

STATE UNIVERSITY OF NEW YORK AT ALBANY
Department of Economics

Ph. D. Preliminary Examination: Macroeconomics
Fall, 2007

Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. In models with multiple equilibria, recessions are periods of lower welfare.
2. According to Keynesian macroeconomic theory, investment demand creates exactly the saving needed to finance it. Consequently the theory cannot explain how a credit crunch can cause a recession.
3. Outside Money in Overlapping Generations Models is simply a store of value. It has no role as a medium of exchange.
4. For any continuous function $f(\cdot)$ it is always true that $\mathbb{E}_x(f(x)) < f(\mathbb{E}_x(x))$, where x is a random variable and $\mathbb{E}_x(\cdot)$ is the expectation with respect to x .
5. Externalities in production are fundamental to the understanding of long-run economic growth.
6. A tax on capital income has no effect on workers (people who own no assets).

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Diamond Overlapping Generations Model with Outside Money

- **Time:** discrete, infinite horizon
- **Demography:** A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.
- **Preferences:** for the generations born in and after period 0,

$$U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \beta u(c_{2,t+1}),$$

where $c_{i,t}$ is consumption in period t and stage i of life. $u(\cdot)$ is increasing, strictly concave and twice differentiable, with $\lim_{c \rightarrow 0} u'(c) = \infty$, $\lim_{c \rightarrow \infty} u'(c) = 0$. For the initial old generation $\tilde{U}(c_{2,0}) = u(c_{2,0})$.

- **Productive technology:** A constant returns to scale technology implies that an $f(k)$ is available to each household. k_t is the per-worker capital stock. The usual “Inada” conditions apply.
 - **Endowments:** Everyone has one unit of labor services when young. The old cannot work, so they have to rely on earnings from renting capital. The initial generation of old people have $(1+n)k_0$ units of capital each. Each old person in this initial generation also has H units (dollars) of money, which provides no benefit *per se*, but is fixed in supply and recognizable as money.
 - **Institutions:** There are competitive markets for labor, physical capital, consumption goods and money. Using the consumption good as the numeraire, let the per unit wage in period t be w_t and the gross return on capital rented in period t be R_t . The price of money in terms of the consumption good is $1/p_t$, or, equivalently, acquiring one unit of the consumption good costs p_t dollars.
- Write out the problems faced by generation t workers and firms in this economy (ignore inside money). What condition on the price level p_t and interest rate R_t must be met if money is to circulate (i.e., be used as means of saving)?
 - Write down the market clearing conditions and define a competitive equilibrium in which money circulates.
 - In a steady-state of the economy where real money demand, $\frac{M_t^d}{p_t}$, is fixed for all t , what condition must be met by the return on holding money, $\frac{p_t}{p_{t+1}}$?
 - Derive the condition that a Social Planner who weights all generations equally puts on the steady-state per capita stock of capital. What does this imply for the optimality of competitive equilibria?
 - What can we conclude from (c) and (d) about the efficiency properties of any steady-state monetary equilibrium? What does this imply for the existence of steady-state equilibria with over-investment? Explain your answer.

8. (From King, Plosser and Rebelo, 1988.) Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = A_t L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (\text{PRF})$$

where: Π_t is profit; L_t is labor; W_t is the real wage; and Y_t is output. Productivity, A_t , is exogenous and follows a random walk in logs:

$$\frac{A_t}{A_{t-1}} = \exp(\varepsilon_t) \Leftrightarrow a_t \equiv \ln(A_t) = a_{t-1} + \varepsilon_t, \quad (\text{TS})$$

where $\{\varepsilon_t\}$ is a zero-mean i.i.d. process. The population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, C_t , and labor are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi \frac{1}{1+\gamma} L_t^{1+\gamma} \right] \right),$$

$$0 < \beta < 1, \quad \gamma > 0, \quad \chi > 0.$$

Households receive labor income and profits from firms. Households earn a gross return of $(1+r)K_t$ on their assets, K_t , with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1} , are chosen in period t . Households also face the usual non-negativity and No-Ponzi-Game conditions.

- (a) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.
- (b) When technology follows a random walk, shocks to technology have permanent effects, and the standard practice is to express non-stationary variables as fractions of the technology level A_t . In particular, define

$$y_t = \frac{Y_t}{A_t}, \quad k_t = \frac{K_t}{A_t}, \quad c_t = \frac{C_t}{A_t}.$$

Note that we do **not** rescale L_t . Using these definitions, rewrite your answer to part (a).

- (c) Let lower-case letters with carats “ $\hat{}$ ” denote deviations of logged, **rescaled** variables around their steady state values. In particular, define

$$\hat{y}_t = \ln\left(\frac{y_t}{y_{ss}}\right), \quad \hat{k}_t = \ln\left(\frac{k_t}{k_{ss}}\right), \quad \hat{c}_t = \ln\left(\frac{c_t}{c_{ss}}\right), \quad \hat{\ell}_t = \ln\left(\frac{L_t}{L_{ss}}\right).$$

Show that the log-linearized expressions for labor hours and output are

$$\hat{\ell}_t = -\theta \hat{c}_t; \quad \hat{y}_t = -\lambda \hat{c}_t,$$

$$0 < \lambda < \theta.$$

- (d) Suppose that the steady state consumption-to-capital ratio, $C_{ss}/K_{ss} = c_{ss}/k_{ss}$, is $\psi > r$. It is then straightforward to show that the steady state output-to-capital ratio, Y_{ss}/K_{ss} , is $\psi - r$. (Take this as given). Using this result, log-linearize the capital accumulation equation to show that

$$\widehat{k}_{t+1} = (1+r)\widehat{k}_t - \omega\widehat{c}_t - \varepsilon_{t+1}. \quad (\text{CA}'')$$

Why is the coefficient on the technology innovation ε_{t+1} negative?

- (e) It is straightforward to log-linearize the Euler equation, and show that

$$\widehat{c}_t \approx E_t(\widehat{c}_{t+1}). \quad (\text{EE}'')$$

Combining equations (EE'') and (CA''), we can express consumption as a function of capital and the technology shock:

$$\widehat{c}_t = \eta\widehat{k}_t + \mu\varepsilon_t, \quad (\text{CF})$$

with

$$\eta = \frac{r}{\omega}; \quad \mu = 0.$$

(Take all of this as given.)

1. Intuitively, why is $\mu = 0$? (**Hint:** Recall that \widehat{c}_t measures logged deviations in relative consumption, $c_t \equiv C_t/A_t$.)
2. Does $\mu = 0$ imply that both relative and absolute (total) consumption are completely invariant with respect to technology? Briefly explain.

9. Optimal Growth with a Consumption Tax.

The economy of Discountia is described as follows:

- **Time:** Discrete; infinite horizon
- **Demography:** Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit-maximizing firms, owned jointly by the households.
- **Preferences:** the instantaneous household utility function over individual consumption, c , is $u(c)$, where $u(\cdot)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.
- **Technology:** There is a constant returns to scale technology over capital and labor such that output per unit of labor employed is $f(k)$, where k is capital input per unit of labor. Capital depreciates at the rate $\delta < 1$.
- **Endowments:** Households' initial capital stock is k_0 , and each household has 1 unit of labor.
- **Institutions:** Every period there are markets for labor and capital and consumption goods. There is a government that has the power to levy taxes and make transfers.

The Prime Minister of Discountia, Tony Brown, recons he can get re-elected if GDP goes up during his current term. He has also learned some growth theory and is particularly taken by the notion of the “golden rule” level of output and how, because of discounting, that “golden rule” output exceeds output in the *Laissez Faire* economy. To this end he decides to levy a proportional consumption tax, τ , on everyone. (One way to think of this is as a sales tax on everything consumed.) To avoid being too disliked, he decides to rebate the the revenues to the households with the lump-sum transfer x_t . The government budget is always in balance. By distorting the households' consumption decisions he hopes to induce greater capital accumulation and therefore increase GDP.

- (a) Write down the problem faced by the representative household, and by the representative firm. (Note that they both take τ and x_t as given.) Define and solve for a characterization of competitive equilibrium.
- (b) Write down and impose the government budget constraint. Solve for the laws of motion for the economy as a whole and for the saddlepath-stable steady-state. To what extent does it depend on τ ? Comment on the prime minister's knowledge of economics and explain your answer. How might the results change if households valued leisure?

10. Consider an economy with a single representative consumer who maximizes

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{\gamma} \exp(-\gamma c_t) \right] \right), \quad 0 < \beta < 1, \quad \gamma > 0.$$

The consumer is also endowed with one unit of labor per period, which he supplies inelastically.

The sole source of the single non-storable good is a representative farm that produces the good using labor and an everlasting tree. The farm's objective is to maximize each period's profits:

$$\begin{aligned} \max_{\ell_t \geq 0} \pi_t &= y_t - w_t \ell_t, \\ \text{s.t. } y_t &= d_t \ell_t^{1-\alpha}, \quad 0 < \alpha < 1, \end{aligned}$$

where: π_t is profit; y_t is the tree's "fruit" or output; ℓ_t is the labor input; and w_t is the real wage.

Output depends on the exogenous shifter d_t , which follows a two-state Markov process. In particular, d_t takes on the values d_L and $d_H > d_L$. We will also assume that the conditional probabilities are symmetric in that

$$\begin{aligned} f(d_L, d_L) &= \Pr(d_{t+1} = d_L | d_t = d_L) = \pi \\ &= f(d_H, d_H), \\ f(d_H, d_L) &= \Pr(d_{t+1} = d_H | d_t = d_L) = 1 - \pi \\ &= f(d_L, d_H). \end{aligned}$$

Consumers receive labor income and profits from any farms that they might own. At the beginning of time 0, each consumer owns one farm. Let $p_t = p(d_t)$ be the price at time t of a title to all future profits (π_t 's) from a farm, and let $q(d', d_t)$ be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when $d_{t+1} = d'$.

- (a) Write down the consumer's problem in recursive form and find the first order conditions.
- (b) Solve the farm's problem, and express profits and labor income as a fraction of output. Impose the equilibrium labor allocation and find equilibrium output and profits, $y_t = y(d_t)$ and $\pi_t = \pi(d_t)$.
- (c) Given our assumptions about the stochastic process for d_t , we can follow Mehra and Prescott (1985) to derive closed-form expressions for $p_L = p(d_L)$ and $p_H = p(d_H)$.
 1. Find the equilibrium Euler equation for stocks.
 2. Using the values specified above, evaluate this Euler equation at p_L and p_H .
 3. Show that your answer to part 2 leaves you with a simple linear system that can be solved for p_L and p_H . You do **not** need to actually solve this system—just set it up.
- (d) Provide a brief argument as to why the pricing function you derived in part (c) generates **equilibrium** prices.