

Instructions: Answer 3 of the following 5 questions. Show all of your work. Write your answer to each question in a separate bluebook. On the cover of the bluebook, write the number of the question under "Section." DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 3 hours.

1. Two individuals invest in a project which takes two periods to complete. At the start of period one, person A invests 4.5 and person B invests 1.5. At the end of period one, each of the investors has a chance to withdraw her investment. The decisions whether to withdraw from the project or not are made simultaneously. If either investor withdraws, the project is scrapped and the scrapped value is 4. If both investors withdraw, they share the scrapped value in proportion to their investment; A gets 3 and B gets 1. If one investor withdraws while the other does not, the one who withdraws gets the first claim on the scrapped value up to the amount of her investment; if A withdraws while B does not, A gets $\min(4.5, 4)$ and B gets $4 - \min(4.5, 4)$. If B withdraws while A does not, B gets $\min(1.5, 4)$ and A gets the rest of the scrapped value. If neither investor withdraws at the end of period one, the investors have another chance to withdraw at the end of period two. If either investor withdraws, the payoffs are the same as before. If neither investor withdraws at the end of period two, the project is completed and the investors get the total of 12 in gross return which they share in proportion to their original investment. There is no discounting.

a. Draw the extensive form of the above game.

b. How many pure strategies are there for investor A ? How many for investor B ?

Give an example of a pure strategy for each investor.

c. What are the pure strategy subgame perfect Nash equilibria? Explain your answer.

d. Interpret the answer to c. Is there any problem with treating subgame perfect Nash equilibrium as a prediction of the outcome of rational play in this game?

2. A profit seeking monopolist has three choice variables: output level q , marketing intensity m , and new equipment e . The cost of producing q units of output is $\frac{cq}{e}$, where $c > 0$ is a constant. Equipment is purchased in a competitive market at the price r . The cost of marketing intensity level m is $h(m, \theta)$, where θ is parameter and h is increasing in both of its arguments. The maximum price at which q units of output can be sold with marketing intensity m is $P(q, m)$, where $P_q < 0$ and $P_m > 0$. (Subscripts denote partial derivatives.)

a. Assume here and below that $h_{m\theta} > 0$. Interpret this condition and give a possible economic interpretation for the parameter θ .

b. Under what condition on P does higher marketing intensity increase marginal revenue? From now on, assume that this condition holds.

c. Suppose that in the short run, the amount of new equipment, $e > 0$ is fixed. How do the firm's short run choices of output level and marketing vary depending on the parameter θ ? depending on e ? Be as specific as possible and interpret your conclusions.

d. In the long run, the firm is free to choose any nonnegative levels of q , m and e . How does the firm's long run choice of e vary depending on θ ? Interpret your conclusion.

e. Compare the short and long run responses of q to a change in θ . Interpret the comparison.

f. Show how the firm's long run profit changes in response to changes in r . Does the answer depend on θ ? If so, in what way. Interpret the results. This answer requires very little computation (why?).

3. Consider an economy with two consumers, A and B ; one commodity, m ; and two possible states of nature, 1 and 2, which are equally likely to occur. In state 1, A is endowed with three units of m and B is endowed with one unit. In state 2, A is endowed with one unit and B is endowed with two. The agents have identical von Neumann - Morgenstern preferences with the Bernoulli utility function $u(m) = \sqrt{m}$.

a. Draw an Edgeworth box depicting the feasible allocations of state 1 consumption of m and state 2 consumption.

b. Identify the ex ante Pareto efficient allocations.

c. Suppose each agent were offered the opportunity to purchase insurance by an outside provider at an actuarially fair premium, how much would each agent buy and how much m would they consume in each state?

d. Alternatively, suppose no such insurance were available but the agents might “self-insure,” that is, trade in contingent claims. Which feasible trades might they agree to?

e. Finally, suppose they were to trade at market determined prices. Contrast each agent’s consumption and level of well-being at the market outcome versus the outcome with outside insurance. Explain the difference.

4. A principal offers an agent an enforceable contract for work on a project. Under the contract, the principal is to pay $s(x)$ for $x \in \{1, 2, 3\}$ units of publicly observable output. The agent’s effort level on the job, $e \in \{0, 1\}$, cannot be observed by the principal. The probability that the output is x given the agent’s effort e is $p(x|e)$, with $p(x|1)/p(x|0)$ increasing in x . Under the contract, if x units are produced, the principal’s utility is $x - s(x)$ and the agent’s utility is $s(x) - e$. The principal and agent are both risk-neutral expected utility maximizers. The agent is willing to accept any contract with a nonnegative payment function $s(\cdot)$. Suppose that the payment function s^* is optimal for the principal among all contracts with nonnegative payments, and suppose that s^* induces the agent to choose $e = 1$.

a. Is it possible that $s^*(1)$ is positive? Is it possible that $s^*(2)$ is positive? Explain.

b. Consider instead the payment function S that is optimal for the principal among nonnegative payment functions s satisfying the additional constraint $[s(x) - s(y)]/(x - y) \leq 1$ for $x \neq y$. Why might it make economic sense to restrict attention to payment functions satisfying this constraint?

c. Assume that the payment function S in part b induces the agent to choose $e = 1$. Is it possible that $S(1)$ is positive?

d. Under the assumption of part c, suppose that $S(2) > 0$. What can be concluded about $S(3) - S(2)$? Be as specific as possible and justify your answer. Give an economic explanation for the difference between the payment functions s^* and S .

5. Consider the following first-price, sealed bid, private value auction of an object to two bidders. In the auction, the highest bidder wins the bid and pays her bid. Suppose that for $i = 1, 2$, bidder i ’s valuation of the object, θ_i , is independently and uniformly distributed on $[0, 1]$. In this case, the common cumulative distribution function of θ_i is $F(\theta_i) = \theta_i$ on $[0, 1]$. The bidders are risk neutral. The above set-up is common knowledge.

a. Explain briefly how the auction can be treated as a Bayesian game. What is a (pure) strategy of a bidder?

b. Define a pure strategy Bayesian Nash equilibrium of the above bidding game.

c. Derive a symmetric Bayesian Nash equilibrium of the game.

d. Now, consider instead a second-price, sealed bid auction. In this auction, the highest bidder wins the object and pays the second highest bid. The rest of the set-up is the same as before. What is the dominant strategy Bayesian Nash equilibrium of this bidding game?

e. Give an expression for the expected revenue of the auctioneer for each of the above auction schemes.

f. Calculate the expected revenue of the auctioneer in each of the above auction schemes. Show that they are the same.