A. Overlapping generations model with discriminatory land rights

**Time:** discrete, infinite horizon

**Demography:** Each period 2 newborns enter the economy and live for two periods. There is no population growth. The newborns comprise one of type A and one of type B. Type A households are allowed to own land and type B are not. The type A household acquires the land (of size $A$) at the beginning of their second period of life by inheritance from the previous generation of type A individual who just died. They can rent the land to a farm but cannot rent it to type B individuals directly. There is a single farm owned by the type A households.

**Preferences:** for the generations born in and after period 1:

$$U_J^t (c_{1,t}, l_{1,t}^J, c_{2,t+1}^J, l_{2,t+1}^J) = u(c_{1,t}, l_{1,t}^J) + \beta u(c_{2,t+1}^J, l_{2,t+1}^J) \quad \text{for } J = A, B$$

where $c_{i,t}^J$ is consumption in period $t$ and stage $i$ of life for a type $J = A, B$ household and $l_{i,t}^J$ is leisure in period $t$ and stage $i$ of life for a type $J$ household. For the initial old generation $\tilde{U}_J^t (c_{2,1}^J, l_{2,1}^J) = u(c_{2,1}^J, l_{2,1}^J)$ for $J = A, B$. The utility function is twice differentiable, increasing in both arguments and strictly concave. It has a positive cross-partial (i.e. $u_{12} > 0$). Moreover, $\lim_{c \to 0} u_1(c, l) = \infty$, $\lim_{l \to 0} u_2(c, l) = \infty$, $\lim_{c \to \infty} u_1(c, l) = 0$, $\lim_{l \to 1} u_2(c, l) = 0$. The first 3 assumptions here are standard but the last one is new.

**Productive technology:** The farm has access to a technology so that output, $y_t = f(a_t, n_t)$ where $a_t$ is the amount of land used and $n_t$ is the amount of labor hired in period $t$. The production function satisfies the standard Inada conditions, is twice differentiable, increasing in both arguments, concave and exhibits constant returns to scale.

**Endowments:** Everyone has one unit of time in both periods of life. The young can work to earn income; the old cannot. Old type A households receive $A$ units of land. As old people cannot work, they have to rely on savings (using inside money) and, for type A only, earnings from renting land.

**Institutions:** There are competitive markets, for labor, land and consumption goods. Using the consumption good as the numeraire, let the per labor unit wage in period $t$ be $w_t$ and the per unit rental rate of land be $q_t$. There are also enforceable IOU contracts so that inside money circulates. Let the gross rate of return on inside money contracts that pay in period $t$ be $R_t$. 

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1. Write out and solve the problems faced by generation $t$ individuals of type $A$ and type $B$ and, the farm in this economy.

2. Write down the market clearing conditions for land, labor and inside money and define a competitive equilibrium.

3. Now, focussing on a steady state (so that we can drop the time subscripts on variables) solve for a system of 4 equations that characterize the steady state equilibrium. What are the four steady state variables that your system can be used to solve for? Note: if you drop arguments, (e.g. from the production function to reduce notation) please indicate at least once, that you know what those arguments are.

4. Now, suppose that preferences are such that second period consumption is the same for both types of household. What does that mean for the amount of work done by the type A young people relative the type B young people?

5. What is the advantage in this model of assuming that $\lim_{l \to 1} u_2(c, l) = 0$?

6. Write down but do not solve the Planners’s problem for this economy.

B. Comparative statics in simple linear technology model

System:

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\begin{align*}
    u(c, l) &= h \\
    zu_1(c, l) &= u_2(c, l).
\end{align*}
\]

Where $u(., .)$ is a standard utility function, $c$ is consumption, $l$ is leisure and, both $z$ and $h$ are arbitrary constants. What is $\frac{dc}{dh}$?