

STATE UNIVERSITY OF NEW YORK AT ALBANY
Department of Economics

Ph. D. Preliminary Examination: Macroeconomics
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Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements*, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.

1. If the data showed that money supply increases have led to higher output and employment, a central bank could stabilize its economy by making the money supply countercyclical.
2. State run pension schemes (e.g. Social Security) achieve nothing as they simply crowd-out private savings one-for-one.
3. Volatility in asset prices greater than the corresponding volatility in the present discounted value of dividends indicates the existence of bubbles in equilibrium asset prices.
4. A permanent increase in the price of oil will cause general inflation (i.e. inflation in all other prices).
5. A decline in employment during recessions indicates the presence of inefficiencies in labor markets.
6. The real business cycle model can explain why output growth is positively correlated.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider an economy with a single representative consumer who maximizes

$$E_t \left(\sum_{j=0}^{\infty} \beta^j \ln(c_{t+j}) \right), \quad 0 < \beta < 1.$$

The consumer is also endowed with one unit of labor per period, which he supplies inelastically.

The sole source of the single non-storable good is a representative farm that produces the good using labor and an everlasting tree. The farm's objective is to maximize each period's profits:

$$\begin{aligned} \max_{\ell_t \geq 0} \pi_t &= y_t - w_t \ell_t, \\ y_t &= d_t \ell_t^{1-\alpha}, \quad 0 < \alpha < 1, \end{aligned}$$

where: π_t is profit; y_t is the tree's "fruit" or output; ℓ_t is the labor input; and w_t is the real wage. Output depends on the exogenous shifter d_t , where d_t is a non-negative random variable governed by a Markov process with the stationary transition density $f(d', d)$.

Consumers receive labor income and profits from any farms that they might own. At the beginning of time 0, each consumer owns one farm. Let p_t be the price at time t of a title to all future profits from a farm, and let $q(d', d_t)$ be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when $d_{t+1} = d'$.

- (a) Write down the consumer's problem in recursive form and find the first order conditions.
- (b) Solve the farm's problem, and express profits and labor income as a fraction of output. Impose the equilibrium labor allocation and find equilibrium output and profits, $y_t = y(d_t)$ and $\pi_t = \pi(d_t)$.
- (c) Derive the equilibrium pricing function for stocks, $p_t = p(d_t)$. (This function should not include any expected future prices.)
- (d) Derive the equilibrium kernel used to price one-step-ahead contingent claims, $q(d_{t+1}, d_t)$. Holding $f(d_{t+1}, d_t)$ fixed, is the pricing kernel increasing or decreasing in d_t ? Why? Holding d_t and d_{t+1} fixed, is the kernel increasing or decreasing in $f(d_{t+1}, d_t)$? Why?

8. Consider the following economy dated in discrete time by the index $t = 1, 2, \dots, T$. The economy is populated by J agents indexed by $j = 1, 2, \dots, J$. Agent j 's preferences over date-contingent consumption are represented by the utility function

$$u^j(c_1^j, \dots, c_T^j) = - \sum_{t=1}^T \beta^{t-1} \exp(-b^j c_t^j), \quad j = 1, 2, \dots, J, \quad 0 < \beta < 1,$$

where c_t^j is the period- t consumption of agent j . Each agent j is endowed with a stream of the single, physically distinct consumption good: $\{y_t^j\}_{t=1}^T$. The aggregate endowment in each period is denoted $Y_t = \sum_{j=1}^J y_t^j$.

- (a) Letting q_t denote the price of period- t consumption, define the Arrow-Debreu competitive equilibrium for this economy. Is the competitive equilibrium allocation Pareto optimal? Explain.
- (b) Solve for the Arrow-Debreu competitive equilibrium prices $\frac{q_t}{q_1}$, $t = 2, \dots, T$ and the corresponding real interest rates r_t , $t = 1, \dots, T$ in the equivalent sequence-of-markets competitive equilibrium.
- (c) Suppose that the aggregate endowment increases in some period $t = s > 2$, and that this increase is known before the beginning of trading in period-1. Determine the effect on the period- t Arrow-Debreu price $\frac{q_s}{q_1}$ and corresponding real interest rate r_s , and explain the intuition behind these results.
- (d) Consider two separate economies: Economy I and Economy II. The economies are in all respects identical except for the distribution of the preference parameters b^j in the population. For simplicity, the preference parameter in each economy is ordered by the index j : $b^1 < b^2 < \dots < b^J$. That is, higher agent index numbers correspond to higher b 's: $b^{j+1} > b^j$ for all $j = 1, \dots, J - 1$. In Economy II, the b^j 's are pairwise greater than those in Economy I. That is, $(b^1)^{II} > (b^1)^I$, $(b^2)^{II} > (b^2)^I, \dots, (b^J)^{II} > (b^J)^I$. Compare the price of consumption in an arbitrary period t in the two economies, *proving* your result.
- (e) Explain the intuition behind your result in part (d).

9. Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\begin{aligned} \max_{L_t \geq 0} \Pi_t &= Y_t - W_t L_t, \\ Y_t &= G_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \end{aligned} \tag{PRF}$$

where: Π_t is profit; L_t is labor; W_t is the real wage; Y_t is output; and G_t is government spending. The population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, C_t , and labor are given by

$$\begin{aligned} E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi \frac{1}{1+\gamma} L_t^{1+\gamma} \right] \right), \\ 0 < \beta < 1, \quad \gamma > 0, \quad \chi > 0. \end{aligned}$$

Households receive labor income and profits from firms. They pay taxes on this income to the government at the rate $\tau_t \in [0, 1)$. Households earn a gross return of $(1+r)K_t$ on their assets, K_t , with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1} , are chosen in period t . Households also face the usual non-negativity and No-Ponzi-Game conditions.

The government follows a balanced budget rule:

$$\tau_t Y_t = G_t. \tag{BB}$$

(Asset income is *not* taxed.) Note that government expenditures are driven by the tax rate τ_t , which follows an AR(1) process around the log of its steady state value:

$$\hat{\tau}_t \equiv \ln(\tau_t/\tau) = \phi \hat{\tau}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \tag{TS}$$

where $\{\varepsilon_t\}$ is an exogenous stationary martingale difference sequence, and $0 < \tau < 1$ is the steady state tax rate. Individual consumers and producers are sufficiently small to take G_t as given.

- (a) Find the equilibrium allocation:

1. Find the first order conditions for profit and utility maximization.
2. Eliminate wages and government spending to find the production function and labor allocation condition.
3. Find the Euler equation and the aggregate resource constraint.

- (b) Let lower-case letters with carats “ $\hat{}$ ” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\begin{aligned} \hat{\ell}_t &= \theta_1 \hat{\tau}_t - \theta_2 \hat{c}_t, \\ \hat{y}_t &= \lambda_1 \hat{\tau}_t - \lambda_2 \hat{c}_t. \end{aligned}$$

What are the signs of θ_1 and λ_1 ? Briefly discuss.

- (c) Suppose that the steady state consumption-to-capital ratio, C_{ss}/K_{ss} , is $\psi > r$. It is then straightforward to show that the steady state output-to-capital ratio, Y_{ss}/K_{ss} , is $(\psi - r)/(1 - \tau)$. (Take this as given). Using this result, log-linearize the capital accumulation equation to show that

$$\hat{k}_{t+1} = (1 + r)\hat{k}_t + \omega_1\hat{\tau}_t - \omega_2\hat{c}_t. \quad (\text{CA}')$$

- (d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS) and (CA')) to express consumption as a function of capital and the tax rate

$$\hat{c}_t = \eta\hat{k}_t + \mu\hat{\tau}_t. \quad (\text{CF})$$

with

$$\eta = \frac{r}{\omega_2}; \quad \mu = \frac{r\omega_1}{\omega_2(1 - \phi + r)}.$$

(Take this as given.) Is consumption increasing or decreasing in the income tax rate? Recalling your answer to part (b) above, briefly discuss.

10. Consider an economy with many identical agents each with preferences given by the utility function

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \quad 0 < \beta < 1, \sigma > 0,$$

where c_t is individual consumption. Each agent has one unit of time available in each period and can produce consumption goods according to the production function:

$$y_t = \alpha h_t^\theta H_t^{1-\theta} u_t, \quad 0 < \theta < 1, \alpha > 0,$$

where u_t is time spent in goods production and H_t denotes average human capital across all agents in the economy. Each agent accumulates human capital according to the equation

$$h_{t+1} = \delta h_t (1 - u_t), \quad \delta > 0.$$

Assume that each agent has the same initial stock of human capital, h_0 . Note that the consumption good cannot be stored.

- Explain the intuition behind the two technologies in this model: goods production and human capital accumulation.
- Write the social planner's problem as a dynamic program identifying all state variables and choice variables.
- Solve for the optimal growth rates of human capital and consumption, and for u_t .
- Solve for the recursive competitive equilibrium balanced growth path, where consumption and human capital grow at constant rates and where u_t is constant.
- Determine whether the competitive equilibrium growth rate is faster or slower than the optimal growth rate. Explain the intuition behind your answer.