(a)

\[
\max_{s_{t+1}, M_{t+1}^d, c_{1,t}, c_{2,t+1}} \{ u(c_{1,t}) + \beta u(c_{2,t+1}) \}
\]

s.t. \( w_t = c_{1,t} + s_{t+1} + \frac{M_{t+1}^d}{p_t} \)

\( c_{2,t+1} = R_{t+1}s_{t+1} + \frac{M_{t+1}^d}{p_{t+1}} \)

where \( M_{t+1}^d \) is the nominal amount of money carried into period \( t+1 \). \( s_{t+1} \) is saving in terms of physical capital.

Substituting for \( c_{1,t} \) and \( c_{2,t+1} \) leaves

\[
\max_{s_{t+1}, M_{t+1}^d} \left\{ u \left( w_t - s_{t+1} - \frac{M_{t+1}^d}{p_t} \right) + \beta u \left( R_{t+1}s_{t+1} + \frac{M_{t+1}^d}{p_{t+1}} \right) \right\}
\]

FOC:

\( s_{t+1} : \)

\[-u'(c_{1,t}) + \beta R_{t+1}u'(c_{2,t+1}) = 0 \]

\( M_{t+1}^d : \)

\[-\frac{u'(c_{1,t})}{p_t} + \frac{\beta u'(c_{2,t+1})}{p_{t+1}} = 0 \]

Condition on prices is that we need \( \frac{p_t}{p_{t+1}} = R_{t+1} \) for both saving devices to be used in equilibrium.

Firms:

\( R_t = f'(k_t), \quad w_t = f(k_t) - k_t f'(k_t) \)

(b) Market clearing:

\[
(1+n)k_{t+1}^* = s_{t+1}
\]

\[
M_{t+1}^d = H/(1+n)^t
\]

Equilibrium: An allocation, \( \{c_{1,t}, c_{2,t+1}, k_{t+1}, M_{t+1}^d\} \) and a sequence of prices \( \{w_t, R_t, p_t\} \) such that given prices households and firms optimize and markets clear.
(c) \( \frac{p_t}{p_{t+1}} = (1 + n) \)

(d) (This is straight from the class notes.) The planner has to solve

\[
\max_{\{c_{1,t}, c_{2,t+1}, k_{t+1}\}} u(c_{2,0}) + \sum_{t=0}^{\infty} u(c_{1,t}) + \beta u(c_{2,t+1})
\]

s.t. \( f(k_t) - (1 + n)k_{t+1} - \frac{c_{2,t}}{1 + n} - c_{1,t} \)

As the planner does not discount the maximized sum is likely to be unbounded but it does not stop him/her solving the per period problem. Using \( \lambda_t \) as the co-state variable on the period \( t \) constraint the Lagrangian is

\[
\mathcal{L} = u(c_{2,0}) + \sum_{t=0}^{\infty} \left\{ u(c_{1,t}) + \beta u(c_{2,t+1}) + \lambda_t \left[ f(k_t) - (1 + n)k_{t+1} - \frac{c_{2,t}}{1 + n} - c_{1,t} \right] \right\}
\]

FOC's:

- \( c_{2,0} : \)
  \[
u'(c_{2,0}) = \frac{\lambda_0}{1 + n} \]

- \( c_{1,t} : \)
  \[
u'(c_{1,t}) = \lambda_t \]

- \( c_{2,t} : \)
  \[
  \beta \nu'(c_{2,t}) = \frac{\lambda_t}{1 + n}
  \]

- \( k_t : \)
  \[
  \lambda_t f'(k_t) - \lambda_{t-1}(1 + n) = 0
  \]

The FOC for \( k_t \) is sufficient to answer the question. In steady-state, the planner sets \( f'(k^*) = (1 + n) \). If in the CE \( f'(k^*) > (1 + n) \) the allocation is efficient. If in the CE, \( f'(k^*) < (1 + n) \) there is over accumulation of capital. Some capital could be consumed by the existing old and subsequent generations made better off.

(e) Any monetary equilibrium will be efficient. In the Diamond economy over accumulation of capital can occur in which \( f'(k_t) < (1 + n) \). In such an environment money would dominate physical capital in rate of return. Savings would be diverted from capital to money increasing the interest rate on capital until \( f'(k_t) = (1 + n) \). If in the Diamond economy without money equilibrium implied \( f'(k_t) > (1 + n) \) capital dominates money and money could not circulate. The over investment equilibria could still exist as long as money does not have value.