1. Diamond Coconut Economy with idiosyncratic preferences

   **Time:** Discrete, infinite horizon

   **Geography:** A trading island and a production island.

   **Demography:** A mass of 1 of ex ante identical individuals with infinite lives.

   **Preferences:** The common discount rate is $r$, consumption of my own produce or the produce of someone whose coconut I do not like yields 0 utils. Consumption of produce from someone whose good I like yields $u$ utils. The share of individuals whose goods I like is $\phi$. Whether an individual likes my good or not is independent of whether I like hers. (So she likes my good with probability $\phi$ too.)

   **Productive Technology:** On the production island individuals come across a tree with a coconut with probability $\alpha$ each period. The cost of obtaining the coconut is $c \sim F$. The distribution function $F(.)$ is continuous over its support, $(0, \bar{c})$ where $\bar{c} > u$ (so the coconuts from some trees will get rejected).

   **Matching Technology:** On the trading island people with coconuts meet each with probability $\gamma$, a constant.

   **Navigation:** Travel between islands is instantaneous.

   **Endowments:** Everyone has a boat and starts off with one of their own coconuts

   (a) Write down the asset value (Bellman) equations for this economy. Define the terms you introduce.

   (b) Define a search equilibrium.

   (c) Solve for an implicit equation that specifies the reservation tree “height”, $c^*$, in terms of the parameters of the model

   (d) How does $c^*$ change with respect to $\phi$ (i.e. obtain sign of the comparative static)?

   (e) Draw a diagram showing the population flows between the islands. Write down the steady-state equations and solve for the population on the trading island as a function of $c^*$. 
2. Optimal growth with proportional labor taxes

**Time:** Discrete; infinite horizon

**Demography:** A single representative consumer/worker household, and a single representative profit maximizing firm, owned by the household. (Both act competitively in the market economy). Both household and firm live for ever.

**Preferences:** The instantaneous household utility function over, individual consumption, $c \in R_+$, and leisure $l \in [0, 1]$, is $u(c, l)$ where $u(., .)$ is strictly increasing in both arguments and strictly concave. The discount factor is $\beta \in (0, 1)$.

**Technology:** There is a constant returns to scale technology over capital, $k$, and labor, $h$, so that output is $f(k, h)$, where $f(., .)$ is strictly increasing in both arguments and strictly concave. Capital is fully used up in production (i.e. depreciation rate is 1).

**Endowments:** Households’ initial capital stock is $k_0$, each household has 1 unit of time which they can allocate in any proportion to work or leisure.

**Institutions:** There is a government that has to meet an exogenous spending stream, $\{g_t\}$. In the market economy the government will levy a proportional tax $\{t_t\}$ on labor income.

**Markets:** Every period households rent out their capital, sell their labor and buy/sell goods in competitive markets. The implied prices are $r_t$ and $w_t$. (The consumption good is the numeraire.)

(a) Write down and solve the household’s problem in recursive form (i.e. using dynamic programming).

(b) Write down the solution to the firm’s problem, the government budget constraint, the market clearing conditions and define a competitive equilibrium.

(c) Solve for a system of equations that characterizes the competitive equilibrium.

(d) If $g_t = \bar{g}$ for all $t$, obtain a system of equations that characterizes the steady-state equilibrium, $(c^*, h^*, k^*)$

(e) Now solve the Planner’s problem with $g_t = \bar{g}$ and solve for the system that characterizes His steady-state solution, $(c_p, h_p, k_p)$.

(f) Briefly discuss any differences between your answers to parts (d) and (e).