**STATE UNIVERSITY OF NEW YORK AT ALBANY**

**Department of Economics**

Ph.D. Comprehensive Examination in Microeconomics, June 9, 1998

Answer problem 1 and answer two of the remaining three numbered problems. Justify your answers (except for definitions). Whenever possible, show why your answers are correct. Partial credit will be awarded in proportion to the difficulty of the parts of the problems you solve. Write your answers to the numbered problems in separate bluebooks. On the cover of each bluebook, beside "SUBJECT", write the number of the problem answered. The exam lasts three hours.

1. Consider an economy with two competitive producers, indexed by $i = 1, 2$, two produced goods, $j = 1, 2$, and two primary inputs. The output of good $j$ by producer $i$ is $q_{ij} = t_{ij} K_{ij}^{\alpha_j} L_{ij}^{1-\alpha_j}$ where $K_{ij}$ and $L_{ij}$ are capital and labor inputs used by $i$ in the production of good $j$ and where $1 > \alpha_j > 0$ and $t_{ij} > 0$ for $i, j = 1, 2$. Producer $i$ wholly owns her business and has strictly positive endowments $\bar{K}_i$ of capital and $\bar{L}_i$ of labor. The factor prices of capital and labor are $r$ and $w$ respectively. The price of good $j$ is $p_j$, $j = 1, 2$. a) Derive the cost function $c_{ij}$ for the production of good $j$ by producer $i$. b) Find the demand for inputs of capital and labor by producer $i$ in the production of one unit of good $j$ (the demand for inputs as functions of $r$ and $w$). c) Suppose that each producer $i$ is also a competitive consumer with the utility function $u_i = x_{1i}^{1/2} x_{2i}^{1/2}$, $i = 1, 2$, where $x_{ji}$ is the consumption of good $j$ by consumer $i$. (Consumer $i$ owns firm $i$.) Find the competitive (Walrasian) demand function of consumer $i$ when the consumer's wealth is $W_i > 0$. d) Find the set of profit maximizing output level(s) of good $j$ by producer $i$ at the prices $p_j$, $r$ and $w$. Show that if there is a unique profit maximizing output level then it must be 0. e) Suppose that $\bar{L}_1 + \bar{L}_2 = 1$, $\bar{K}_1 + \bar{K}_2 = 1$, $\alpha_1 = 1/2$, $\alpha_2 = 1/4$, $t_{12} = t_{21} = t_{22} = 1$, $t_{11} = 2$. The wealth of each consumer comes from the profit of the consumer's firm and from the sale of the consumer's endowments. Find the competitive equilibrium output of good 1 by producer 2. Find an expression for the equilibrium price of good 1 as a function of the equilibrium values of $r$ and $w$. Find an expression for the total amount of good 1 produced in equilibrium in terms of $r$ and $w$ and exogenous parameters. Find an expression for the total amount of capital used in the production of good 1 in terms of $r$ and $w$ and exogenous parameters. Do the same for the total amount of capital used in the production of good 2. Find the competitive equilibrium factor price ratio $r/w$. Explain why all of these equilibrium values can be computed without knowing the distribution of the consumers' endowments. Is a competitive equilibrium allocation of this economy Pareto optimal? f) Suppose instead that $t_{1j} = t_{2j} = 1$ for each good $j$ and that $\bar{L}_i = \bar{K}_i = 1/2$ for $i = 1, 2$, and that. Suppose that there is a competitive market for capital but none for labor. The producers' demands for labor are the same as their endowments of labor. Is an equilibrium of this economy Pareto optimal? Explain.

2. Consider the class of two-player noncooperative extensive form games with perfect information in which (1) each player has only one turn to move, (2) each player has only two possible choices of moves, and (3) the second player has the opportunity to move after either move chosen by the first player. In answering the following, define clearly any notation you introduce. a) Specify the set of pure strategies for the player who moves second. b) Define a pure strategy *Nash equilibrium* (NE) for games in the above class. c) Define a pure strategy *subgame perfect Nash equilibrium* (SPNE) for games in the above class. d) Is it possible that there is more than one SPNE for such games? e) One reason for introducing SPNE was to rule out NE containing threats that are not credible. Give an example of a game from the above class in which a player chooses a noncredible threat as part of a NE, but does not do so in any SPNE. Show that your answer is correct. f) Is the concept of SPNE needed to rule out noncredible promises of cooperation in NE of the above class of games? In answering this question, first explain what strategies might naturally be interpreted as noncredible promises of cooperation. Then determine whether there is a NE for a game in the above class that contains a noncredible promise of cooperation and is not a SPNE. Give an intuitive explain why such a NE does or does not exist and give an intuitive explanation for this result.

3. A risk-neutral owner of a firm needs to hire a manager to run it. The profit of the firm is publicly observable, but the owner cannot observe the effort level of the manager. The manager is capable of providing any effort level $e \in \mathbb{R}_+$. If she chooses effort level $e$ and is paid a wage $w$, her utility will be $w - (e^2/2)$, and the profit of the firm (prior to any wage payment to the manager) will be $\pi = e + \epsilon$, where $\epsilon$ is a random variable with mean 0. The manager seeks to maximize her expected utility, and her reservation expected utility level is 0. The owner only cares about the money he gets from the firm (i.e., the profit, $\pi$, minus the wage paid to the manager). a) Suppose that the owner offers the manager a contract paying the wage $w(\pi) = a\pi + b$ when the profit of the firm is $\pi$. Find values of $a$ and $b$ that are optimal for the owner subject to the requirement that the manager is willing to accept the contract. b) Can the owner obtain higher expected utility by offering a contract with a nonlinear wage function $w(\pi)$? c) Find the expected utilities of the owner and manager and the level of effort chosen by the manager under the optimal contract in part a). Compare the values of these variables to what they would be under an optimal contract for the owner if the manager's effort level were publicly observable. d) Suppose that the owner and the manager both must pay the fraction $t$ of their earnings in income tax. What is the effect of the tax on the optimal values of $a$ and $b$ in part a) and on the effort level chosen by the manager under the optimal contract when the owner cannot observe the manager's effort level? (Assume that the manager's reservation expected utility is still 0.) Interpret your answer and give an intuitive explanation for it.

4. Consider an industry with an incumbent firm and a potential entrant. The inverse demand function for the industry is $p(q) = 10 - q$. Both firms have marginal cost equal to 1 and seek to maximize their expected profit. The incumbent has a fixed output capacity (maximum possible output) of $\bar{q}$ that equals either 2 or 4. The incumbent knows its value, but the entrant initially believes that the two possible values each have probability 1/2. If the entrant enters, it must pay an entry cost of 10, but it learns the capacity of the incumbent. Then the two firms play a Cournot duopoly game, choosing their output levels simultaneously, with the incumbent's output constrained by its capacity, but the entrant's output unrestricted. If the entrant does not enter, its profit is 0 and the incumbent receives the monopoly profit, given its output capacity constraint. All of the above information is common knowledge for the two firms. a) Show that it is possible for the demand for the industry output to be treated as if it were the demand of a single competitive consumer. Find a possible utility function for this consumer (a function of money and the output of the industry). b) Draw a game tree for the extensive form game played by the two firms. c) Show that the entrant chooses to enter. d) Show that the entrant would choose not to enter if it knew before entering that $\bar{q} = 4$. e) Suppose that before the entrant decides whether to enter, the incumbent can advertise at a cost of $c$. Assume first that the entrant can see if the incumbent advertises, but the advertising has no other effect. Is there any value of $c$ such that the new game has a "separating" weak perfect Bayesian equilibrium (WPBE), i.e., a WPBE in which the incumbent chooses to advertise if and only if its capacity is 4 and the entrant enters if and only if the incumbent does not advertise? f) Answer the question in part e) assuming that advertising changes the industry inverse demand function to $10 - aq$ with $0 < a < 1$. Interpret the conclusion.