Sidrauski model with special people.

Time: Discrete; infinite horizon

Demography: Continuum of mass 1 of infinite lived consumer/worker households. A proportion $\lambda$ of the households are special in a way made clear below. There is also a large number of profit maximizing firms, owned jointly by the households.

Preferences: the instantaneous household utility function over, individual consumption, $c$, and real money holding, $m$ is $u(c,m)$ where $u(\cdot,\cdot)$ is increasing in both arguments and concave. We have $\lim_{c \to 0} u_1(c,m) = \infty$ and $\lim_{m \to 0} u_2(c,m) = \infty$ where $u_i(c,m)$ is the derivative of $u$ with respect to its $i$th argument. The discount factor is $\beta \in (0,1)$.

Technology: Each firm has access to a technology which has constant returns to scale with respect to the capital and labor it hires. Thus output per unit of labor employed is $f(k^f)$ where $k^f$ is capital hired per unit of labor. Capital depreciates at the rate $\delta < 1$.

Endowments: Households' initial capital stock is $k_0$. Each household has 1 unit of labor each period. Each household has an initial endowment of money, $H_0$. The stock of money grows at the rate $\sigma$ so that the nominal amount of money in circulation in period $t$ is $H_t = (1 + \sigma)H_0$. The additional money is distributed by helicopter drop but only to the special people. If you are one of the $\lambda$ you get cash transfers, $\tau_t$ (if $\sigma < 0$, $\tau_t < 0$) if you are one of the $1 - \lambda$ nothing happens to your cash balances over night.

Institutions: Every period there are markets for capital, labor and money. (Firms behave competitively).

(a) Write down and characterize the solution to both the special and non-special households’ problems in terms of wages, $w_t$, the rental rate of capital, $r_t$, the price of goods, $p_t$ and transfers, $\tau_t$. You can write out one of the problems if you like and simply point out how they will differ from each other.

(b) Write down and solve the firm’s problem given prices.

(c) Write down the market clearing conditions for money and capital, and the government budget constraint. Define a competitive equilibrium. Hint you will need to think about each group holding different amounts of money and having different allocations.

(d) Assuming that $\frac{p_{t+1}}{p_t} = 1 + \sigma$, use the household budget constraints to derive expressions for the steady state consumption of the special and non-special households in terms of the other steady state variables.
(2) Mortensen-Pissarides with closed-hut bargaining

**Time:** Discrete, infinite horizon

**Demography:** A mass of 1 of ex ante identical workers with infinite lives and a large mass of firms who create individual vacancies.

**Preferences:** Workers and firms are risk neutral (i.e. $u(x) = x$). The common discount rate is $r$. The value of leisure for workers is $b$ utils per period. The cost of holding a vacancy for firms is $a$ utils per period.

**Productive Technology:** Matched firm/worker pairs produce $p$ units of the consumption good per period. With probability $\lambda$ each period, jobs experience a catastrophic productivity shock and the job is destroyed. Assume $p > 2b$

**Matching Technology:** Unemployed workers encounter vacancies at the rate $m(\theta)$ where $\theta = v/u$, $v$ is the mass of vacancies and $u$ is the mass of unemployed workers. The function $m(.)$ is increasing concave and $m(\theta) < 1$ for all $\theta$. Also $\lim_{\theta \to 0} m'(\theta) = 1$, $\lim_{\theta \to \infty} m'(\theta) = 0$, and $m(\theta) > \theta m'(\theta)$. The rate at which vacancies encounter unemployed workers is then $m(\theta)/\theta$.

**Institutions:** The terms of trade are determined by symmetric Rubinstein type bargaining which means that the wage $w = p/2$ (as long as the outside option of the worker does not bind which will not happen while $p > 2b$).

(a) Write down the set of flow value equations or Bellman equations for workers and firms.

(b) Define a steady state free-entry equilibrium and solve for a single equation in $\theta$.

(c) Obtain an expression for the unemployment rate, $u$ in terms of $\theta$.

(d) How do changes in $b$ and $a$ affect the level of unemployment? Provide intuition for your results.