

Instructions: Answer 3 of the following 4 numbered questions. Show all of your work. Write your answer to each numbered question in a separate bluebook. On the cover of the bluebook, write the number of the question under "Section." DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER ON THE BLUEBOOKS. The exam lasts 3 hours.

1. A fast food chain manufactures output, y , according to a fixed coefficient (Leontief) technology, requiring α units of labor and β units of ground beef to produce a unit of y . In addition, one unit of a manager's time is required to oversee the production process no matter how much is produced.

a. Suppose the demand for the firm's output is downward sloping and its revenue function, $R(y)$, is strictly concave. Let c denote the price of ground beef, w the wage rate, and m the cost of the manager's time. Derive the first order conditions for the optimal production/supply of y and briefly discuss the properties of the profit function $\pi(c, w, m)$.

b. Suppose the chain were to sell the franchise to the manager for an amount F . The manager would then choose y to maximize profits subject to the requirement that it must purchase all of its beef from the parent company at price p . What choice of p and F would maximize the profits of the parent company (which consist of the fee F plus profits from the sale of beef)? What will happen to the price of fast food to consumers?

c. Now suppose the requirement of labor per unit of output decreases with the amount of effort, e , the manager expends monitoring the workers ($\alpha'(e) < 0$ and $\alpha(0) = \underline{\alpha}$, for all e) and that each unit of effort costs the manager \$1. Characterize the optimal choices of output and effort on the part of the manager if the manager were to own the franchise.

d. Again assuming α varies with e as in part c, compare the price of fast food paid by the consumers if the parent company owns the store and pays the manager a fixed wage versus that in which the manager owns the store. (For the manager-owned store, assume $p = c$.)

2. Consider an L -good private ownership economy with more than one consumer and more than one firm. Let Y be the aggregate production set (the sum of the production sets of the individual firms). We call a vector *strictly positive* [respectively, *strictly negative*] if every component of the vector is strictly positive [resp., strictly negative].

a. Provide notation to describe such an economy formally, and define a competitive (Walrasian) equilibrium for the economy. Assume that in equilibrium there is no disposal of goods, so that supply equals demand for all goods.

b. For such an economy, a competitive equilibrium exists if each of the following conditions is satisfied:

(1) Every consumer has a strictly positive endowment vector and has a continuous, strictly increasing, strictly quasiconcave utility function defined on \mathbb{R}_+^L .

(2) The vector 0 is in the production set of every firm.

(3) Y is closed and convex.

(4) Y contains every strictly negative vector.

(5) Y contains no strictly positive vector.

Interpret conditions (3), (4) and (5).

c. Describe an economy in which all but one of the conditions (1) through (5) are satisfied, but no competitive equilibrium exists. You may assume that some or all of the firms are not capable of producing any output. Consider the way in which excess demands depend on prices. Illustrate your answer with a diagram. Explain in economic terms the reason for the nonexistence of equilibrium.

d. A popular textbook claims that the problem of existence of competitive equilibrium is irrelevant for economic analysis. "Clearly something is happening in the real world. The object of science is to explain these events. Dwelling on the possible nonexistence of these events is of questionable empirical value." Do you agree? Can anything useful for economic analysis be learned from existence theorem stated above? Justify your answers.

e. According to theorems of Sonnenschein, Mantel and Debreu, the competitive model is essentially indeterminate in the sense that the conditions in part b above do not imply useful restrictions on the way in which aggregate excess demands depend on prices, beyond continuity, homogeneity and Walras' Law. Explain why, despite this result, general equilibrium theory might still be useful for practical economic analysis.

3. A tourist resort has only one restaurant. The manager of the restaurant seeks to maximize its profit. A fixed fraction λ of the restaurant customers are tourists and the rest are locals ($1 > \lambda > 0$). The net utility function of a meal at price $p \geq 0$ and quality $q \geq 0$ is $u_t = q - \theta_t p$ for a tourist and $u_l = q - \theta_l p$ for a local, where $0 < \theta_t < \theta_l$.

a. Interpret this last inequality.

During the period under consideration, each customer eats one meal if the net utility is nonnegative, and no meal otherwise. The cost of producing a meal of quality q is q^2 .

b. If the restaurant could distinguish between tourists and locals and could offer them different menus, what combinations of prices and qualities would it offer its two types of customers?

Assume from now on that the restaurant cannot discriminate among its customers. In parts c through g below, assume also that the restaurant finds it profitable to serve both tourists and locals, and that it offers a menu with only two types of meals.

c. An optimal menu for the restaurant can be represented as a solution to a maximization problem. Formulate such a problem.

d. Show that at the optimum, the locals are indifferent between having a meal at the restaurant and doing without it.

e. Show that at the optimum, the tourists are indifferent between the two types of meals on the menu.

f. Show that at the optimum, tourists are charged at least as much as locals for a meal.

g. Find an optimal menu for the restaurant and compare it to the answer in part b.

h. Under what circumstances would the restaurant serve only locals or only tourists?

4. An American company (\mathcal{A}) and a Japanese company (\mathcal{J}) are planning to develop high definition televisions (HDTVs) using either digital or analog technology. The American Company first decides whether or not to develop the HDTVs at all. If the American Company does not develop them, then the Japanese company knows this and develops either an analog or a digital HDTV. The corresponding payoffs are (for \mathcal{A} , for \mathcal{J}) = $(0, 3)$ if \mathcal{J} develops analog, and (for \mathcal{A} , for \mathcal{J}) = $(0, 4)$ otherwise. If the American Company decides to develop HDTV, then so does the Japanese company, and the companies simultaneously choose either digital or analog technology. In that case, let μ be \mathcal{J} 's belief of the probability that \mathcal{A} chooses digital HDTV. Company \mathcal{J} 's behavioral strategies corresponding to (digital, analog) are denoted by $(\sigma_D, 1 - \sigma_D)$. Company \mathcal{A} 's behavioral strategies corresponding to (not develop, digital, analog) are denoted by $(\alpha_1, \alpha_2, \alpha_3)$. The payoffs (\mathcal{A} 's first) are $(3, -1)$ if \mathcal{A} chooses digital and \mathcal{J} chooses analog, and the payoffs are $(-1, 3)$ if \mathcal{A} chooses analog and \mathcal{J} chooses digital. Both companies get payoff 3 if both choose digital, and both get payoff 2 if both choose analog. The companies are risk-neutral.

a. Draw a game tree for the game played by \mathcal{A} and \mathcal{J} .

b. List all the subgames of the above game. What are the pure strategies of \mathcal{J} ?

c. Find optimal choice(s) of σ_D by Company \mathcal{J} given its belief μ .

d. Find optimal choice(s) of $(\alpha_1, \alpha_2, \alpha_3)$ by Company \mathcal{A} as a reaction to σ_D .

e. Find all Nash equilibria of the above game corresponding to beliefs μ such that $\mu = \alpha_2 / (\alpha_2 + \alpha_3)$ if $\alpha_2 + \alpha_3 > 0$. What is the significance of this restriction on μ ?