Diamond Overlapping generations model with log utility

**Time:** discrete, infinite horizon

**Demography:** A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.

**Preferences:** for the generations born in and after period 1;

$$U_t(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period $t$ and stage $i$ of life. For the initial old generation $\bar{U}(c_{2,1}) = \ln(c_{2,1})$.

**Productive technology:** Firms have access to the technology $F(K, L) = zK^\alpha L^{1-\alpha}$ where $K$ is the capital stock, $L$ is labor and and $\alpha \in (0, 1)$. (This means that period $t$ output per worker will be $zk_t^\alpha$ where $k_t$ is per worker capital stock at the firm.)

**Endowments:** Everyone has one unit of labor services when young. (Old people cannot work so they have to rely on earnings from renting capital.) The first generation of old have $(1+n)k_1$ units of capital each.

**Institutions:** There are competitive markets, for labor, physical capital and consumption goods. Using the consumption good as the numeraire, let the per unit wage in period $t$ be $w_t$ and the gross return on capital rented in period $t$ be $R_t$.

1. Write out and solve the problems faced by generation $t$ individuals and firms in this economy (ignore inside money).

2. Comment on the relative magnitude of the income and substitution effects (i.e. which one dominates?).

3. Write down the market clearing conditions and define a competitive equilibrium.

4. Solve for the (non-trivial) steady state level of the capital stock.

5. What are the dynamic properties of the steady state?

6. Write down and solve the problem faced by a Social Planner who weights all generations equally.

7. Under what condition on parameters does the first welfare theorem fail to hold?