear that workers will always choose $\hat{w} > rV$. As $F(.)$ has finite support, $\hat{w}$ has to exist. This is all that matters for this exercise.

For a given expected queue length q, the value to holding a vacancy, V_f is now

$$rV_f = -a + \frac{qe^{-q}}{r}\int_{\hat{w}}^{\bar{p}}(y - \hat{w} - rV_f)dF(y) + \frac{(1 - e^{-q} - qe^{-q})}{r}\int_{rV}^{\bar{p}}(y - rV - rV_f)dG(y|q) \tag{23}$$

In this model a *free-entry steady-state equilibrium* is a tuple, $\{V_f, V, q, \hat{w}\}$ such that q solves (23) with $V_f = 0$, and $V = V(\hat{w})$. A necessary condition for equilibrium is that $ra < \bar{p} - b$. This is because firms need to be assured of covering their up-front advertising cost, a, even when the number of applicants is expected to be unbounded. In the absence of restrictions on F, multiple equilibria cannot be ruled out.[15]

For any equilibrium consider the value, $\tilde{V}_f$, to an individual firm of a one-time option to offer a minimum wage which exceeds rV and to which the firm can commit. As $V_f = 0$,

$$r\tilde{V}_f = -a + \frac{\tilde{q}e^{-\tilde{q}}}{r}\int_{\hat{w}}^{\bar{p}}(y - \hat{w})dF(y) + \frac{(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})}{r}\int_{\bar{w}}^{\bar{p}}(1 - G(y|\tilde{q}))dy \tag{24}$$

[15] A sufficient condition for $V(w)$ to be concave is that F has a non-decreasing hazard, $f(y)/(1 - F(y))$. Concavity of $V(w)$ ensures uniqueness of equilibrium.

where $\tilde{q}$, is the expected queue length associated with offering the minimum wage.[16] Workers will adjust their search behavior so as to be indifferent between applying to the firm offering the minimum wage and all other firms so that $\tilde{q}$ is obtained from

$$rV = b + \frac{e^{-\tilde{q}}}{r}(1 - F(\hat{w}))(\hat{w} - rV) + \frac{(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})}{r\tilde{q}}(1 - G(\bar{w}|\tilde{q}))(\bar{w} - rV) \quad (25)$$

The continuation value of the worker and the firm are not affected by this option so, as long as $\hat{w} > \bar{w}$, neither is $\hat{w}$.

Following the analysis of the basic model, we can now ask when firms would voluntarily adopt the minimum wage by evaluating

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} = \left.\left(\frac{\partial \tilde{V}_f}{\partial \bar{w}} + \frac{\partial \tilde{V}_f}{\partial \tilde{q}}\frac{d\tilde{q}}{d\bar{w}}\right)\right|_{\bar{w}=rV} \quad (26)$$

From (24)

$$r^2 \left.\frac{\partial \tilde{V}_f}{\partial \bar{w}}\right|_{\bar{w}=rV} = -(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})(1 - G(\bar{w}|\tilde{q}))$$

and

$$\begin{aligned} r^2 \left.\frac{\partial \tilde{V}_f}{\partial \tilde{q}}\right|_{\bar{w}=rV} &= (1 - \tilde{q})e^{-\tilde{q}} \int_{\hat{w}}^{\bar{p}} (y - \hat{w})dF(y) + \tilde{q}e^{-\tilde{q}} \int_{\bar{w}}^{\bar{p}} (y - \bar{w})dG(y|\tilde{q}) \\ &\quad -(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}}) \int_{\bar{w}}^{\bar{p}} \frac{\partial G(y|\tilde{q})}{\partial \tilde{q}} dy \end{aligned}$$

From (25),

$$\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})(1 - G(\bar{w}|\tilde{q}))}{r^2V - rb - (1 - \tilde{q})e^{-\tilde{q}}(1 - F(\hat{w}))(\hat{w} - rV)}$$

[16] The last integral is obtained by integration by parts from

$$\int_{\bar{w}}^{\bar{p}} (y - \bar{w})dG(y|\tilde{q})$$

and using (22) to substitute for $r^2V - rb$,

$$\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{(1-e^{-\tilde{q}}-\tilde{q}e^{-\tilde{q}})(1-G(\bar{w}|\tilde{q}))}{\tilde{q}e^{-\tilde{q}}(1-F(\hat{w}))(\hat{w}-rV)}$$

which is positive. Substituting back into (26) yields

$$\begin{aligned}\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} &= \left[\frac{(1-e^{-\tilde{q}}-\tilde{q}e^{-\tilde{q}})(1-G(\bar{w}|\tilde{q}))}{\tilde{q}e^{-\tilde{q}}(1-F(\hat{w}))(\hat{w}-rV)}\right]\times \\ &\left\{\begin{array}{l} -\tilde{q}e^{-\tilde{q}}(1-F(\hat{w}))(\hat{w}-rV)+(1-\tilde{q})e^{-\tilde{q}}\int_{\hat{w}}^{\bar{p}}(y-\hat{w})dF(y) \\ +\tilde{q}e^{-\tilde{q}}\int_{\bar{w}}^{\bar{p}}(y-\bar{w})dG(y|\tilde{q})-(1-e^{-\tilde{q}}-\tilde{q}e^{-\tilde{q}})\int_{\bar{w}}^{\bar{p}}\frac{\partial G(y|\tilde{q})}{\partial\tilde{q}}dy \end{array}\right\}\end{aligned}$$

The sign of $\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV}$ clearly depends on the sign of the contents of the curly brackets. The last term is positive from because $\frac{\partial G(y|\tilde{q})}{\partial\tilde{q}} < 0$ as established above. The first 3 terms can be written as

$$\begin{aligned}&\tilde{q}e^{-\tilde{q}}\left[\int_{\bar{w}}^{\bar{p}}(y-\bar{w})dG(y|\tilde{q})-(1-F(\hat{w}))(\hat{w}-\bar{w})-\int_{\hat{w}}^{\bar{p}}(y-\hat{w})dF(y)\right] \\ &+e^{-\tilde{q}}\int_{\hat{w}}^{\bar{p}}(y-\hat{w})dF(y)\end{aligned}$$

in which the contents of the square brackets can be written as

$$\begin{aligned}&\int_{\bar{w}}^{\bar{p}}(y-\bar{w})dG(y|\tilde{q})-\int_{\hat{w}}^{\bar{p}}(y-\bar{w})dF(y) \\ > \ &\int_{\bar{w}}^{\bar{p}}(y-\bar{w})dG(y|\tilde{q})-\int_{\bar{w}}^{\bar{p}}(y-\bar{w})dF(y) \\ = \ &\int_{\bar{w}}^{\bar{p}}(F(y)-G(y|\tilde{q}))dy > 0.\end{aligned}$$

the last line comes from integration by parts. The upshot is that

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} > 0. \tag{27}$$

That is, for low enough values of the minimum wage, universal non-adoption of the minimum wage cannot be an equilibrium.

The foregoing implies that some firms will adopt a binding minimum wage sufficiently close to rV. Following analysis similar to that of Section 2, it is straightforward to show that when the adoption choice is fully incorporated into this version of the model, similar results transpire. That is, the intersection of set of parameter values at which all firms adopt the minimum wage with the set of parameter values at which no firms adopt the minimum wage is non-generic in the set of all permissible parameter values. In practical terms this, combined with result (27), means that holding all other parameters fixed, there is a critical value of the minimum wage below which all firms will adopt and above which no firms will adopt.

4 Conclusion

This paper provides a model of the labor market in which firms use the minimum wage as a commitment device. The point is to shed light on why firms appear to voluntarily bind themselves to paying higher wages than they would otherwise pay.

The essence of the argument is that firms face a trade-off between higher labor costs from adopting a binding minimum wage and the improved application rate that it implies. In the baseline model (homogenous jobs, workers and matches) for minimum wages that just bind, the trade-off works in the firms' favour. This is because when there are multiple applicants, the firms only forfeit a small fraction of the match surplus while workers move from receiving none of the surplus to receiving some of it. This means that the application rate rises very quickly with the minimum wage and the improved matching rate for firms offsets the higher labor cost. When there is match specific heterogeneity, there is an additional benefit from a larger pool of applicants - higher expected match quality. In both cases, the benefits accrue to the firms because the adjustment in application rates leave workers indifferent as to where to apply.

A number of potential extensions of this framework have been alluded to in the text. The possibility that firms might offer the minimum wage as a signal of productivity or job security may well be worth investigating. To verify the validity of such theories, though, requires more comprehensive data on how firms actually advertise their openings.

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