STATE UNIVERSITY OF NEW YORK AT ALBANY
Department of Economics

Ph. D. Comprehensive Examination: Macroeconomics
Spring, 2000

Instructions. This exam consists of two parts. The first part contains extended problems; please answer any two of the four problems provided. The second part contains short-answer-type questions; please answer any three of the six questions provided. Be sure to write the numbers of the questions you answer on the covers of your bluebooks.

You have 180 minutes to complete this exam. You should allocate about one hour to each extended problem, and the remaining time to the shorter questions. Good luck!

Part I. Please answer any 2 of the following 4 questions.

1. Consider the following economy consisting of a representative consumer and a representative firm. The consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \left(\ln c_t + \ln x_t\right),$$

where c_t is consumption of a market good, purchased in a competitive market, x_t is consumption of a home-produced good, and $0 < \beta < 1$. The agent is endowed with one unit of time in each period to allocate between market labor, n_t, and home production activities, a_t: $n_t + a_t = 1$. Production of both the market good and the home good require the use of capital, k_t, and labor. The firm produces output according to the function

$$y_t = k_t^{\alpha} n_t^{1-\alpha},$$

where $0 < \alpha < 1$. Similarly, production of the home good is given by

$$x_t = k_t^{\phi} a_t^{1-\phi},$$

where $0 < \phi < 1$. Although capital is used by both the firm and the household, it can only be produced by the firm so that

$$c_t + k_{t+1} = y_t.$$

Note that there is 100% depreciation of capital within a period.

(a) Set up the social planner's problem as a dynamic programming problem and determine the decision rules for k_{t+1}, n_t, and a_t. (**Hint:** Guess the value function $v(k_t) = A + B \ln k_t$ and solve for the decision rules in terms of the constant B; then compute the expression for B.)

(b) Compute the steady state value of k_t. What is the relationship between this value and the parameter ϕ? Explain.

(c) What is the long-run growth rate of this economy? How does this model explain cross-country differences in per capita income growth?

2. Consider a one-tree, one-good Lucas tree model. The preferences of the representative agent are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left[c_t^{1-\sigma}-1\right]\right),\quad 0<\beta<1,\quad \sigma>0.$$

Each tree gives off a non-storable "fruit" or dividend d_t, where d_t is a non-negative random variable governed by a Markov process with the stationary one-step transition density $f(d',d)$. The economy starts off with each household owning one such tree. Let p_t be the price at time t of a title to all future dividends from the tree, and let $q(d',d)$ be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when $d_{t+1}=d'$ and $d_t=d$.

(a) Write down Bellman's functional equation for the consumer's problem. You can assume that d_t summarizes the state of the aggregate economy. Let $y(d')$ denote the "purchasing kernel" that identifies how many state-d' contingent claims the consumer purchases.

(b) Using a recursive approach, define an equilibrium in this economy. Assume that *in equilibrium* the consumer's "purchasing kernel" for contingent claims, $y(x,d',d)$, equals zero for all values of d'.

(c) Derive the equilibrium pricing function for stocks, $p_t=p(d_t)$. (This function should not include any expected future prices.) Be sure to point out any restrictions that you impose in deriving this function.

(d) Suppose that d_t follows a two-state Markov chain. In particular, the dividend either equals d_L or d_H, where $d_L=(1/\gamma)\,d_H$, $d_H>0$, and $\gamma>1$. The transition probabilities for d_t are given by

$$f(d_H,d_H)=\Pr\left(d_{t+1}=d_H\mid d_t=d_H\right)=\pi,$$

$$f(d_L,d_H)=\Pr\left(d_{t+1}=d_L\mid d_t=d_H\right)=1-\pi,$$
$$f(d_H,d_L)=\Pr\left(d_{t+1}=d_H\mid d_t=d_L\right)=1-\pi,$$
$$f(d_L,d_L)=\Pr\left(d_{t+1}=d_L\mid d_t=d_L\right)=\pi,$$

with $\pi\in(0,1)$. For each potential (d_{t+1},d_t) pair, find the price of the one-step-ahead contingent claim, $q(d_{t+1},d_t)$, in terms of d_H, γ, σ and π.

(e) Let $R_t^{-1}=R(d_t)$ be the time-t price of a risk-free discount bond that pays one unit of consumption at time $t+1$ under any state. Use the pricing kernel to find $R^{-1}(d_H)$. Now suppose that when dividends are high ($d_t=d_H$) the representative agent can also buy an "insurance policy" that pays $1/(1-\pi)$ units of the consumption good if $d_{t+1}=d_L$ but nothing if the $d_{t+1}=d_H$. What is the price of this insurance policy?

(f) When $d_t=d_H$ the discount bond and the insurance policy both have an *expected* payment of 1. Why, then, does one of the assets have a higher market value?

3. Consider an individual that maximizes the present value of her utility from consumption and leisure over an infinite lifetime:

$$\max_{\{c_t,k_{t+1},\ell_t\}_{t=0}^{\infty}} E_0\left(\sum_{t=0}^{\infty}\beta^t\left(\ln\left(c_t\right)+\ln\left(1-\ell_t\right)\right)\right),$$

$$s.t.\quad \left(1+\tau_t^C\right)c_t+k_{t+1}=Rk_t+w_t\ell_t, \qquad \text{(FBC)}$$

$$\ell_t\in[0,1],$$

$$\lim_{J\to\infty}E_t\left(R^{-J}k_{t+J}\right)=0, \qquad \text{(ENPG)}$$

$$k_0 \ given,$$

$$c_t\geq 0,$$

where

$c_t =$ consumption,
$\ell_t =$ labor hours,
$k_t =$ capital,
$R =$ gross return on capital, $R>1$,
$w_t =$ real wage,
$\tau_t^C =$ tax rate on consumption spending,
$\beta =$ discount factor, $\beta\in(0,1)$, $\beta R\leq 1$.

The revenues raised by the consumption taxes are used by the government to purchase goods and services that provide the consumer no utility, and have no effect on real wages.

(a) Find the first order conditions for an interior solution to the consumer's problem, namely the Euler equation and the labor allocation condition.

(b) Derive the consumer's expected present value budget constraint (EPVBC). Be sure to point out any restrictions that you impose in deriving this equation.

(c) Now suppose that the consumer has perfect foresight. Express consumption and labor at time t, c_t and ℓ_t, as functions of current capital, current and future wages and current and future consumption taxes. Briefly explain how c_t and ℓ_t depend on the consumption tax sequence $\left\{\tau_{t+j}^C\right\}_{j=0}^{\infty}$.

(d) Now suppose that the consumer faces uncertainty in period 0, but in period 1 she acquires perfect foresight. In particular, while $w_1=w_2=w_3=...$, and $\tau_1^C=\tau_2^C=\tau_3^C=...$, w_1 and τ_1^C are not known at time 0. You can also assume that $\beta R=1$.

   1. Using your answer to part (c), express consumption at time 1, c_1, as a function of k_1, w_1 and τ_1^C. Insert this result into the time-0 Euler equation.
   2. Given your result for part (1), will uncertainty about the wage w_1 cause the consumer to engage in a form of precautionary saving at time 0? Briefly justify your answer.

3. Given your result for part (1), will uncertainty about the consumption tax τ_1^C cause the consumer to engage in a form of precautionary saving at time 0? Briefly justify your answer.

4. Consider the following endogenous growth model populated by a dynastic household. The preferences of the household are represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t N_t \ln c_t,$$

where $0 < \beta < 1$, N_t is population and c_t is per capita consumption. Population grows at rate n so that the period-t population is given by $N_t = (1+n)^t$ (the initial population is normalized to 1). Per capita output of the consumption good is given by

$$c_t = \alpha h_t u_t,$$

where $\alpha > 0$, h_t is per capita human capital and u_t is the time spent producing the consumption good. The household is endowed with one unit of time in each period and must allocate v units of time just getting to and from work in each period. The remainder of the time endowment can be allocated to production of the consumption good or human capital accumulation. Finally, human capital is accumulated by combining existing human capital with the time allocated to human accumulation according to the expression

$$h_{t+1} = \gamma h_t \left(1 - v - u_t\right),$$

where $\gamma > 0$ and h_0 is given.

   (a) Set up the social planner's problem as a dynamic programming problem. Show that per capita output and human capital grow at the same rate and determine this growth rate.

   (b) Determine u_t.

   (c) How does the economy's growth rate and time devoted to human capital accumulation depend on v? Explain your result.

   (d) Which growth fact(s), if any, does this model help us to understand? Explain.

Part II. Please answer any 3 of the following 6 questions.

5. Consider exogenous and endogenous growth models.

   (a) What determines long-run growth in the Solow (or Ramsey) model? Explain using model details.

   (b) What determines long run growth in the endogenous (for example, $Y = AK$) model? Explain using model details.

   (c) Compare and contrast the implications of the two models for differential in cross-country growth rates. Include the role of the saving rate in your answer.

(d) How do endogenous growth advocates justify the absence of a diminishing marginal product for capital in their framework?

6. Using the guidance of a neoclassical growth model (exogenous growth), evaluate the following statement as either being **true**, **false**, or **uncertain**: "An increase in the population growth rate increases growth in output and increases welfare." Carefully explain your answer.

7. In answering the following questions, please refer to specific models. While they are not necessary, graphical analyses will be useful.

   (a) Explain what is meant by rational expectations? How do we model them?

   (b) Can systematic monetary policy, i.e., policy conducted according to a feedback rule, have real economic effects? Explain in detail using specific examples.

   (c) Can you use rational expectations to justify a policy of inflation-targeting? (Recall that under inflation targeting, the central bank sets the money supply in order to achieve a certain inflation rate.) Explain.

8. **Has the Federal Reserve helped Ed Prescott?** It has been argued that the Real Business Cycle model provides a good description of the real postwar U.S. economy because the Federal Reserve's monetary policies have minimized the real effects of financial shocks and frictions. Critique this claim.

9. Consider the following proposition: "A reduction in current taxes will have no effect on aggregate consumption."

   (a) Carefully describe a theoretical economy for which this proposition is **true**.

   (b) Carefully describe a theoretical economy for which this proposition is **false**.

   Note: Here the word "describe" means that you should come as close as possible to writing down a complete and "fully articulated" model economy. While a mere discussion (that is accurate!) will receive some credit, an explicit model will receive more.

10. **Martyrs and Deadbeats.** Consider the plight of some macroeconomics students under various grading options. In particular, while each student must hand in answers to each homework, homework grades are assigned on a group basis, with each member of the group receiving a common grade. Assume that the preferences of each student are given by

$$u(g_i, e_i) = \alpha g_i - e_i, \quad \alpha > 1,$$

where g_i is student i's grade and $e_i \in [0,1]$ is student i's effort level.

   (a) Suppose that the grade for group J is given by

$$g_J = \min_{i \in J} \{e_i\},$$

   i.e., the grade for the entire group will be the grade of its laziest member. Under this grading scheme, can there be coordination failure, i.e., a Nash equilibrium where the group receives a socially inefficient grade? Briefly explain.

(b) Now suppose that the grade for group J is given by

$$g_J = \max_{i \in J} \{e_i\},$$

i.e., the grade for the entire group will be the grade of its most diligent member. It can be shown that under this grading scheme, any pure strategy Nash equilibrium will be Pareto optimal. (**Note**: equilibria will be asymmetric.) Briefly explain how and why this result differs or does not differ from your answer to part (a).

(c) Do you think the economy is better approximated by the grading scheme in part (a) or the scheme in part (b)? Explain.