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Sponsor: National Science Foundation

Dates: August 1, 2008 – July 31, 2011

Amount: \$85,555

Algebraic and Number Theoretic Aspects of Vertex Algebra Theory

Vertex algebras are relatively new fundamental algebraic structures with many applications in representation theory, conformal field theory, finite group theory, topology, etc. Vertex algebras were introduced in 1986 by R. Borcherds [Bo] who also initiated general theory of vertex operators for purposes of constructing the Moonshine Module vertex algebra, which was later accomplished by I. Frenkel, J. Lepowsky and A. Meurman [FLM].

This project focuses on certain algebraic and number theoretic aspects of vertex algebra theory. On the algebraic side we propose new methods for studying various families of vertex algebras important in conformal field theory. On the number theoretic side we propose new modular, q -series and constant term identities.

The potential intellectual merit of the proposed project are the following:

- Motivated by ideas of logarithmic conformal field theory we propose to study new families of quasi-rational vertex operator algebras and their enhanced (e.g., tensor) categories of modules. These objects are currently of great interest among physicists and our work is one of the first rigorous studies of these important objects in mathematics. At the more technical level, we construct new series of irrational C_2 -cofinite vertex operator (super)algebras, we examine their representation theory, graded dimensions of irreducible modules and closely related correlation functions, discuss modular invariance, and relate them to quantum groups. In particular, these considerations led us to discoveries of new constant term and q -series identities.
- We show that modular invariance in rational conformal field theory is a fundamental concept underlying many constructions in the theory of modular forms and studies of modular q -series identities. It has been known that there is a close connection between rational conformal field theory and modular functions but up to now, with some notable exceptions, there were almost no attempts to use vertex algebras to prove results of interest to number theorists.
- We propose to develop tools for construction of (new) formulas for graded dimensions of modules for certain rational vertex (super)algebras. In particular, we propose a new method for constructing fermionic formulas for graded dimensions of principal subspaces of integrable highest weight modules for both twisted and untwisted affine Kac-Moody Lie algebras. This will be done through an extensive use of vertex algebra theory, their intertwining operators and related q -difference equations. We hope that this will lead us to new fermionic expressions for characters of standard modules and to combinatorial bases for standard modules.

The **broader impacts** of this research proposal is to expand the horizons of applications of vertex algebras and to build new links between different areas of mathematics and physics. It may well be that some ideas and results related to this project will lead to advances in other areas of mathematics and theoretical physics.

At the undergraduate level, we develop an URP related to constant term identities (Section 2.5), which could be possibly used for an REU. Previously, some ideas from the proposal have already been used for an REU (see [D]). At the graduate level, we expect that this project will lead to the Ph.D. thesis of one graduate student. We believe that the scope of the project will allow more students to be involved at both graduate and undergraduate level. Finally, the PI has developed two advanced graduate courses: *Infinite-dimensional Lie algebras* (Spring 2007) and *Introduction to vertex algebra* (Fall 2007) at the University at Albany, closely related to the main ideas in the proposal.