Digital Technologies, Knowledge Spillovers, Innovation Policies, and Economic Growth in a Creative Region

by

Amitrajeet A. Batabyal

and

Peter Nijkamp

---

1
We thank participants in the Tinbergen Institute Workshop, Amsterdam, May 2014, for their helpful comments on a previous version of this paper. In addition, Batabyal acknowledges financial support from the Gosnell endowment at RIT. The usual disclaimer applies.

2
Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA. Phone 585-475-2805, Fax 585-475-5777, Internet aabgsh@rit.edu

3
Department of Spatial Economics, VU University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Internet p.nijkamp@vu.nl
Digital Technologies, Knowledge Spillovers, Innovation Policies, and Economic Growth in a Creative Region

Abstract

We theoretically study the impact of two innovation policies on economic growth in a region that is creative in the sense of Richard Florida and that uses digital technologies to produce a final consumption good. The use of these digital technologies in our creative region gives rise to incomplete knowledge spillovers. Our analysis generates three salient findings. First, we characterize the balanced growth path (BGP) equilibrium. Second, we solve the social planner’s problem, describe the Pareto optimal allocation of resources, and then compare the Pareto optimal allocation with the BGP equilibrium allocation. Finally, we study the impacts that a research subsidy and a particular patent policy have on economic growth in our creative region and then we relate our findings to the incompleteness of the above mentioned knowledge spillovers.

Keywords: Creative Capital, Economic Growth, Knowledge Spillover, Patent Policy, Research Subsidy

JEL Codes: R11, O33, L52
1. Introduction

1.1. The two central research questions

The theory of regional economic growth and development has thus far been substantially influenced by Schumpeterian innovation theory as developed by Aghion and Howitt (1992) and others. The introduction and acceptance of advanced technology on the one hand and the effective utilization of new knowledge on the other have played a critical role in current regional economic development strategies (see Griliches (1992)). In the past few years, digital technology has increasingly been recognized as a radical and new technology with far reaching implications for the economic and geographical profile of regions. A given system is said to be using digital technology if it uses parts which contain or make use of binary or digital logic. This use can either be in hardware or in software. Basic binary logic has only two states such as one or zero and on or off. A combination of logic cells can make simple to highly complex circuits or integrated circuits. Most systems making use of digital technology in contemporary times have a micro controller or a processor, some form of storage, and a program running in it for decision making or processing. Viewed a little differently, Beekman and Beekman (2011) tell us that digital technology generates, stores, and processes data as a string of binary digits or “bits.”

The role of digital technologies in modern society since the 1990s has been compared to that of the railroad during the Industrial Revolution. Today, digital technologies can be found not only in computers but in a wide range of consumer electronic goods including smart phones, digital cameras, camcorders, and MP3 players. Koten (2013) is surely right when he points out that digital technologies have not only transformed manufacturing but that they have also made manufacturing leaner and smarter. In addition, like the railroad system, digital technologies have an unprecedented
space opening characteristic to them.

The work of De Berranger and Meldrum (2000), Koten (2013) and others tells us that the production and the subsequent utilization of digital technologies—for instance, to produce a smartphone—occurs in creative regions. This brings us to the work of Richard Florida who has been the leading contemporary proponent of the many virtues of creative regions. According to Florida (2002, 2005a, 2005b), creative regions are important because they are the key drivers of economic growth and development. In this regard, creative regions are able to play this vital instrumental role because they are populated by members of the creative class who possess creative capital.

Florida (2002, p. 8) defines the core of the creative class “to include people in science and engineering, architecture and design, education, arts, music and entertainment, whose economic function is to create new ideas, new technology and/or new creative content.” This creative class possesses creative capital which refers to “the intrinsically human ability to create new ideas, new technologies, new business models, new cultural forms, and whole new industries...” (Florida, 2005a, p. 32).

Recently, Florida (2013a, 2013b) has commented on the salience of digital technologies in creative regions. Even so, to the best of our knowledge, neither he nor anyone else has theoretically studied the working and the economic growth prospects of a creative region that produces digital technologies to manufacture a final consumption good such as a smart phone, a digital camera or a television. Given this lacuna in the literature, we have two basic objectives in this paper. First, we construct and analyze a model of economic growth in a creative region that uses digital technologies to produce a final good. The use of these digital technologies gives rise to knowledge spillovers that are limited and hence incomplete. Second, we study the impact of two innovation policies, namely,
a research subsidy and a particular patent policy, on economic growth in our creative region. Before proceeding to the specifics of our model, we now briefly survey the literature on digital technologies and the creative class in creative regions.

1.2. Review of the literature

We concentrate first on digital technologies. Mu and Lee (2005) examine the growth of technological capability in the telecommunications industry in China. They find that technological learning and “catching up” is significantly affected by the extent of knowledge diffusion and by industrial promotion by the government. Liu and San (2006) point out that countries with high levels of “social learning” are the ones that are best able to promote the sharing of internet technologies and speed up the rate of internet technology diffusion. De Laurentis (2006) contends that peripheral regions that have knowledge based economies can leverage cultural industries to promote economic growth and development.

Vinciguerra et al. (2011) study infrastructure networks in European regions and point out that internet connectivity is more important than airline connectivity in determining the extent of innovative activities in the regions under study. Focusing on the North American music industry, Hracs (2012) studies the ways in which digital technologies have challenged what he calls the “entrenched power” of the major record labels. A recent study by Tranos and Nijkamp (2013) provides a new modeling perspective on the “death of distance” assumptions in global connectivity patterns in the internet. Finally, Carter (2013) examines urban regeneration in Manchester, England. He shows that if Manchester is to become a more creative and sustainable city then it must imaginatively use digital technologies and have a commitment to open innovation.

Moving on to the creative class in creative regions, Markusen (2006) concentrates on artists
and criticizes the creative class notion. She contends that although artists make valuable contributions to the diversity and vitality of cities, it is unlikely that they have much in common with other members of Florida’s creative class. In contrast, McGranahan and Wojan (2007) focus on rural counties in the United States and point out that their measure of the creative class is strongly associated with regional development. Donegan and Lowe (2008) show that by deepening traditional labor market institutions and legislative supports, income inequality in cities with a large and creative talent pool can be ameliorated.

Lorenzen and Andersen (2009) concentrate on 444 city regions in eight European nations and study the interplay between the older notion of “centrality” and the newer notion of the “creative class.” Their analysis shows that the concept of centrality exerts a strong influence on what they call “urban hierarchies of creativity.” In turn, “the study of creative urban city hierarchies yields new insights into the problem of centrality” (Lorenzen and Andersen, 2009, p. 363). The statistical analysis conducted by Andersen et al. (2010) shows that Florida’s creative class thesis is supported for larger Nordic cities but not as well for smaller Nordic cities. Disputing aspects of Florida’s creative class thesis, Comunian et al. (2010) conduct an empirical analysis and point out that it is certainly not obvious that bohemian graduates can be agents of knowledge spillovers. Finally, Olfert and Partridge (2011) contend that it would be misguided to set policy to increase the livability of a community by increasing its cultural footprint.

We now proceed to analyze the two central research questions of this paper that were stated in the last paragraph of section 1.1. To do so, we shall organize the remainder of this paper as follows. Section 2 adapts the prior work of Jones (1995, 1999) and Acemoglu (2009, pp. 446-448)
and describes our theoretical model of a stylized creative region. Section 3 delineates the balanced growth path (BGP) equilibrium. Section 4 solves the social planner’s problem, describes the Pareto optimal allocation of resources, and then compares the Pareto optimal allocation with the BGP equilibrium allocation. Section 5 first studies the effects that a research subsidy and a particular patent policy have on economic growth in our creative region and then this section relates our findings to the incompleteness of the knowledge spillovers mentioned in section 1.1. Finally, section 6 concludes and then discusses potential extensions of the research delineated in this paper.

2. The Theoretical Framework

2.1. Preliminaries

Consider a stylized creative region that is populated by members of the creative class and that has an infinite horizon economy. The working of this region is marked by the use of creative capital and innovative activity (on which more below in section 2.2). The stock of creative capital or \( K_c(t) \) grows exponentially at rate \( h > 0 \) and hence we have \( \frac{dK_c(t)}{dt} = K_c(t) = hK_c(t) \). The representative household in this creative region displays constant relative risk aversion (CRRA) and its CRRA utility function is denoted by

\[
U(c(t)) = \int_0^{\infty} \exp\{-\rho \int \}
\]

\[
\left\{ c(t) \left(\frac{1}{1-\theta} - 1\right) \right\} dt, \quad \theta > 1,
\]

where \( c(t) = C(t)/K_c(t) \) is consumption per unit of creative capital in time \( t \), \( \rho > 0 \) is the time discount rate, and \( \theta \geq 0 \) is the constant coefficient of relative risk aversion.

The single final good for consumption such as a smart phone or a television is produced competitively with the production function

---

4 To the best of our knowledge, the two papers by Batabyal and Nijkamp (2010, 2011) are the only papers in the literature that have shed *theoretical* light on some of Florida’s ideas. Having said this, the reader should note that the Batabyal and Nijkamp (2010) paper studies the creative capital accumulation decision faced by individuals in a regional economy. In contrast, the Batabyal and Nijkamp (2011) paper analyzes the effects of neutral and non-neutral productivity growth on creative capital in a trading regional economy. In other words, there is no overlap between the questions studied in the above two papers and the two central research questions we analyze in this paper.
where $Y(t)$ is output at time $t$, $\beta \in (0,1)$ is a parameter of the production function, $N(t)$ denotes the total number of the varieties of digital technologies that are used to produce the final good at time $t$, $x(v,t)$ is the total amount of the digital technology of variety $v$ used at time $t$, and $K_{cR}(t)$ is the portion of the total stock of creative capital that is employed in the final good sector at time $t$. In the remainder of this paper, we normalize the price of the final consumption good to equal unity at all time points. Our next task is to discuss how digital technologies in our creative region are first invented and then produced.

### 2.2. Invention and production of digital technologies

The work of the research and development (R&D) sector in our creative region gives rise to the invention and then the production of new digital technologies. We shall describe the outcome of the R&D process with a so called “innovation possibilities frontier.”\(^5\) Mathematically, this frontier is given by

$$
\frac{dN(t)}{dt} = \dot{N}(t) = \eta N(t)^\zeta K_{cR}(t),
$$

where $\eta > 0$ is a flow parameter, $K_{cR}(t)$ is the portion of the available creative capital stock that is employed in the R&D sector, and $\zeta$ is a parameter that captures the extent of the knowledge spillovers arising from past R&D. Let $X(t)$ denote the total expenditure incurred on the production of digital technologies. Once the blueprint for a particular digital technology has been invented, we

\(^5\) See Batabyal and Nijkamp (2013) for additional details on the innovation possibilities frontier.
The assumption that patents are perpetual is dispensed with in our study of the effects of a particular patent policy in section 5.2 of the paper.

Suppose that one unit of this technology can be produced at marginal cost equal to $\psi>0$ units of the final consumption good.

In equation (2), if $\zeta=1$ then this means that the knowledge spillovers are linear or complete. In many growth models, it is this linearity in the spillover of knowledge that is the source of endogenous economic growth. However, our reading of the extant literature—see Mu and Lee (2005) and De Laurentis (2006)—tells us that while the invention and the production of digital technologies are certainly likely to give rise to knowledge spillovers, these spillovers are unlikely to be complete. Therefore, in the remainder of this paper, we suppose that $\zeta$ is positive but less than one and hence the knowledge spillovers under consideration here are partial or incomplete. Specifically, the term $N(t)^\zeta$ on the RHS of equation (2) captures the partial spillovers from the existing stock of ideas about digital technologies. Put differently, the greater is $N(t)^\zeta$, the more productive is a unit of creative capital employed in the R&D sector or $K_{cR}(t)$.

The market for creative capital must clear and hence, in equilibrium, we will have

$$K_{cE}(t) + K_{cR}(t) = K_c(t). \quad (3)$$

There is free entry into R&D activities in our creative region. The number of initial varieties of digital technologies $N(0)>0$ is supplied by monopolists. A firm in our creative region that invents a new digital technology of variety $v$ receives a fully enforced perpetual patent and is the monopolistic supplier of this variety. With this theoretical framework in place, our next task is to characterize the BGP equilibrium in our creative region. While undertaking this exercise, we shall adapt some results in Peters and Simsek (2009, pp. 150-159) to our case of a creative region that

---

6 The assumption that patents are perpetual is dispensed with in our study of the effects of a particular patent policy in section 5.2 of the paper.
uses digital technologies.

3. The BGP Equilibrium

We begin by first concentrating on the equilibrium allocation of resources in our creative region. To this end, let $r(t)$ and $w(t)$ denote the interest rate and the wage paid to creative capital at any time $t$. Maximizing the representative household’s CRRA utility function, we know that two conditions must be satisfied. The first condition is that the evolution of consumption per creative capital unit is described by the familiar Euler equation. That equation is

\[
\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta} \{r(t) - \rho\}. \tag{4}
\]

The second condition says that the transversality condition must be satisfied. Modifying equation 13.17 in Acemoglu (2009, p. 437) to our problem, the transversality condition is

\[
\lim_{t \to \infty} \exp \left\{ -\int_0^t r(b) db \right\} \int_0^{N(t)} V(v,t) dv = 0, \tag{5}
\]

where $V(v,t)$ denotes the net present discounted value of owning the blueprint for a digital technology of variety $v$.

Equilibrium in the creative capital market requires that the wage be equated to the marginal product of creative capital. Using equation (1) and the fact that in equilibrium we have $x(v,t) = K_{ce}(t)$ for all varieties $v$, we get

\[
w(t) = \frac{\partial Y(t)}{\partial K_{ce}(t)} = \frac{\beta}{1 - \beta} \int_0^{N(t)} x(v,t)^{1-\beta} dv K_{ce}^{\beta - 1} = \frac{\beta N(t)}{1 - \beta}. \tag{6}
\]
The production function for the final consumption good $Y(t)$ given by equation (1) can now be written more compactly as

$$Y(t) = \frac{1}{1-\beta}N(t)K_{cE}(t).$$  \hspace{1cm} (7)

Similarly, we can write the total expenditure on the production of digital technologies in our creative region as

$$X(t) = \int_0^{N(t)} \psi x(v,t) dv = (1-\beta)N(t)K_{cE}(t).$$  \hspace{1cm} (8)

Given equation (8), the resource constraint confronting our creative region tells us that the final good $Y(t)$ is allocated to consumption $C(t)$ and to expenditure on the production of digital technologies $X(t)$. In symbols, we have $Y(t) = C(t) + X(t)$ and this last expression can also be written as

$$Y(t) - X(t) = \frac{\beta(2-\beta)K_{cE}(t)N(t)}{1-\beta} = C(t) = c(t)K_c(t).$$  \hspace{1cm} (9)

Since the use of creative capital is an important aspect of R&D in our creative region, we can write the BGP free entry condition into R&D as

$$\eta N(t)^{\zeta} V(v,t) = w(t).$$  \hspace{1cm} (10)

Equation (10) tells us that a unit of creative capital employed in the invention and production of digital technology of variety $v$ costs the wage rate given by $w(t)$ and this unit gives rise to a flow
rate of $\eta N(t)^{\xi}$ innovations, each of which have value $V(v,t)$.\footnote{Now, modifying equations 13.8 and 13.27 in Acemoglu (2009, p. 436, p. 445) to our case, the above value function can be written in the form of a Hamilton-Jacobi-Bellman equation given by

$$r(t)V(v,t) - \dot{V}(v,t) = \pi(v,t) = \beta K_{ce}(t),$$

where $\pi(v,t)$ is the profit of the monopolist producing digital technology of variety $v$ at time $t$.

Consistent with the discussion in footnote 7, let us focus on an equilibrium in which there is a positive amount of R&D in the creative region under study. Then, equations (6) and (10) together tell us that

$$\eta N(t)^{\xi}V(v,t) - \omega(t) = \frac{\beta N(t)}{1 - \beta}, \forall t.$$

Differentiating both sides of equation (12) with respect to time $t$, we get $\dot{V}(t)/V(t) = (1 - \zeta)\dot{N}(t)/N(t)$. Substituting this last expression in equation (11) and then simplifying the resulting outcome tells us that the equilibrium value function can be written as

$$V(v,t) = \frac{\beta K_{ce}(t)}{r(t) - (1 - \zeta)\{\dot{N}(t)/N(t)\}}.$$

Substituting for $V(v,t)$ from equation (13) into the free entry condition given by equation (12), we get
\[
\frac{V(v,t)}{N(t)^{1-\xi}} = \frac{\beta K_{ce}(t)}{(1-\beta)\eta} \left[ \rho R(t) - (1-\zeta)\{N(t)/N(t)\}N(t)^{1-\xi} \right].
\] (14)

Differentiating equations (4) and (9) with respect to time \( t \) and then simplifying the resulting expression, we get

\[
\frac{\dot{K}_{ce}(t)}{K_{ce}(t)} + \frac{\dot{N}(t)}{N(t)} = \frac{\dot{c}(t)}{c(t)} + \frac{\dot{K}_c(t)}{K_c(t)} = r(t) - \frac{\rho}{\theta} + h. \tag{15}
\]

We can now solve equation (15) for the equilibrium interest rate \( r(t) \). The solution of interest is

\[ r(t) = \rho + \theta \left[ \frac{\dot{K}_{ce}(t)}{K_{ce}(t)} + \frac{\dot{N}(t)}{N(t)} \right]. \]

Substituting this last expression in equation (14) gives us

\[
\frac{\beta}{(1-\beta)\eta} \left[ \frac{\beta K_{ce}(t)}{(1-\beta)\eta} \right] = \frac{\beta K_{ce}(t)}{(1-\beta)\eta} \left[ \rho + \theta \left( \frac{\dot{K}_{ce}(t)}{K_{ce}(t)} + \frac{\dot{N}(t)}{N(t)} \right) - (1-\zeta)\{N(t)/N(t)\}N(t)^{1-\xi} \right]. \tag{16}
\]

Note that the rate of technological progress in our creative region is given by the innovation possibilities frontier described by equation (2). Therefore, using equation (3) to simplify equation (2), we get

\[
\frac{\dot{N}(t)}{N(t)} = \eta N(t)^{\zeta-1} K_{ce}(t) = \eta N(t)^{\zeta-1} \{K_c(t) - K_{ce}(t)\}. \tag{17}
\]

Observe that equations (16) and (17) together comprise a system of two differential equations in the two unknowns \( K_{ce}(t) \) and \( N(t) \). Therefore, knowing the initial number of digital technologies in our creative region \( N(0) \) and the transversality condition given in equation (5), we can in principle solve these two equations for the trajectories of the unknown variables or \( \{K_{ce}(t), N(t)\}_{t=0}^{\infty} \).
Knowing these two trajectories, we can ascertain the equilibrium interest rate from equation (15) and equation (9) gives us the equilibrium value of consumption. Finally, the equilibrium value function $V(v,t)$ is given by equation (12).

Having determined the equilibrium values of the different variables of interest in the preceding paragraph, we are now in a position to focus on the BGP. From the definition of a BGP, it follows that the number of digital technology varieties in our creative region grows at a constant rate, say, $g^{BGP}$. Equation (17) then tells us that the rate of growth of creative capital employed in R\&D is given by

$$\frac{\dot{K}_{cR}(t)}{K_{cR}(t)} = (1 - \zeta)g^{BGP}. \quad (18)$$

An implication of equation (18) is that $\frac{\dot{K}_{cE}(t)}{K_{cE}(t)} = \frac{\dot{K}_{cR}(t)}{K_{cR}(t)} = \frac{\dot{K}_{c}(t)}{K_{c}(t)} = h > 0$. Using this last result, equation (18) tells us that the BGP growth rate of digital technology varieties in our creative region is given by

$$g^{BGP} = \frac{h}{1 - \zeta}. \quad (19)$$

What about total consumption on the BGP? Equation (9) tells us that total consumption $C(t)$ in our creative region grows at rate $\dot{C}(t)/C(t) = \frac{\dot{K}_{cE}(t)}{K_{cE}(t)} + \frac{\dot{N}(t)}{N(t)} = g^{BGP} + h$. Using this last expression, we deduce that consumption per unit of creative capital or $c(t)$ also grows at the rate $g^{BGP}$ given by equation (19). To determine the BGP interest rate, we simplify the consumption Euler equation given by (4) with equation (19). This gives us
and hence the BGP interest rate is clearly constant.\(^8\)

Inspecting equation (19), two aspects of the growth of the economy of our creative region are worth emphasizing. First, we see that this growth rate is positively affected by the growth rate of the total stock of creative capital \(h\). Second, the effect of the parameter \(\zeta\) measuring the extent of knowledge spillovers from past R&D is more nuanced. We know that \(\zeta \in (0,1)\). Therefore, as the knowledge spillovers increase, i.e., as \(\zeta \to 1\), the economic growth rate of our creative region also increases. In contrast, when the knowledge spillovers become more limited, i.e. as \(\zeta \to 0\), our creative region’s growth rate decreases. This completes our discussion of the BGP equilibrium. We now first study the Pareto optimal allocation of resources and then compare this Pareto optimal allocation with the BGP equilibrium allocation.

4. The Pareto Optimal Allocation

We begin with the social planner’s problem for our creative region. This planner maximizes the utility of a representative household of the creative class subject to a set of constraints. In full generality, the planner solves

\[
\max_{c(t), k_{cdt}, k_{cdt}, (x(t),z(t))_{t=0}^{\infty}} \int_0^\infty \exp \left\{ - (\rho - h) r \right\} \frac{c(t)^{1-\theta} - 1}{1-\theta} \, dr
\]

subject to

\[r^{BGP} = \rho + \frac{h\theta}{1-\zeta} = \rho + g^{BGP} \theta \tag{20}\]

---

\(^8\) Straightforward but tedious algebra gives us an inequality condition involving the various parameters of the model that must hold for the transversality condition in equation (5) to be satisfied. The specific inequality condition is that \((1-\theta)g^{GP} < \rho - h\).
As stated above, the social planner’s problem is too complex to solve in a tractable manner. Therefore, we simplify this problem by first supposing that the total number of digital technologies $N(t)$ and the creative capital employed in the final good sector $K_{ce}$ are both given. Then, we can focus on a simpler problem which requires the social planner to solve

$$Y(t) = X(t) + c(t)K_{ce}(t),$$  \hspace{1cm} (22)

$$Y(t) = \frac{1}{1-\beta} \int_{0}^{N(t)} x(v,t)^{1-\beta} dv K_{ce}(t)^{\beta},$$  \hspace{1cm} (23)

$$X(t) = \int_{0}^{N(t)} \psi x(v,t) dv,$$  \hspace{1cm} (24)

$$\frac{\dot{N}(t)}{N(t)} = \eta N(t)^{\zeta-1} K_{ce}(t),$$  \hspace{1cm} (25)

and

$$K_{ce}(t) = K_{ce}(t) + K_{cr}(t).$$  \hspace{1cm} (26)

In words, in the above simpler problem, the social planner is first allocating digital technologies optimally in our creative region. The first order necessary condition for an optimum to the problem in (27) is given by $x^{SP}(v,t) = \psi^{-1/\beta} K_{ce}(t) = (1-\beta)^{-1/\beta} K_{ce}(t)$, where the superscript $SP$ denotes social planner. This last expression tells us that

$$Y(t) - X(t) = \beta (1-\beta)^{-1/\beta} K_{ce}(t) N(t).$$  \hspace{1cm} (28)
Substituting equation (28) into the social planner’s full problem stated above in this section, we get the following simplified problem:

\[
\max_{(c(t), \kappa_{c}(t), N(t))} \int_0^{\infty} \exp\{-(\rho-h)t\} \frac{c(t)^{1-\theta}-1}{1-\theta} \, dt
\]  

(29)

subject to

\[c(t)K_c(t) = \beta(1-\beta)^{-\beta}K_{ce}(t)N(t)\]  

(30)

and

\[\frac{\dot{N}(t)}{N(t)} = \eta \{K_c(t) - K_{ce}(t)\} N(t)^{\xi-1}.\]  

(31)

Let \(f_{ce}(t) = K_{ce}(t)/K_c(t)\) denote the fraction of the stock of creative capital that is employed in the final good sector. Using this fraction and the meaning of \(c(t)\) in the maximization problem described in equations (29)-(31) gives us the current value Hamiltonian function for this problem. That function is

\[\mathcal{H}(f_{ce}(t), N(t), \alpha(t)) = \frac{\beta(1-\beta)^{-\beta}f_{ce}(t)N(t)^{1-\theta}-1}{1-\theta} + \alpha(t)\eta \{1-f_{ce}(t)\} K_c(t)N(t)^{\xi},\]  

(32)

where \(\alpha(t)\) is the costate variable associated with the constraint in (31) and the control variable is the fraction \(f_{ce}(t)\). The first order necessary conditions for an optimum are

\[\frac{\partial \mathcal{H}}{\partial f_{ce}} = \beta(1-\beta)^{-\beta}N(t)^{1-\theta}f_{ce}(t)^{\theta} - \alpha(t)\eta K_c(t)N(t)^{\xi} = 0\]  

(33)

and
\[ \partial U / \partial N = (\beta (1 - \beta)^{-1/\beta} f_{cE}(t))^{1 - \theta} N(t)^{-\theta} + \zeta \eta \alpha(t) K_c(t) (1 - f_{cE}(t)) N(t)^{\zeta - 1} = (\rho - h) \alpha(t) - \dot{\alpha}(t). \] (34)

Substituting from equation (33) into (34) and then simplifying the resulting expression gives us

\[ \frac{\dot{\alpha}(t)}{\alpha(t)} = \eta K_c(t) N(t)^{\zeta - 1} \{ \zeta + (1 - \zeta) f_{cE}(t) \} - (\rho - h). \] (35)

In order to meaningfully compare the equilibrium allocation of section 3 with the Pareto optimal allocation of this section, we will need to derive an equation that is similar to the consumption Euler equation (4). To do this, we substitute from equation (30) into equation (33) and then simplify the resulting expression. This gives us

\[ \beta (1 - \beta)^{-1/\beta} c(t)^{-\theta} = \eta \alpha(t) K_c(t) N(t)^{\zeta - 1}. \] (36)

Now, differentiating both sides of equation (36) with respect to time and then simplifying using equation (35) gives us the analog of the consumption Euler equation (4) we seek. Specifically, we get

\[ \frac{\dot{\alpha}(t)}{c(t)} = \frac{1}{\theta} \left[ \eta K_c(t) N(t)^{\zeta - 1} \{ \zeta + (1 - \zeta) f_{cE}(t) \} - \rho + (1 - \zeta) \frac{\dot{N}(t)}{N(t)}. \right]. \] (37)

Rewriting the innovation possibilities frontier given by equation (25), we can tell that

\[ \frac{\dot{N}(t)}{N(t)} = \frac{\eta K_c(t) (1 - f_{cE}(t))}{N(t)^{1 - \zeta}} \frac{\eta K_c(t) (1 - f_{cE}(t))}{N(t)^{1 - \zeta}}. \] (38)

Using equation (38), the analog of the consumption Euler equation given by equation (37) can be written as
To facilitate the comparison of the Pareto optimal allocation of this section with the equilibrium allocation of the previous section, let us consider a solution to the social planner’s problem that displays balanced growth. In such a solution, consumption grows at a constant rate and, in addition, equation (39) tells us that the creative capital to digital technologies ratio $\frac{K_c(t)}{N(t)^{1-\zeta}}$ is constant. This tells us that the growth rate of digital technology varieties or $g^{SP}$ in the Pareto optimal allocation will be the same as the corresponding growth rate in the BGP equilibrium described by $g^{BGP}$ in section 3. Using equation (19), we infer that

$$g^{SP} = g^{BGP} = \frac{h}{1-\zeta}. \quad (40)$$

Using equation (40) and our previous observation that the ratio $\frac{K_c(t)}{N(t)^{1-\zeta}}$ is constant, the innovation possibilities frontier in equation (38) tells us that the fraction $f_{cE}(t) = f_{cE}^{SP}$, also a constant. Using this result along with equations (22) and (28) gives us an expression for consumption per creative capital unit or $c^{SP}(t)$. That expression is

$$c^{SP} = \beta(1-\beta)^{-1/\beta} f_{cE}(t) N(t) = \beta(1-\beta)^{-1/\beta} f_{cE}^{SP} N(t). \quad (41)$$

Taking logarithms and then differentiating both sides of equation (41) with respect to time, we see that consumption per creative capital unit grows at the rate $g^{SP}$ which is also the rate at which digital technology varieties in our creative region grow.

Even though the growth rates of the digital technology varieties and consumption per creative capital unit are identical in the BGP equilibrium and in the social planner’s problem, it is
important to understand that the BGP equilibrium is not Pareto efficient. This is because the digital technology varieties to creative capital ratio in the BGP equilibrium or \( \{ N^{1-\zeta}/K_c \}^{BGP} \) is always lower than the corresponding ratio \( \{ N^{1-\zeta}/K_c \}^{SP} \) selected by the social planner. To see this, note from equation (39) that

\[
\frac{\{ N^{1-\zeta}/K_c \}^{SP}}{\rho + h\theta/(1-\zeta)}.
\]  

Algebraically manipulating the free entry condition (equation (14)) and equations (17) and (19), it can be shown that \( \{ K_c/N^{1-\zeta} \}^{BGP} = (1/\eta)[h/(1-\zeta)+\{h(1-\zeta)^{-1}(\theta+\zeta-1)+\rho\}/(1-\beta)] \). Simplifying this last expression further, we get

\[
\frac{\{ N^{1-\zeta}/K_c \}^{BGP}}{\eta} = \frac{\{\{(\theta+\zeta-1)h/(1-\zeta) + \rho\}/(1-\beta) + h/(1-\zeta)\}}{\eta}.
\]  

Comparing the RHSs of equations (42) and (43), it is clear that the two digital technology varieties to creative capital ratios are not identical and hence it follows that the BGP equilibrium and the solution to the social planner’s problem are distinct. In addition, comparing the \( \{ N^{1-\zeta}/K_c \}^{SP} \) and \( \{ N^{1-\zeta}/K_c \}^{BGP} \) ratios we see that the digital technology varieties to creative capital ratio in the BGP equilibrium is lower than the corresponding ratio chosen by the social planner as long as the inequality

\[
\frac{\rho + h\theta/(1-\zeta)}{1-\zeta} < \frac{h}{1-\zeta} + \frac{\rho + (\theta+\zeta-1)h/(1-\zeta)}{1-\beta}
\]  

holds. Algebraic manipulations show that the above inequality holds whenever
is satisfied. Now, because the inequality describing the transversality condition in footnote 8 holds, we deduce that the inequality in (45) also holds. This demonstrates the validity of our claim that the digital technology varieties to creative capital ratio in the BGP equilibrium is always lower than the corresponding ratio picked by the social planner.

From an intuitive standpoint, when solving his problem, the social planner avoids the static monopoly distortions present in our discussion of the BGP equilibrium in section 3 and he internalizes (i) the positive knowledge spillovers and (ii) the benefits arising from innovations on the future employment of creative capital. As a result of this internalization, relative to the section 3 scenario, the social planner employs more creative capital in R&D. This additional use of creative capital leads to a higher level of digital technology varieties but not to a higher growth rate of these same technologies. As discussed in the preceding paragraph, this is also why the digital technology varieties to creative capital ratio in the social planner’s problem is always higher than the corresponding ratio in the BGP equilibrium. This concludes our discussion of the social planner’s problem. We now analyze the impacts that a research subsidy and a particular patent policy have on economic growth in our creative region.

5. Innovation Policies and their Effects

We begin by pointing out that the research subsidy as well as the particular patent policy do not affect the final consumption good sector in our creative region. Therefore, we can safely omit this sector from our subsequent analysis and focus exclusively on the R&D sector. Let us first consider a subsidy to research.
5.1. Research subsidy

The government in our creative region subsidizes R&D by paying each R&D conducting firm a fraction \( \lambda \in (0,1) \) of the total cost of employing creative capital for R&D. With this governmental payment, the actual per unit wage paid by a R&D conducting firm is \( w^{RS}(t) = (1-\lambda)w(t) \), where the superscript \( RS \) denotes the research subsidy. As such, the free entry condition in R&D given by equation (12) needs to be modified to account for the research subsidy. The new free entry into R&D condition is

\[
\eta N(t)^{\zeta}V(\nu,t) = w^{RS}(t) = (1-\lambda)w(t) = (1-\lambda)\frac{BN(t)}{1-\beta}, \quad \forall t. \tag{46}
\]

The value function is still given by equation (13). We know that on the BGP, the interest rate is constant and so is the rate \( g^{BGP} \) at which the number of digital technology varieties is growing. Using this last result along with equations (13) and (46), we get

\[
\frac{\beta \eta K_{ce}(t)N(t)^{\zeta}}{r - (1-\zeta)g^{BGP}} = \frac{\beta(1-\lambda)N(t)}{1-\beta}. \tag{47}
\]

Solving for \( K_{ce}(t) \) from equation (47), taking logarithms, and then differentiating the resulting expression with respect to time gives us

\[
\frac{\dot{K}_{ce}(t)}{K_{ce}(t)} = (1-\zeta)\frac{\dot{N}(t)}{N(t)}. \tag{48}
\]

Recall from the discussion right after equation (18) in section 3 that on the BGP, \( K_{ce}(t) \) grows at the rate \( h>0 \). Substituting this in the LHS of equation (48) tells us that
This growth rate is clearly the same as the growth rate given in equation (19) in our discussion of the BGP equilibrium. Because the two growth rates are unchanged, we conclude that the government’s research subsidy has no impact on the growth rate of the digital technology varieties. In addition, because the consumption Euler equation must continue to hold, the interest rate with the subsidy is still given by equation (20). This tells us that the research subsidy has no effect on the interest rate either.

Recall that \( K_{eE} = f_{eE} K_e \), \( K_{eR} = f_{eR} K_e \), and that \( f_{eE} + f_{eR} = 1 \). Using this information, we rewrite equation (46). This gives us

\[
\frac{\beta f_{eE}}{r_{BGP} - h} \left( \frac{K_c}{N^{1-\xi}} \right) = \frac{\beta (1-\lambda)}{\eta (1-\beta)}. \tag{50}
\]

Next, we use the fact that \( f_{eE} + f_{eR} = 1 \) and then rewrite equation (38). We get

\[
\eta (1-f_{eE}) \left( \frac{K_c}{N^{1-\xi}} \right) = g_{BGP} = \frac{h}{1-\zeta}. \tag{51}
\]

Equations (50) and (51) comprise a system of two equations in the two unknowns \( f_{eE} \) and \( K_c/N^{1-\xi} \). Note that this last ratio is constant along the BGP. As such, these two equations can be solved for the two unknowns. To this end, let us solve for \( K_c/N^{1-\xi} \) from equation (51), substitute the result in equation (50), and then simplify to isolate the ratio \( f_{eE}/(1-f_{eE}) \). This gives us

\[
\frac{f_{eE}}{1-f_{eE}} = \frac{(1-\zeta)(1-\lambda)(r_{BGP} - h)}{h(1-\beta)}. \tag{52}
\]
We now want to ascertain the sign of the partial derivative $\partial f_{cE}/\partial \lambda$. As such, we differentiate equation (52) with respect to $\lambda$ to get

$$\frac{\partial f_{cE}}{\partial \lambda} = \frac{(1-\zeta)(r^{BG} - h)(1-f_{cE})^2}{h(1-\beta)} < 0. \quad (53)$$

Equation (53) tells us that the fraction of the available creative capital that is employed in the final consumption good sector decreases in the research subsidy $\lambda$. Since $f_{cR} = 1 - f_{cE}$, it follows that the research subsidy increases $f_{cR}$ or the fraction of the available creative capital that is employed in the R&D sector of our creative region. Let us now focus on equation (51). Since the fraction $f_{cR}$ rises with the research subsidy, inspection of equation (51) tells us that in order for the LHS of this equation to equal the RHS, the constant ratio $K_c/N^{1-\zeta}$ on the LHS must decline. This tells us that after the implementation of the research subsidy, our creative region reaches a new BGP equilibrium in which the new technology to creative capital ratio $N^{1-\zeta}/K_c$ is higher than in the previous steady state with no research subsidy. In other words, although the research subsidy has no impact on the growth rate of our creative region, this subsidy does lead to a positive level effect in the sense that it leads to a long run reallocation of creative capital between the final good and the R&D sectors and this reallocation gives rise to a higher technology to creative capital ratio. We now study the effects that a particular patent policy has on economic growth in our creative region.

5.2. Patent policy

In the R&D sector, we now suppose that in contrast with the assumption employed in section 2.2, a firm that discovers the blueprint for a new digital technology does not receive a fully enforced perpetual patent on this particular technology. Instead, the government in our creative region implements a policy in which each patent expires at the rate $\epsilon > 0$. With this policy in place, instead
of equation (11), the value function $V(v,t)$ now satisfies a different Hamilton-Jacobi-Bellman equation and that equation is

$$r(t)V(v,t) - \dot{V}(v,t) = \beta K_{eb}(t) - \epsilon V(v,t).$$  

(54)

Compared to equation (11), the RHS in equation (54) is different because of the government’s patent policy. Specifically, with a flow rate of $\epsilon$, the patent on the blueprint for a new digital technology is lost and hence competition in the R&D sector of our creative region will reduce the value of the blueprint in question to zero.

Algebra and reasoning similar to that employed in the discussion preceding equation (13) in section 3 tells us that along the BGP, the value function in equation (54) satisfies the condition $V(v,t) = \beta K_{eb}(t)/(\epsilon + r - h)$. Using this last condition, the free entry into R&D condition or the analog of equation (12) can be written as

$$\eta N(t)^{\frac{\beta}{1-\beta}}V(v,t) = \frac{\beta \epsilon N(t)^{\frac{\beta}{1-\beta}}}{\epsilon + r - h} = \omega(t) = \frac{\beta N(t)}{1-\beta}, \ \forall t.$$  

(55)

Simplifying equation (55) tells us that $K_{eb} = N(t)^{1-\zeta}(\epsilon + r - h)/\eta(1-\beta)$. Taking the logarithm of both sides of this last equation and then differentiating the resulting expression with respect to time tells us that the growth rate of the digital technology varieties with the patent policy under consideration equals $g^{BGP} = h/(1-\zeta)$. Clearly, this growth rate is identical to the equation (19) growth rate describing the BGP equilibrium. Since these two growth rates are unchanged, we conclude that like the research subsidy, the government’s patent policy also has no effect on the growth rate of the digital technology varieties in our creative region.

Using the discussion in the preceding paragraph and the expression $f_{eb} = K_{eb}$, the free entry
into the R&D sector condition given in equation (55) can be re-written as

\[
\left\{ \frac{\beta f_cE}{\varepsilon + r - h} \right\} \left\{ \frac{K_c}{N^{1/\zeta}} \right\} = \frac{\beta}{\eta(1 - \beta)}.
\] (56)

Similarly, the innovation possibilities frontier given by equation (2) now simplifies to

\[\eta f_{cr} \left( \frac{K_c}{N^{1/\zeta}} \right) = g^{BGF} = \frac{h}{1 - \zeta} \] (57)

As in section 5.1, we now want to determine the sign of the partial derivative \( \partial f_cE / \partial \varepsilon \).

Differentiating equation (56) with respect to \( \varepsilon \), we get

\[
\frac{\partial f_cE}{\partial \varepsilon} = \frac{N^{1/\zeta}}{\eta K_c(1 - \beta)} > 0.
\] (58)

Equation (58) tells us that the faster the rate \( \varepsilon \) at which patents in our creative region expire, the higher is the fraction of the total stock of creative capital that is employed in the final good sector and hence the lower is \( f_{cr} \) or the fraction employed in the R&D sector. Now, inspecting equation (57), we see that the RHS of this equation is constant. Therefore, if \( f_{cr} \) in the LHS declines then, to ensure that this equation continues to hold, the ratio \( K_c/N^{1/\zeta} \) in the LHS must rise. This last finding tells us that the constant technology to creative capital ratio \( N^{1/\zeta}/K_c \) must fall, i.e., the patent policy under study gives rise to a level effect. This result make intuitive sense. From the standpoint of a firm in the R&D sector of our creative region, owning a patent that expires faster makes this patent less valuable. Put differently, at any given wage \( w(t) \), there is now less incentive to conduct R&D.
Hence, to encourage more R&D in the creative region under study, the value of patents has to increase. This can be done by ensuring that a greater fraction of the existing creative capital stock is employed in the final good sector. When this happens, the demand for digital technologies rises and this, in turn, raises the profits of the monopolistic suppliers of the various digital technologies. So, as in section 5.1, the government’s patent policy has no growth effect but it does have a level effect.

5.3. Discussion

From the analysis in sections 5.1 and 5.2, we have seen that the research subsidy and the patent policy have no growth effects. This means that the government of our creative region will not be able to raise economic growth in this region by using innovation policies of the sort studied in this paper. All that the government can hope to accomplish with these two policies is to bring about a level effect in which the long run technology to creative capital ratio changes.

Why is this the case? The answer lies in the partial knowledge spillovers described by the parameter $\zeta$ in the innovation possibilities frontier in equation (2). Because $\zeta<1$, equation (19) tells us that the economic growth rate of the creative region under study is determined completely by the rate of growth of the stock of creative capital $h$ and the extent to which there are decreasing returns to present knowledge $N(t)$ described by the parameter $\zeta$. Given this state of affairs, innovation policies in our creative region are able to influence only (i) the allocation of the stock of creative capital in the final good and in the R&D sectors and (ii) the technology to creative capital ratio $N(t)^{1-\zeta}/K_c$. Innovation policies of the sort studied in this paper would be able to positively impact the economic growth rate of our creative region if and only if the knowledge spillovers are complete ($\zeta=1$).

The veracity of the claims made in the preceding paragraph can be verified by inspecting
equation (19). We see that when the knowledge spillovers are very weak or limited \((\zeta \to 0)\), for all practical purposes, they do not matter for economic growth and the BGP growth rate of our creative region is determined entirely by the rate of growth of the stock of creative capital. In symbols, we have \(\lim_{(\zeta \to 0)} g^{BGP} = h\). In contrast, as the knowledge spillovers become strong, the BGP growth rate \(g^{BGP}\) also rises. In the limit, as the knowledge spillovers become complete \((\zeta \to 1)\) the economic growth rate of our creative region becomes arbitrarily large \((g^{BGP} \to \infty)\).

6. Conclusions

In this paper, we theoretically studied the impact of two innovation policies on economic growth in a region that was creative in the sense of Richard Florida and that used digital technologies to produce a final consumption good such as a smart phone or a television. The use of these digital technologies in our creative region gave rise to knowledge spillovers that were partial and not complete. Our analysis led to three significant findings. First, we characterized the balanced growth path (BGP) equilibrium. Second, we solved the social planner’s problem, described the Pareto optimal allocation of resources, and then compared the Pareto optimal allocation with the BGP equilibrium allocation. Finally, we analyzed the effects that a research subsidy and a particular patent policy had on economic growth in our creative region and then we related our findings to the incompleteness of the above mentioned knowledge spillovers.

The analysis in this paper can be extended in a number of directions. One interesting extension that would also yield insights into phenomena occurring over space involves the analysis of a multi-region generalization of the model studied in this paper in which trade between the different regions is explicitly modeled. To see the kinds of questions that might be studied in such a scenario, consider first a simplified model with two regions \((i=1,2)\) that trade with each other.
Now, modifying equation (2), if we specify the germane innovation possibilities frontier as
\[ \dot{N}_i(t) = \eta_i N(t)^2 K_{eRi}(t), \quad i=1,2, \]
where \( N(t) = \sum_{i=1}^{2} N_i(t) \), then two questions that would be interesting to study concern the impact of opening each region to trade. In particular, does trade openness lead to more or less innovation in each of the two regions? In addition, what impact does trade have on the long run growth rate of these same two regions? A second interesting extension would involve an analysis of the implications of the introduction and the acceptance of a new digital technology by a particular creative region. This question is significant because depending on utilization conditions in a region, the use of a new digital technology may lead to both an accelerated growth path and to a rise in spatial disparities either within a region or between regions. Studies that incorporate these aspects of the problem into the analysis will increase our understanding of the ways in which the interactions between innovative activities and economic interactions influence the growth and development of dynamic, creative regions.
References


Markusen, A. 2006. Urban development and the politics of a creative class: Evidence from a study


