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Ph.D. Preliminary Examination in Microeconomics, September 19, 2003

Instructions Answer 3 of the following 4 questions. Show all of your work. Write your answer to each question in a separate bluebook. On the cover of the bluebook, write the number of the question under "Section." DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 3 hours.

[1] A buyer and a seller would like to trade an object. Before trading, the buyer can make some investment that increases his value of the product. The investment can not be observed by the seller, and it does not change the seller's reservation value of the product, which is normalized to zero. If the buyer invests $I \geq 0$, then the buyer's value becomes $v + I$, where $v > 0$ is the initial value without investment. The cost of investment is $I^2/2$. The timing of the game is as follows. First, the buyer chooses investment level I. Second, the seller offers a fixed price p for the product. Finally, the buyer chooses to accept or reject the offer: if the buyer accepts, then the buyer's payoff is $v + I - \frac{I^2}{2} - p$ and the seller's payoff is p; if the buyer rejects, then the buyer's payoff is $-\frac{I^2}{2}$ and the seller's payoff is 0. You may assume that the buyer always accepts an offer when it is indifferent.
(a) Define a subgame perfect equilibrium (SPE) for this game.
(b) Does there exist a pure-strategy SPE? Explain your answer.
(c) Suppose now the buyer can only choose between two levels of investment: $I_1 > I_2 \geq 0$ with probabilities α and $1 - \alpha$ respectively. Find an optimal pricing strategy for the seller.
(d) Suppose the buyer can *only* choose between investment levels $I_1 = 1$ or $I_2 = 0$. Find a mixed strategy SPE in which the buyer randomizes between these two investment levels and the seller randomizes between two prices $\{p_1, p_2\}$. Justify your answer.

For the last two parts of problem 1, assume there is no restriction on the investment choice.
(e) Show that there exist two prices $p_1 > p_2 \geq v$ such that if the seller randomizes between these two prices with some probabilities $\beta > 0$ and $1 - \beta > 0$ respectively, then the buyer's best pure strategy response correspondence contains two values: there exist two investment levels $I_1 > I_2 > 0$ that yield the buyer (same) optimal expected payoff.
(f) Do answers in part (e) lead you to think that there is a mixed strategy SPE in which the buyer put probabilities on two investment levels and the seller randomizes between two prices $\{p_1, p_2\}$? Explain carefully whether this conjecture is correct.

[2] Consider the following pricing problem for a monopoly selling an indivisible good. There are three buyer types, and $n_i > 0$ is the fraction of buyers of type i. Each buyer consumes at most one unit of the good and obtains utility that depends on the quality of the good. A buyer of type i gets utility $\theta_i x - p$ by buying one unit of quality $x \geq 0$ at price p ($0 < \theta_1 < \theta_2 < \theta_3$). A buyer who does not buy the good gets utility 0. Let $C(x)$ be the cost of producing one unit of the good with quality $x \geq 0$. Assume that $C(0) = 0, C'(0) = 0, C'(x) > 0$ and $C''(x) > 0$ for $x > 0$ and $\lim_{x\to\infty} C'(x) = \infty$.
(a) Suppose the monopolist can directly observe buyer type and can offer contracts of the form $(p_i; x_i)$ contingent upon type (i.e. the incentive constraints are not imposed). Characterize the profit-maximizing set of contracts for the monopolist.
(b) Suppose the types are not observable to the monopolist. The monopolist offers a menu of contracts of the form $(p_i; x_i)$ where a type i contract is meant for type θ_i consumers. Write down the constrained monopolist pricing problem.
(c) Solve the relaxed problem where only the downward adjacent incentive constraints (DAIC) and participation constraint (P1) for type 1 are imposed,

$$\begin{aligned} \theta_i x_i - p_i &\geq \theta x_{i-1} - p_{i-1},\ i = 2, 3. \qquad \text{(DAIC)} \\ \theta_1 x_1 - p_1 &\geq 0 \qquad\qquad\qquad\qquad\quad \text{(P1)}. \end{aligned}$$

(d) Under what sufficient condition on the population distribution $(n_1; n_2; n_3)$ does the solution to the relaxed problem satisfy all of the incentive constraints in the original problem?

[3] Suppose an agent's state of health affects its capacity for enjoyment. Its von Neumann - Morgenstern utility function is given by $u(c, h) = c(h + 1)$, where c denotes consumption and h indicates the agents' health status. In particular, h takes on the value 0 if the agent is ill and 1 if it is healthy. Initially, these occur with probability p° and $1 - p^\circ$, respectively. The agent is endowed with ω units of the consumption good, which can either be consumed directly or can be spent on healthcare which reduces the likelihood of illness. If the agent devotes x units to healthcare, the ultimate likelihood of illness is reduced to $p(x) = \frac{p^\circ}{1+x}$.
(a) Formulate the choice problem facing the agent.
(b) Fully characterize the solution as a function of the initial probability p° and ω and discuss the role of each. In particular, argue that healthcare is *weakly normal* in that, for given p°, expenditure on healthcare is nondecreasing in ω.
(c) Suppose the agent were offered two jobs that differed in their pay, ω, and their initial risk of injury, p, as above. In particular, suppose $\omega^1 > \omega^2$ and $p^1 > p^2$, where each p would be subject to the reduction rule $p(x)$. Specify precisely the selection criterion according to which the agent would choose between the two.
(d) Returning to the original model, suppose commercial health insurance were available at the fixed premium $\pi = p^\circ$, measured in units of the consumption good, such that in return for paying πy, the consumer would receive y in the event of illness. Formulate the agent's choice problem in this case and determine the amount of insurance it would purchase. What, if any, complications arise due to the fact that the probability of illness is endogenous whereas the premium is fixed?
(e) Next consider two identical individuals, each with the utility function, u; endowment, ω; and initial likelihood p° as described above, where each of the likelihoods can be reduced according to the rule $p(x)$. Suppose the health status of the two agents is independent. In the absence of commercial insurance, is there scope for the two agents to mutually insure one another against illness? Explain.
(f) Discuss how your answers to parts (d) and (e) would change if the utility function were given instead by $u(c, h) = v(c)(h + 1)$, where $v' > 0$, $v'' < 0$.

[4] Consider an economy with two consumers, two inputs, three produced goods, and with free disposal of all goods. Consumer i initially owns $e_i > 0$ units of input i, $i = 1, 2$, and none of the other input. Consumer i has the utility function $\sqrt{x_{ii}x_{i3}}$, where x_{ij} is consumer i's consumption of produced good j, $j = 1, 2, 3$. Each consumer owns a firm capable of producing the three produced goods according to the following technology. Produced good 1 requires one unit of input 1 per unit of output. Produced good 2 requires 1/4 unit of input 1 and one unit of input 2 per unit of output. Produced good 3 requires one unit of input 1 and 2 units of input 2 per unit of output. Justify your answers to the following problems whenever possible.
(a) Describe the returns to scale and possibilities for input substitution in production in this economy.
(b) Find the cost function for production of each separate produced good.
(c) Specify one or more equations that can be used to determine competitive equilibrium prices, w_i for input i and p_j for produced good j, as functions of the endowments e_i.
(d) Under what conditions on e_1 and e_2 is there a competitive equilibrium in which w_1, the price of input 1, is 0? Under what conditions is there a competitive equilibrium in which w_2, the price of input 2, is 0?
(e) Find every competitive equilibrium price vector. Under what condition(s) on e_1 and e_2 does a competitive equilibrium exist? Under what conditions is there more than one competitive equilibrium allocation? Compare the latter conditions with the conditions under which there is a unique equilibrium allocation.
(f) Consider a competitive equilibrium in which both input prices are positive. What can be concluded about e_1 and e_2? Compare the utility level of consumer 1 in such an equilibrium to the utility level if e_1 were slightly higher (or much higher), with e_2 fixed? Be as specific as possible and interpret your answer.
(g) What can be said about the efficiency of the competitive equilibrium allocations for this economy?