

STATE UNIVERSITY OF NEW YORK AT ALBANY
Department of Economics

Ph. D. Preliminary Examination: Macroeconomics
Spring, 2005

Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. A persistent current account deficit, perhaps one that is everlasting, is not socially optimal and should be corrected by the government.
2. A country's long-run growth rate does not depend on its income tax structure.
3. A deficit-financed expansion of government spending has no effect on aggregate output.
4. Changes in the structure of income tax can affect the distribution of (pre-tax) incomes.
5. Consumers with linear-quadratic preferences will not engage in precautionary saving.
6. Temporary increases in the government budget deficit designed to increase economic activity, i.e. "fiscal stimulus" packages, are welfare-enhancing.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider a world with T periods and J agents. There is a single perishable consumption good. All agents have the same utility function

$$u(c_1^j, c_2^j, \dots, c_T^j) = \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\alpha}}{1-\alpha}, \quad j = 1, 2, \dots, J.$$

Each agent is given a lifetime endowment of the consumption good $(y_1^j, y_2^j, \dots, y_T^j)$.

- Characterize as completely as you can, the set of Pareto optimal allocations in this economy.
 - Carefully define the Arrow-Debreu competitive equilibrium for this economy and characterize, as completely as you can, the equilibrium Arrow-Debreu prices: $\frac{q_t}{q_1}$, $t = 2, \dots, T$.
 - Carefully define the sequence of market (Radner) competitive equilibrium for this economy and characterize, as completely as you can, the equilibrium price of a one-period pure discount bond purchased in period t for $t = 1, \dots, T - 1$. What is the period- t real interest rate?
 - State the two welfare theorems. Do the equilibria in parts (b) and (c) satisfy one or both of the theorems? Explain.
8. Consider the following variant of the Lucas tree model. There are two types of trees: apple trees, which will be indexed by “A”; and banana trees, indexed by “B”. Apple trees have a constant yield, but the yield of the banana trees varies with the intensity of the “banana blight”. The amount of fruit produced by each type of tree is given by

$$\begin{aligned} d_t^A &= \frac{1}{2}d, \quad d > 0, \\ d_t^B &= d \left(\frac{1}{2} + e_t \right). \end{aligned}$$

The effect of the banana blight, e_t , follows a symmetric two-state Markov chain with the values $\{-\varepsilon, \varepsilon\}$, where $0 < \varepsilon < 1/2$. The transition density $f(e', e)$ is given by

$$\begin{aligned} f(-\varepsilon, -\varepsilon) &= \Pr(e_{t+1} = -\varepsilon | e_t = -\varepsilon) = \pi \\ &= f(\varepsilon, \varepsilon), \\ f(\varepsilon, -\varepsilon) &= \Pr(e_{t+1} = \varepsilon | e_t = -\varepsilon) = 1 - \pi \\ &= f(-\varepsilon, \varepsilon). \end{aligned}$$

The preferences of the representative consumer are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [c_t^{1-\sigma} - 1] \right), \quad 0 < \beta < 1, \quad \sigma > 0.$$

where c_t denotes total consumption of apples and bananas; agents are indifferent between the two. The economy starts off with each agent owning one apple and one banana tree apiece.

Let $p_t^i = p^i(e_t)$ be the price at time t of a title to all future dividends from a tree of type i , $i \in \{A, B\}$. Let $q(e', e)$ be the price of a one-step-ahead contingent claim that delivers one unit of fruit when $e_t = e$ and $e_{t+1} = e'$. Finally, let x_t denote the consumer's financial resources, which she allocates between stocks, contingent claims, and consumption.

- (a) Write down the consumer's problem in recursive form and find the first order conditions. Let $z(e')$ denote the "purchasing kernel" that identifies how many state- e' contingent claims the consumer purchases.
 - (b) For each potential (e', e) pair, express the equilibrium price $q(e', e)$ as a function of π , ε and σ . Suppose $\pi = 1 - \pi = 1/2$. Under this assumption, which (e', e) pair has the highest contingent claim price? Briefly discuss.
 - (c) Given our assumptions about the stochastic process for e_t , we can follow Mehra and Prescott (1985) to derive closed-form expressions for $p^A(e_t)$ and $p^B(e_t)$. To see how this works, define $p^{A-} = p^A(-\varepsilon)$ and $p^{A+} = p^A(\varepsilon)$.
 1. Imposing the equilibrium allocation, write down the Euler equation for $p^A(e_t)$.
 2. Evaluate this Euler equation at $e_t = -\varepsilon$ and $e_t = \varepsilon$.
 3. Show that your answer to part 2 leaves you with a simple linear system that can be solved for p^{A-} and p^{A+} . You do *not* need to actually solve this system—just set it up.
 4. Is this solution the only one that satisfies the Euler equation? If not, what additional information could you use to rule out other solutions?
9. Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (\text{PRF})$$

where: Π_t is profit; L_t is labor; W_t is the real wage; and Y_t is output. The population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, C_t , and labor are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} C_t^{1-\gamma} (1-L_t) \right),$$

$$0 < \beta < 1, \quad \gamma \geq 0.$$

Households receive labor income and profits from firms and pay lump-sum taxes to the government. Households earn a gross return of $(1+r)K_t$ on their assets, K_t , with

$\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1} , are chosen in period t . Note that in this economy, capital is used only as a storage device. Households also face the usual non-negativity and No-Ponzi-Game conditions.

The government's flow budget constraint is given by

$$H_t = G_t,$$

where H_t denotes total lump-sum taxes. Taxes are driven by government spending, which follows an AR(1) process around the log of its steady state value:

$$\hat{g}_t \equiv \ln(G_t/G_{ss}) = \phi\hat{g}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \quad (\text{TS})$$

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process, and G_{ss} is steady state government spending. (You can also assume that $G_t < Y_t, \forall t$.) Note that government spending has no effect on the utility households receive from private consumption, and no effect on the private production function.

- (a) Write down the social planner's problem for this economy as a dynamic programming problem. (The planner takes G_t as given.) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.
- (b) Let's consider how one might calibrate some of the parameters of the model.
 1. How might one calibrate α ?
 2. Rewrite the labor allocation condition in the form

$$\frac{Y_t}{C_t} = F(L_t; \alpha, \gamma).$$

Briefly describe how one could combine U.S. data with this equation to calibrate γ .

- (c) Let lower-case letters with carats " $\hat{\cdot}$ " denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\hat{\ell}_t = -\theta\hat{c}_t, \quad \hat{y}_t = -\lambda\hat{c}_t, \quad \theta > \lambda > 0.$$

- (d) Log-linearize the Euler equation, and show that

$$\hat{c}_t \approx E_t(\hat{c}_{t+1}). \quad (\text{EE}')$$

Hint: At low levels of variance, we have:

$$\ln(E_t(X_t)) \approx E_t(\ln(X_t)).$$

- (e) Suppose that the steady state consumption-to-capital ratio, C_{ss}/K_{ss} , is $\psi > 0$, while the steady state government spending-to-capital ratio, G_{ss}/K_{ss} , is $\chi > 0$, with $\psi + \chi > r$. One can log-linearize the capital accumulation equation and solve

the resulting system (which also includes (TS) and (EE')) to express consumption as a function of capital and government spending:

$$\begin{aligned}\widehat{c}_t &= \eta \widehat{k}_t - \mu \widehat{g}_t, \\ \eta &= r/\omega > 0, \\ \mu &= \frac{r\chi}{(1-\phi+r)\omega} > 0.\end{aligned}\tag{CF}$$

(Take all of this as given.)

1. Express \widehat{y}_t and $\widehat{\ell}_t$ as functions of capital and government spending.
 2. Is the effect of government spending on output increasing or decreasing in the persistence parameter ϕ ? Explain.
10. Consider a deterministic version of a Lucas tree economy inhabited by a representative agent. There is a single tree that bears a constant amount of fruit equal to $y_t = y$ units in each period. There is a market in equity shares entitling the owner to a fraction of the tree's fruit yield. For convenience, the number of shares is normalized to one. Let a_t denote the number of shares owned by the agent at the end of period t ; let p_t denote the period t price of the tree. In equilibrium, p_t adjusts so that the agent is willing to hold all shares: $a_t = 1$. The gross return on the asset purchased in period t is defined as $R_t = (p_{t+1} + y_{t+1})/p_t$. The agent has preferences given by the utility function:

$$\sum_{t=1}^{\infty} \beta^{t-1} \gamma_t u(c_t)$$

where c_t is period- t consumption of the agent and $u(\cdot)$ is increasing and strictly concave. The agent's preferences are peculiar in that he receives more utility from a given quantity of consumption in periods that are multiples of four. In these periods, momentary utility is given by $\gamma u(\cdot)$. Thus, $\gamma_t = \gamma > 1$ for $t = 4, 8, \dots$ and $\gamma_t = 1$ otherwise. The fruit cannot be stored.

- (a) Assuming that the economy is not on a bubble path, derive formulas for the the gross return in periods 7 through 10: $R_t, t = 7, 8, 9, 10$. Sketch a time-series graph that depicts the pattern of returns.
- (b) Explain the economic intuition behind the time-series pattern of returns in part (a).
- (c) Introduce to the above environment another agent with the same endowment but with preferences that are linear in momentary utility and not affected by seasonal preference shifts:

$$\sum_{t=1}^{\infty} \beta^{t-1} c_t.$$

Derive formulas for the the gross return in periods 7 through 10: $R_t, t = 7, 8, 9, 10$ and sketch the new time-series pattern of returns.

- (d) Explain the economic intuition behind any differences that you observe between your part (a) and part (c) answers.