Partial Answers to Micro Theory Comprehensive of Fall 1999

1(c) The question does not say if the transportation cost is for moving the goods or the consumers or both. One possibility is that the consumers do not move and M delivers x from A to B and y from B to A. If M starts at location 0 then half of the amount of x delivered to B disappears and M keeps the fraction γ of what remains. Also, half of the amount of y delivered to A disappears and M keeps the fraction γ of what remains. An efficient allocation maximizes $x(1-y_B)(1-\gamma)/2$ subject to $(1/2)(1-x)(1-\gamma)y_B^2 \geq u_2$ and $\gamma(x+y_B)/2 \geq u_M$, where x is the amount of good x consumed by A and y_B is the amount of good y consumed by B. An alternative assumption is that the only transportation cost is for moving the goods. Then it is efficient for the consumers to meet at location t while M moves the entire endowment of each good at a cost equal to half the distance traveled. M delivers $1-(t/2)$ units of x and $(1+t)/2$ units of y. M gets utility $3\gamma/2$. An efficient allocation maximizes xy subject to $([1-(t/2)](1-\gamma)-x)[(1/2)(1+t)(1-\gamma)-y]^2 \geq u_2$ since the fraction $1-\gamma$ of what is delivered of each good is available for the two consumers. Differentiating the Lagrange function with respect to t, x and y and rearranging terms as in 1(a) we obtain $x=y$ and $2x_B = y_B$ and $x=(3/2)(1-t)(1-\gamma)$. The requirement $y_B \geq 0$ implies $(1/2)(1+t)(1-\gamma)-y \geq 0$, so with $x=y$ we have $1/2 \leq t \leq 1$. The new technology adds new efficient locations closer to B. Assume that γ is small enough so that the total amount delivered is at least as great as in part (a). If the location is $t \in [1/2, 2/3]$ then both consumers gain. If the location is closer to B then B gains and A could get lower utility than in (a).

2. Part (a) is a special case of (b), so consider (b). Smith maxes $p(a)(\sqrt{60+R-q}-a)+(1-p(a))(\sqrt{100-q}-a)$. The first order condition is $\gamma p'(a)-1 \leq 0$ with equality of the optimal a is positive, where $\gamma = \sqrt{60+R-q}-\sqrt{100-q}$. Since $p'(a) = -1/(8\sqrt{a})$, the optimal a, denoted $a^*(R,q)$, satisfies $\sqrt{a} = -\gamma/8$ if this term is positive. The term is nonpositive if $R \geq 40$, so in that case the optimal a is 0. If $R=q=0$ then the optimal a is $a_0 \equiv (10-\sqrt{60})^2/64$. If Smith buys full or more than full coverage, then the optimal effort a is 0. When $R < 40$, $\gamma < 0$ and the optimal effort is positive. In that case, a rise in coverage R reduces $-\gamma$ and the optimal a. Since $\partial(-\gamma)/\partial q > 0$, a rise in premium q raises the optimal effort. (c) If the insurance company knows Smith's utility function then defining $p^*(R,q) \equiv p(a^*(R,q))$, the company chooses (R,q) to maximize its expected net revenue $q - p(a^*(R,q))R$ subject to the requirement that Smith gets at least his reservation expected utility:

$$p^*(R,q)(\sqrt{60+R-q}-a^*(R,q))+(1-p^*(R,q))(\sqrt{100-q}-a^*(R,q)) \geq p(a_0)(\sqrt{60}-a_0)+(1-p(a_0))(10-a_0).$$

3. (a) The firms have increasing returns to scale since multiplying the input by the factor $t > 1$ multiplies the output by the factor $t^2 > t$. (b) Assuming that the consumption set for each consumer is $\mathbb{R}^2_+$, an efficient allocation (with consumption x^i_j of good j by consumer i and input L_i used by firm i owned by i) maximizes u_1 subject to $u_1 \leq x^1_1$, $u_1 \leq x^1_2$, $x^2_1 \geq u_2$, $x^2_2 \geq u_2$, $x^1_2 + x^2_2 + L_1 + L_2 \leq 2$ and $x^1_1 + x^2_1 \leq L_1^2 + L_2^2$ for some $u_2 \geq 0$. From the first order conditions or by inspection, we see that the allocation is inefficient unless $x^i_1 = x^i_2$ for each i. Otherwise is possible to reduce the larger term and increase the smaller by changing the total labor input and output. The first order conditions also imply that if $L_i > 0$ for each i then $L_1 = L_2$. But then the total output is $2L_1^2$, whereas the output is $(2L_1)^2$ if the same total input is used but only firm 1 produces. This shows that at most one firm produces. (The same conclusion can be obtained by considering the second order conditions for the above maximization problem when the firms produce the same amount.) Let L be the input level and let y be the total consumption of rice. From above, total leisure consumption is y, so $y+L=2$ and $y=L^2$, which imply $y=L=1$. By varying u_2 in the maximization problem we see that if a single firm produces one unit of rice, then every allocation with $x^i_1 = x^i_2$ for $i=1,2$ and $x^1_j + x^2_j = 1$ for $j=1,2$ is efficient. (c) There is no competitive equilibrium. If equilibrium exists, then the allocation is Pareto optimal since the consumers are locally nonsatiated. But the firm that produces cannot be maximizing profit at any equilibrium prices since it has increasing returns to scale. (d) The second fundamental theorem is stated in the text by Mas-Colell, et. al. The theorem requires that the firms have convex production sets, so it does not apply to the economy in this problem. (e) If Firm 1 produces L^2 units of output at a cost of L, it sets its output price p equal to its average cost L/L^2. Both firms make 0 profit. Consumer i maxes utility

subject to the budget constraint $px_1^i + x_2^i \leq 1$. The choice is $x_1^i = x_2^i = 1/(p+1)$ and supply equals demand for labor when $(2/(p+1)) + L = 2$. Using $p = 1/L$, we find that the equilibrium input is $L = 1$, and the allocation is efficient. Normally when price is above marginal cost the output is inefficiently low. In this case, the consumers do not substitute away from rice since they have kinked indifference curves.

4. (b) A pure strategy for manager 2 is a list of moves, one for every information set at which manager 2 moves. There are three such information sets and manager 2 has two choices in each, so there are $2^3 = 8$ pure strategies for 2. An example is (AB, BA, CB), where AB means that if manager 1 plays A, manager 2 plays B. (c) The responses in the strategy above are the only optimal ones for 2, therefore it is the only SPNE strategy for 2. The best response to that strategy by manager 1 is B, so $(B, (AB, BA, CB))$ is the unique SPNE. (d) Yes. If 1 plays A then 2 cannot do better than with the strategy (AB, BC, CB); and against that strategy 1 cannot do better than to play A; so that is a NE. (e) C cannot be selected in NE. It is not part of a best response strategy for 2 after either A or B played by 1. (f) The NE in (d) arises because 2 commits to a choice that is suboptimal for 2 if 1 plays B. Without commitment, after 1 plays B there is nothing to prevent 2 from reacting optimally by playing A. The equilibrium outcome depends on whether or not commitment is possible. The SPNE outcome differs from the NE in (d) only in distribution, not in efficiency, but the total payoff could be changed by raising or lowering the payoff to 2 at node AB. In that way, the total payoff could be higher at the SPNE or instead at the other NE. (g) The answer depends on whether or not commitment is possible. If it is then we could expect the NE of part (d) to be played when 2 plays second, and 2 would be worse off playing first. If commitment is not possible then the SPNE would be played and 2 would be better off playing first. (h) There are now two pure SPNE's: 1 plays A or C, and 2 plays (AB, BC, CB) in both SPNE's. There are also SPNE's where 1 randomizes between A and C. The answers to (d) and (e) do not change. The payoff to 2 does not depend on whether or not 1 can commit. If 2 plays first without commitment, then 1 plays (AB, BA, CA) and 2 plays A and does no better than when playing second. When θ changes, the SPNE outcome changes because now 2 can credibly threaten to choose C after 1 plays B. The interesting thing is that without commitment, a change in 2's preference for C (a project that cannot be selected in any NE) changes the equilibrium outcome.