STATE UNIVERSITY OF NEW YORK AT ALBANY

Department of Economics

Ph.D. Comprehensive Examination in Microeconomics

September 4, 1992

Answer either three or four of the following five numbered problems. Justify your answers and, whenever possible, show why they are correct. Partial credit will be awarded in proportion to the difficulty of the parts of the problems you solve. You will get more credit by answering all parts of a single numbered problem than by answering half of each of two problems. Write each numbered problem answer in a separate bluebook. On the cover of each bluebook, beside "SUBJECT", write the number of the problem answered. Beside "NAME" write the identification number distributed with your exam, but **DO NOT WRITE YOUR NAME**. The exam lasts three hours.

1. In a competitive economy the aggregate production function is $y = L$, where L is the input of labor and y is the output of food. The total endowment of labor is $\bar{L} > 0$, held by consumers. There is no endowment of food. The welfare function of the economy is $u(L, y) = (\bar{L} - L)^{1/2} y^{1/2}$ for $\bar{L} - L \geq 0$. For simplicity, assume that there is a single consumer in the economy and that that consumer's utility function is the above welfare function. The wage rate of labor is fixed at 1. However the price q that the consumer pays for food is different from the price p that food producers receive. The difference between p and q is a unit tax $t = q - p$.

a) For what values of p do profit maximizing choices by producers exist? What are these choices if they exist?
b) Suppose for simplicity that profit (if there is any) and tax revenue (the difference between what the consumer pays for food and what the producers receive) are returned to the consumer in the form of a lump sum transfer. This means that the consumer treats the transfer as fixed (unaffected by the consumer's actions) and chooses optimal consumption given the food price q. What is the consumer's budget constraint, given that the total food output is $\bar{y}$?
c) What is the consumer's optimal choice of (L, y) at the prices $(1, q)$ in the budget set of part b?
d) Given t, define (p, q) to be a *distortion equilibrium* for t if $t = q - p$ and if the profit maximizing output of the producers leads to a consumption choice as in part c such that excess demands for both goods are zero. What is the consumer's choice of (L, y) at a distortion equilibrium for t?
e) If $t \neq 0$ the corresponding distortion equilibrium cannot be Pareto optimal unless the equilibrium is at a "corner". Why not?
f) Suppose that t is reduced slightly, starting from a positive level. What is the effect on the consumer's welfare in the distortion equilibrium for t?

2. Public University of New York (PUNY) offers a two-year Ph.D. program in Economics in which all graduate students are fully supported as TAs or RAs. The PUNY faculty controls the number of graduate students by determining (i) the number of first-year students admitted each year and (ii) the number of first-year students allowed to continue through the second year. The policy instrument that determines how many students continue is called a comprehensive exam.

Each year the PUNY Economics Department needs graduate students to fill at least 25 TA positions and at least 40 RA positions. A collective bargaining agreement between the University and the PUNY Graduate Student Association establishes that (1) students have the right to choose whether they will be TAs or RAs and (2) second-year students cannot be brought in from outside the Department. Exactly one-third of the first-year students choose to be TAs; the rest become RAs. Second-year students have different preferences: one-half become TAs and one- half become RAs. Student wages are independent of job classification: whether they are TAs or RAs, first-year students earn \$6500 and second-year students earn \$7000.

The PUNY Economics Faculty wants to determine the number of entering students and the pass rate of the comprehensive exam which minimize the total amount paid to students.

a) Set up the appropriate optimization problem. Clearly identify the constraints and variables. Draw the constraint set.
b) What is the optimal pass rate on the comprehensive exam?
c) What would be the optimal pass rate if students could be assigned jobs without regard for their preference?

3. A consumer has the utility function $u(x, \ell) = \alpha \, ln \, x + (1 - \alpha) \, ln \, \ell$, where x is the number of bottles of soda consumed per day and ℓ is the number of hours of leisure consumed, with $0 < \alpha < 1$. The price of a bottle of soda is \$1. In an effort to curb pollution, the state passes a law requiring that a deposit of $\$t$ be placed on each bottle of soda purchased. This deposit is to be returned to the consumer only if the bottle is returned. Suppose that it costs the consumer s hours of leisure to return one bottle of soda. The daily income of the consumer is $\$I$. To obtain this income the consumer works a fixed 8 hour day, leaving 16 hours to be divided between leisure and returning bottles.

a) Let y be the number of bottles purchased and returned and let z be the number of bottles purchased but not returned, so that $x = y + z$. Set up the consumer's maximization problem and solve for the optimal z. Be careful to consider possible corner solutions.
b) Could it ever be the case that an increase in the deposit t increases the consumer's choice of z?
c) What happens to the number of bottles of soda consumed and the number of bottles purchased and returned if the deposit t is increased?

4. A single parent, Smith, has an only child named Liz. Smith cares for Liz deeply, but Liz only cares about herself. Smith's utility is $U = (W - T)^{1/2} + u$, where u is Liz's utility, given by $u = (w + T)^{1/2}$. Here, W and w ($w < W$) denote the wealth of Smith and Liz, respectively, and T denotes the tranfer (gift) that Smith gives to Liz. Smith and Liz both know both utility functions.

a) Derive the optimal transfer from Smith's point of view, as a function of W and w.

Suppose now that Liz can spend money on a vacation prior to Smith choosing the transfer. If Liz spends V on the vacation and receives the transfer T later, her utility will be $u = V + (w - V + T)^{1/2}$.

b) Find the first order condition necessarily satisfied by the transfer chosen by Smith after Liz has chosen to spend V on her vacation.

c) Find the first order condition necessarily satisfied by Liz's choice of V. Liz knows that after she makes her choice, Smith will choose T to maximize his utility given Liz's choice of V.

d) Assume that Liz's choice of V in part c), denoted V^*, satisfies $0 < V^* < w$. Is V^* efficient from the point of view of the family? Discuss why or why not, explaining carefully what is meant by efficiency in this case. You need not compute V^* in answering this question.

e) Suppose that Smith chooses T and Liz chooses V as their strategies in a noncooperative game. Describe the Nash equilibria of this game, being as specific as possible. You need not compute them. Are they efficient? What can be said about a Nash equilibrium value of V compared to V^* obtained in part d)?

f) Suppose Smith could make a binding commitment concerning his choice of T *prior* to Liz's choice of V. Is the resulting equilibrium efficient? Compare it to the equilibria described in parts d) and e).

5. In a pure exchange economy with two goods, x and y, there are two consumers, A and B, with utility functions $U_A(x_A, y_A) = x_A y_A$ and $U_B(x_B, y_B) = x_B + ln\ y_B$, respectively, and with endowments $W_A = (1/2, 1/3)$ and $W_B = (1/2, 2/3)$, respectively.

a) Define and compute the core of this economy, and draw the core and the initial endowment point in an Edgeworth box.

b) Which points in the core in part a) are competitive allocations? What are the associated price vectors?

c) Suppose that the economy is the same as above except that the utility function of A is $U_A(x_A, y_A) = x_A y_A + x_A - x_A y_B$. Compare the core of this economy to the core found in part a) and explain the comparison. Hint: use the feasibility constraints to compare the utility of A here to the utility in part a).

d) Suppose that in the economy of part c) each consumer behaves competitively taking the demand of the other consumer as fixed. Compare the resulting competitive equilibrium allocation(s) with the core in part c). Explain the comparison.