Math 461: Topology, Fall 2009 Quiz # 2, September 14

Name: ....

First Problem.

Consider a function $f\colon X \to Y$. Are the following conditions equivalent to the injectivity of f?

1] $\forall\, x_1, x_2 \in X$, if $x_1 = x_2$ then $f(x_1) = f(x_2)$ .... YES | NO

2] $\exists\, g\colon Y \to X$ such that $g \circ f = \mathrm{id}_X$ .... YES | NO

3] $\exists\, g\colon Y \to X$ such that $f \circ g = \mathrm{id}_Y$ .... YES | NO

4] f^{-1} is surjective .... YES | NO

5] $\forall\, y \in f(X)$, $\exists!\, x \in X$ such that $f(x) = y$ .... YES | NO

6] $\forall\, A, B \subset X$, $f(A \cap B) \subset f(A) \cap f(B)$ .... YES | NO

7] $\forall\, A \subset X$, $\forall\, x \in X$, if $x \in f^{-1}\big(f(A)\big)$ then $x \in A$ .... YES | NO

Second Problem.

Consider a function $\varphi\colon X \to Y$. Write the definitions of:

A] left inverse of φ;

B] $\varphi(W)$, where W is a subset of X;

C] $\varphi^{-1}(Z)$, where Z is a subset of Y.

Third Problem.

Complete the following sentence.

If a function $f\colon A \to B$ has both a left inverse $g\colon B \to A$ and a right inverse $h\colon B \to A$, then ....

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