Math 461: Topology, Fall 2009 Quiz # 1, September 9

Name: ....

First problem.
For each of the following six questions, four possible answers are provided, but only one of them is correct: write the corresponding letter in the box!

1. Let $f\colon X \to Y$ be a function. Let x and x' be elements of X such that $f(x) = f(x')$.
   What do we need to know about f to conclude that $x = x'$? .... $\boxed{}$
   A] Nothing: this is true for all functions f.
   B] We need f to be injective.
   C] We need f to be surjective.
   D] We need f to be bijective.

2. Let $f\colon X \to Y$ be a function. Let x and x' be elements of X such that $x = x'$.
   What do we need to know about f to conclude that $f(x) = f(x')$? .... $\boxed{}$
   A] Nothing: this is true for all functions f.
   B] We need f to be injective.
   C] We need f to be surjective.
   D] We need f to be bijective.

3. Let $f\colon X \to Y$ be a function. Let y be an element of Y.
   What do we need to know about f to conclude that $y = f(x)$ for some $x \in X$? .... $\boxed{}$
   A] Nothing: this is true for all functions f.
   B] We need f to be injective.
   C] We need f to be surjective.
   D] We need f to be bijective.

4. Let $f\colon X \to Y$ be a function. Let y be an element of Y.
   What do we need to know about f to conclude that $y = f(x)$ for exactly one $x \in X$? .... $\boxed{}$
   A] Nothing: this is true for all functions f.
   B] We need f to be injective.
   C] We need f to be surjective.
   D] We need f to be bijective.

5. Let $f\colon X \to Y$ be a function. Let y be an element of Y.
   What do we need to know about f to conclude that $y = f(x)$ for at most one $x \in X$? .... $\boxed{}$
   A] Nothing: this is true for all functions f.
   B] We need f to be injective.
   C] We need f to be surjective.
   D] We need f to be bijective.

6. Let $f\colon X \to Y$ be a function. Let x be an element of X.
   What do we need to know about f to conclude that $f(x) = y$ for exactly one $y \in Y$? .... $\boxed{}$
   A] Nothing: this is true for all functions f.
   B] We need f to be injective.
   C] We need f to be surjective.
   D] We need f to be bijective.

Second problem.
Let X and Y be sets, and $\varphi\colon X \to Y$ a function. Suppose that W is a subset of X and Z is a subset of Y. Write the definitions of $\varphi(W)$ and of $\varphi^{-1}(Z)$.

Third problem.
Let A and B be sets, and let $f\colon A \to B$ be a function.
Suppose that A' and A'' are subsets of A, and that B' is a subset of B.
Are the following implications true or false? Prove or disprove them.

(1) $\quad B' \subset f(A' \cap A'') \quad \Rightarrow \quad B' \subset f(A')$ and $B' \subset f(A'')$ TRUE | FALSE

(2) $\quad B' \subset f(A')$ and $B' \subset f(A'') \quad \Rightarrow \quad B' \subset f(A' \cap A'')$ TRUE | FALSE