

STATE UNIVERSITY OF NEW YORK AT ALBANY
Department of Economics

Ph. D. Preliminary Examination: Macroeconomics
Spring, 2007

Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. Risk-neutral agents do not save.
2. Models that allow for sunspot equilibria typically contain production functions with unrealistically high returns to scale.
3. Price inflation is bad.
4. The fact that real wages are procyclical indicates that technology shocks are the primary source of business cycles.
5. Capital taxation leads to worse economic outcomes (e.g. output *per capita*, employment) than raising the same revenue through income taxes.
6. An increase in current income will always lead to an increase in current consumption.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. (Following Christiano and Eichenbaum, 1992) Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (\text{PRF})$$

where: Π_t is profit; L_t is labor; W_t is the real wage; and Y_t is output. The population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, C_t , government spending, G_t , and labor are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t [\ln(C_t + \gamma G_t) - \chi L_t] \right),$$

$$0 < \beta < 1, \quad 0 < \gamma < 1, \quad \chi > 0.$$

Households receive labor income and profits from firms. They pay lump-sum taxes, H_t to the government. Households earn a gross return of $(1+r)K_t$ on their assets, K_t , with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1} , are chosen in period t . Note that in this economy, capital is used only as a storage device. Households also face the usual non-negativity and No-Ponzi-Game conditions.

The government's flow budget constraint is given by

$$H_t = G_t,$$

where H_t denotes total lump-sum taxes. Taxes are driven by government spending, which follows an AR(1) process around the log of its steady state value:

$$\hat{g}_t \equiv \ln(G_t/G_{ss}) = \phi \hat{g}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \quad (\text{TS})$$

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process, and $G_{ss} > 0$ is steady state government spending. (You can also assume that $G_t < Y_t, \forall t$.) Note that government spending has no effect on the private production function. However, government spending can substitute for private consumption at the rate γ .

- (a) Write down the social planner's problem for this economy as a dynamic programming problem. (The planner takes G_t as given.) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.

- (b) Let lower-case letters with carats “ $\hat{}$ ” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\begin{aligned}\hat{\ell}_t &= -\frac{1}{\alpha} [\theta \hat{c}_t + \lambda \hat{g}_t], \\ \hat{y}_t &= -\frac{1-\alpha}{\alpha} [\theta \hat{c}_t + \lambda \hat{g}_t].\end{aligned}$$

Show that $\lambda, \theta \in (0, 1)$. Why is λ positive? Provide an intuitive justification.

- (c) Suppose that $C_{ss}/K_{ss} = \psi$, while $G_{ss}/K_{ss} = \pi$, with $\psi + \pi > r$, which implies that Y_{ss}/K_{ss} is $\psi + \pi - r$. Using this result, log-linearize the capital accumulation equation to show that

$$\hat{k}_{t+1} = (1+r)\hat{k}_t - \omega_1 \hat{g}_t - \omega_2 \hat{c}_t, \quad \omega_1, \omega_2 > 0.$$

- (d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS)) to express consumption as a function of capital and government spending:

$$\hat{c}_t = \eta \hat{k}_t - \mu \hat{g}_t, \quad (\text{CF})$$

with

$$\eta = \frac{r}{\omega_2} > 0, \quad \mu = \frac{1}{\theta(1-\phi+r)} \left[\lambda(1-\phi) + \theta r \frac{\omega_1}{\omega_2} \right] > 0.$$

(Take all of this as given.)

1. Express $\hat{\ell}_t$ as a function of capital and government spending.
2. It can be shown that $\lim_{\gamma \rightarrow 1} \mu = \frac{\lambda}{\theta}$. (Take this as given.) In the limit, as $\gamma \rightarrow 1$, what is the effect of government spending on labor?
3. Intuitively, as $\gamma \rightarrow 1$, how should government spending affect labor? Why?

8. Search with unemployment benefits that expire

Time: Discrete, infinite horizon.

Demography: Single worker who lives forever.

Preferences: The worker is risk-neutral (i.e. $u(x) = x$). He discounts the future at the rate r .

Endowments:

- *When unemployed:* Each period of unemployment, with probability α the worker samples a wage from the continuous distribution F with support $(0, \bar{w}]$. Each period, he also receives leisure with value z . Additionally, when first unemployed he receives unemployment benefits, b , with $b + z < \bar{w}$. In each period with probability γ the benefits expire. Once benefits have expired there is no way to start getting them again except by getting a job. To clarify, an unemployed worker whose benefits have not expired receives z and b . In any period with probability α he gets a job offer, or with probability γ his benefits expire, or with probability $1 - \alpha - \gamma$ nothing happens. (Getting a job offer and losing benefits are exclusive events.) Once benefits have expired he continues to get z and continues to sample wages with probability α .
 - *When employed:* The worker receives his current wage and is subject to lay-off. The probability he is laid-off in any period is λ . Getting a job automatically re-entitles the worker to benefits in the event of lay-off.
- (a) Write down the relevant flow asset value equations for V_b , V_n and $V_e(w)$, the values to being unemployed with benefits, unemployed with no benefits, and employed at wage w respectively.
- (b) Let w_b^* and w_n^* be the reservation wages for the worker when he gets benefits and when he does not get benefits respectively. Given that $w_b^* > w_n^*$ show that $rV_b = w_b^*$ but that $rV_n > w_n^*$. Explain why this happens. **Note:** For this part you do **not** need to derive the reservation wage equations.
- (c) Suppose now that there are a large number of people (of mass 1) in this situation. Draw the diagram that shows the flow rates of population between states when the market is in steady state. Write down (but do not solve) a set of equations that can be used to obtain steady-state populations in each state.

9. Overlapping generations with a minimum wage

Time: Discrete, infinite horizon

Demography: A mass 1 of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people (no population growth).

Preferences: for the generations born in and after period 0;

$$U_t(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period t and stage i of life. $u(\cdot)$ is increasing, strictly concave and twice differentiable. For the initial old generation

$$\tilde{U}(c_{2,0}) = \ln(c_{2,0}).$$

Technology: $F(K, L) \equiv AK^\alpha L^{1-\alpha}$ is a neoclassical production technology which uses capital and labor to produce a consumption good. (Capital depreciates 100% in use.) Consumption goods can be consumed or they can be used as the capital input to the production.

Endowments:

- Everyone has one unit of labor services when young. When old no one can work.
 - Every one among the initial old generation has the same K_0 units of capital which they can rent out to firms. (We can think of a single collectively-owned firm that takes wages and interest rates as given. The firm will make no profits in equilibrium.)
- (a) So far this is the standard Diamond OG model (without population growth). Solve the model for the steady-state per worker capital stock, k_{ss} , and the steady state wage w_{ss} in terms of A , α and β .

Now consider an alternative institutional arrangement:

The government imposes a binding minimum wage, $\tilde{w} > w_{ss}$. With \tilde{w} in place the labor market will not clear. Instead, a proportion $e_t < 1$ of the younger generation is employed. All young people earn the same amount of money so the assignment of individuals to work is by lottery. That is, wage income is shared equally among all the young people, but the probability that a young person in generation t will be required to work is e_t . The market for capital remains competitive.

- (b) Write down and solve the generation t individual's maximization problem and solve for s_t as a function of R_{t+1} , \tilde{w} and e_t .
- (c) Solve the firm's profit maximization problem (it takes wages and interest rates as given). Solve for an expression for the interest rate, R_t , in terms of $\tilde{k} \equiv \frac{K_t}{e_t}$, the ratio of the aggregate capital stock to the employment level. (**Hint:** Whenever $e_t < 1$, \tilde{k} is fixed.)
- (d) Use market clearing in the capital market to obtain a dynamical equation for the employment level. What is the steady state value of e_t ? (**Note: Problem continues on next page.**)

- (e) From what we know about the steady-state of the laissez faire economy, what does $\tilde{w} > w_{ss}$ imply for the stability of the steady state of e_t ? What does the model predict would happen to employment if a binding minimum wage were imposed? (**Note:** $e_t > 1$ is impossible.)
10. Consider the following variant of the Lucas tree model. The preferences of the representative consumer are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [C_t^{1-\sigma} - 1] \right), \quad 0 < \beta < 1, \quad \sigma > 0.$$

where C_t denotes consumption.

Output is produced by an infinite-lived tree: each period, the tree produces d_t units of non-storable output. Output follows a non-stationary Markov process. In particular, if output at time t is d , then with probability $\alpha \in (1/2, 1)$ output at time $t+1$ will be $\gamma d/\alpha$, with $\gamma > 0$, and with probability $1-\alpha$, output at time $t+1$ will be $\gamma d/(1-\alpha)$.

Let $P_t = P_t(d_t)$ be the price at time t of all future dividends from the tree, and let $q_t(d', d)$ be the price of a one-step-ahead contingent claim that delivers one unit of fruit when $d_t = d$ and $d_{t+1} = d'$.

- (a) Consider the process for output **growth**, $G_t \equiv d_t/d_{t-1}$:
1. Show that $G_t \in \{G_L, G_H\}$ follows a stationary Markov chain.
 2. Find the conditional mean $\mu \equiv E_t(G_{t+1})$. Using this result, describe a way to calibrate γ .
- (b) Let s_t denote the number of trees that the consumer owns at the beginning of time t , and let $y(d', d_t)$ denote the “purchasing kernel” for contingent claims. Letting x_t denote the consumer’s financial resources, write down Bellman’s functional equation for the consumer’s problem, and derive the Euler equations associated with stocks and contingent claims.
- (c) Imposing equilibrium, show that

$$\begin{aligned} q_t(d_{t+1}, d_t) &= q(G_{t+1}), \\ p_t &\equiv P_t/d_t = p. \end{aligned}$$

Briefly explain the intuition behind this result.

- (d) Suppose that there is no uncertainty about output growth, so that $G_t = \mu, \forall t$. Using the fact that $p_t = p$, find a closed form solution for the stock price. Is p increasing or decreasing in μ ? Provide an intuitive explanation.