

University at Albany, State University of New York, Department of Economics  
Ph.D. Preliminary Examination in Microeconomics, June 13, 2005

Instructions: Answer three of the following four numbered questions. Whenever possible, justify your answers. Write your answer to each question in a separate bluebook. Write the number of the question on the cover of the bluebook. DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 3 hours.

1. Two farmers, 1 and 2, grow rice at opposite ends of an island. Every year, rain falls on one end of the island, and not on the other (at different ends different years). At the beginning of any given year, meteorologists say that the rain is equally likely to fall on either end of the island that year. If rain falls at one end, the farm there harvests 3 units of rice and the other farm harvests 1 unit. The farmers consume only rice and need rice to survive. They are expected utility maximizers, obtaining utility  $\ln x$  by consuming  $x > 0$  units in a year. At the beginning of one year, one of the farmers proposes that they sign a contract according to which, whoever has the larger harvest will give some to the other.

(a) What notation can be used to describe the contingent consumption resulting from such a contract? In the rest of the problem, assume that the farmers are able to commit themselves to such contracts.

(b) Find all the Pareto efficient allocations of contingent consumption for the two farmers that can be attained through the contracts of part (a). Describe the Pareto efficient contracts in English. Draw the allocations in an Edgeworth box, including indifference curves of the farmers.

(c) Suppose that the farmers both believe that rain will fall with probability  $1/2$  on their side of the island and with probability  $1/2$  on the other side. They sign a contract that yields contingent consumption arising in a competitive equilibrium. What can be said about the equilibrium prices and allocation? Be as specific as possible. Is the allocation Pareto efficient? Justify your answers.

(d) Suppose instead that both farmers are optimists. They both think that the rain will fall on their side of the island with probability  $\pi \in (1/2, 1)$ . Compare the possible outcome(s) of competitive trading of contingent consumption to the outcome in part (c). Be as specific as possible with the given information. What can be said about the efficiency of a competitive equilibrium allocation in this case? Does it matter that the farmers' beliefs cannot both be correct?

(e) Suppose now that farmer 1 is an optimist, but farmer 2 is a pessimist. Both believe the probability to be  $\pi \in (1/2, 1)$  that the rain will fall on farmer 1's farm. Compare the outcome(s) of competitive trading of contingent consumption to the outcomes in parts (c) and (d) and explain the comparisons. Be as specific as possible with the given information. What can be said about the efficiency of a competitive equilibrium allocation in this case? Can it be advantageous to have unrealistic beliefs in this situation? Explain.

2. Consider an economy with  $I$  rational agents, each endowed with one unit of time. Agents contribute labor to the production of a single consumption good,  $y$ , which is produced according to the technology  $y = f(L_1, \dots, L_I)$ , where  $L_i$  denotes the amount of labor time contributed by agent  $i$ . In addition to contributing time to the productive activity, agents can contribute to a nonproductive, *conflict* activity. If each agent  $i$  devotes  $c_i$  units of time to conflict,  $i = 1, \dots, I$ , then agent  $i$  receives the share  $\theta_i(c_1, \dots, c_I)$  of the total output of the consumption good, where  $\sum_i \theta_i(c_1, \dots, c_I) = 1$ . The agents choose their pairs  $(L_i, c_i)$  independently.

- (a) Assume first that the agents care only about their consumption of the produced good. Set up the maximization problem that determines the optimal time allocation for agent  $i$ , given the choices made by the other agents.
- (b) State sufficient conditions on  $f$  and  $\theta$  that ensure the existence of a solution to the time allocation problem in part (a). Try to make the conditions as weak as possible.
- (c) Find what you expect to be all the agents' time allocations for the case in which

$$f(L_1, \dots, L_I) = \sum_i L_i \quad \text{and} \quad \theta_i(c_1, \dots, c_I) = \frac{c_i}{\sum_h c_h}, \forall i,$$

being as specific as possible with the given information. Justify your answer.

- (d) How would the equilibrium allocation of time between the two activities change as  $I$  increases? Comment on the efficiency properties of the outcome.

(e) Next, assume that the agents also care about leisure and, in particular, that they have preferences represented by  $u(y_i, \ell_i) = y_i \ell_i$ , where  $y_i$  denotes  $i$ 's consumption of the produced good and  $\ell_i$  denotes  $i$ 's leisure. As above, the share of aggregate consumption received by agent  $i$  is  $\theta_i(c_1, \dots, c_I)$ , where  $c_i$  is the time  $i$  devotes to the conflict activity. Set up the individual optimization problem that will determine the agents' optimal allocation of time.

(f) Are the conditions described in part (b) still sufficient to ensure the existence of a solution to the optimization problem in (e)? Explain.

(g) Again for the functions  $f$  and  $\theta_i$  specified in part (c), determine an equilibrium and study its behavior as the population size varies.

(h) Assume that the agents care about leisure, as in part (e). Describe an alternative way of sharing the total output that yields a more efficient time allocation than the allocation in (g). Try to make your alternative sharing rule as efficient as possible. Find the resulting time allocation under your sharing rule and compare it to the outcome in part (g), assuming  $f(L_1, \dots, L_I) = \sum_i L_i$ .

3. A continuum of consumers of measure 1 is uniformly distributed along a street of length 1. Two stores, 1 and 2, selling the same good are located at the two ends of the street respectively. Each store can acquire one unit of the good at constant marginal cost  $c$ . Consumers incur travelling cost  $t$  per unit distance of travel. Assume  $c > t > 0$ . Each consumer needs to purchase one unit of the good and will buy the good from a store where the consumer's total cost (price plus transportation cost) is the lowest. The stores simultaneously post nonnegative prices.

(a) Formulate the interaction described above as a normal form game played by the two stores, i.e. specify strategy sets and payoff functions for the stores.

(b) When the store  $i$  sets the price  $p_i$ ,  $i = 1, 2$ , which consumers buy from store 1?

(c) Find the best response correspondences of the two stores.

(d) Is there a dominant strategy for either player? Justify your answer.

(e) Show that the strategies in set  $[0, t)$  are strictly dominated for both players.

(f) What is the result of Iterated Deletion of Strictly Dominated Strategies in the limit as the rounds of deletion go to infinity? Justify your answer.

(g) Find all pure-strategy Nash equilibria. Justify your answer.

4. Firm B has patented a process for making a new drug and wants to sell its patent to a single drug manufacturer. Every drug manufacturer knows that if it is given the patent for free, it could produce the drug and make a net profit of either  $P_1$  or  $P_2$  ( $0 < P_1 < P_2$ ) depending on the complexity of the production process ( $P_1$  if the process is more complex). Firm B knows the complexity of the production process, but the drug manufacturers initially do not. Each manufacturer initially believes that the process would be less complex (and the net profit would be  $P_2$ ) with probability  $\lambda \in (0, 1)$ . (Firm B would want to tell them if the process is less complex, but would not necessarily be believed.)

By spending  $x$  on related research and legal fees, firm B can get more patents related to its new drug and can gain net profit of either  $\pi_1(x)$  or  $\pi_2(x)$  (possibly negative). The net profit will be  $\pi_1(x)$  if the production process for the new drug is more complex. Each  $\pi_i$  is strictly concave, with  $\pi_i(0) = 0$ ,  $\pi'_i(0) > 0$ ,  $i = 1, 2$ ,  $\pi'_2(x) > \pi'_1(x), \forall x > 0$ , and with  $\pi'_1(1) = 0$  and  $\pi'_2(2) = 0$ .

Firm B chooses its expenditure  $x$  and offers its patent for sale. Drug manufacturers, knowing the information above and knowing  $x$  chosen by B, bid independently for the patent, and B sells it to a highest bidder. The firms are rational expected profit maximizers, and all the above information is common knowledge.

(a) Explain briefly why firm B might want to sell its patent to only *one* firm instead of allowing several firms to pay to use the patent.

(b) Interpret the restrictions on the functions  $\pi_i$  above.

(c) Define a perfect Bayesian equilibrium (PBE) for the game played by the firms.

(d) The model above is most similar to which one of the asymmetric information models in *Microeconomic Theory*, by Mas-Colell, et. al. (the competitive labor market model with hidden productivities; the signaling model with hidden productivities; the screening model with hidden productivities; the principal-agent model with hidden effort; the principal-agent model with hidden preferences)? Explain briefly which variables in the model above correspond to which variables in the model in Mas-Colell, et. al.

(e) Explain which pooling and/or separating outcomes can arise in pure strategy PBE in the game above. Justify the claim that these are PBE outcomes. Illustrate them in one or more graphs, with  $x$  on the horizontal axis. Label your graph(s) clearly and explain what the important points in the graph(s) represent.

(f) Under what, if any, conditions is there a Pareto efficient pure strategy PBE outcome? Explain.

(g) Which pure strategy PBE outcome is most plausible? Explain.

(h) Under what conditions is there a constrained efficient pure strategy PBE outcome in this model? (In such an outcome it is impossible for a planner, using only the information available to the drug manufacturers, to obtain a Pareto improvement by having some firm(s) change strategies.) Explain.

(i) Could firm B do better by setting the price of its patent instead of letting the drug manufacturers bid for it? Explain.