Credit Card Acceptance and Product Quality

Adrian Masters

SUNY Albany

August 2013
Credit card network merchant discount fees run from 1% to 3%.
Credit card issuers act as a buyer’s attorney in the case of defective merchandise.
Credit card acceptance indicates product quality.
Paying in full with a credit card gives useful protection against faulty goods costing between £100 and £30,000. If you have a compliant about a purchase you have made with your NatWest Credit Card, please contact the retailer first. If the retailer can’t resolve the issue or has gone out of business, contact us and we will take up the matter on your behalf.
Network Effects
Network Effects

- Masters and Rodrigues-Reyes [2005], Markose and Loke [2003]
Network Effects
  - Masters and Rodrigues-Reyes [2005], Markose and Loke [2003]

Customer stealing: monopolistic competition and heterogeneous buyers
LITERATURE ON MERCHANT FEES

- Network Effects
  - Masters and Rodrigues-Reyes [2005], Markose and Loke [2003]
- Customer stealing: monopolistic competition and heterogeneous buyers
  - Rochet and Tirole [2002], Wright [2010]
LITERATURE ON MERCHANT FEES

- **Network Effects**
  - Masters and Rodrigues-Reyes [2005], Markose and Loke [2003]

- **Customer stealing: monopolistic competition and heterogeneous buyers**
  - Rochet and Tirole [2002], Wright [2010]

- **Customer stealing: Lay-away**
LITERATURE ON MERCHANT FEES

- Network Effects
  - Masters and Rodrigues-Reyes [2005], Markose and Loke [2003]
- Customer stealing: monopolistic competition and heterogeneous buyers
  - Rochet and Tirole [2002], Wright [2010]
- Customer stealing: Lay-away
  - Rochet and Wright [2010]
Basic framework: Rocheteau and Wright [2005]
Basic framework: Rocheteau and Wright [2005]
Competitive search used to abstract from network effects
Basic framework: Rocheteau and Wright [2005]
Competitive search used to abstract from network effects
Wrinkle: inflation makes cards better than cash (for buyers)
Basic framework: Rocheteau and Wright [2005]
Competitive search used to abstract from network effects
Wrinkle: inflation makes cards better than cash (for buyers)
Firms face moral hazard problem: product quality unobserved until sale occurs
ENVIRONMENT: Time and Demography

- **Time:**
  - Discrete, infinite horizon
  - Each period divided into 2 subperiods: centralized (CM) and decentralized (DM)

- **Demography:**
  - Mass \( \bar{n} \) of sellers (in the DM)
  - Infinite lives
ENVIRONMENT: Time and Demography

- **Time:**
  - Discrete, Infinite horizon

- **Demography:**
  - Mass 1 of buyers, mass $\bar{n}$ of sellers (in the DM)
  - Infinite lives
ENVIRONMENT: Time and Demography

- **Time:**
  - Discrete, Infinite horizon
  - Each period divided into 2 subperiods - centralized (CM) and decentralized (DM)

- **Demography:**
  - Mass 1 of buyers, mass $\bar{n}$ of sellers (in the DM)
  - Infinite lives
**ENVIRONMENT: Time and Demography**

- **Time:**
  - Discrete, Infinite horizon
  - Each period divided into 2 subperiods - centralized (CM) and decentralized (DM)

- **Demography:**
ENVIRONMENT: Time and Demography

- **Time:**
  - Discrete, Infinite horizon
  - Each period divided into 2 subperiods - centralized (CM) and decentralized (DM)

- **Demography:**
  - Mass 1 of buyers, mass $n$ of sellers (in the DM)
ENVIRONMENT: Time and Demography

- **Time:**
  - Discrete, Infinite horizon
  - Each period divided into 2 subperiods - centralized (CM) and decentralized (DM)

- **Demography:**
  - Mass 1 of buyers, mass $\bar{n}$ of sellers (in the DM)
  - Infinite lives
Sellers produce goods at the end of the CM
- Sellers produce goods at the end of the CM
- If a seller incurs high effort the good is high quality with probability $\lambda_h$
Sellers produce goods at the end of the CM
If a seller incurs high effort the good is high quality with probability $\lambda_h$
If a seller incurs low effort the good is high quality with probability $\lambda_l < \lambda_h$
Sellers produce goods at the end of the CM.

If a seller incurs high effort the good is high quality with probability $\lambda_h$.

If a seller incurs low effort the good is high quality with probability $\lambda_l < \lambda_h$.

High effort costs a seller $k$, low effort costs 0.
Buyers:

\[ U_t^b = x_t - c(y_t) + \begin{cases} u, & \text{if high quality good consumed} \\ 0, & \text{otherwise} \end{cases} \]
Buyers:

\[ U_t^b = x_t - c(y_t) + \begin{cases} 
  u, & \text{if high quality good consumed} \\
  0, & \text{otherwise}
\end{cases}. \]

\( c(\cdot) \) twice differentiable, \( c'(\cdot) > 0, c''(\cdot) > 0. \)
Buyers:

\[ U_t^b = x_t - c(y_t) + \begin{cases} u, & \text{if high quality good consumed} \\ 0, & \text{otherwise} \end{cases} \]

- \( c(.) \) twice differentiable, \( c'(.) > 0, c''(.) > 0 \).
- That there exists an \( x^* \) such that \( c'(x^*) = 1 \),
Buyers:

\[ U^b_t = x_t - c(y_t) + \begin{cases} 
  u, & \text{if high quality good consumed} \\
  0, & \text{otherwise}
\end{cases} \, . \]

- \( c(.) \) twice differentiable, \( c'(.) > 0, c''(.) > 0 \).
- That there exists an \( x^* \) such that \( c'(x^*) = 1 \),
- \( c(.) \) normalized so \( c(x^*) = x^* \)
Sellers:

\[ U_t^s = x_t - c(y_t) - \begin{cases} 
  k & \text{if the seller incurs high effort} \\
  0 & \text{otherwise} 
\end{cases} \]
Sellers:

\[ U_t^s = x_t - c(y_t) - \begin{cases} 
  k & \text{if the seller incurs high effort} \\
  0 & \text{otherwise} 
\end{cases} \]

- Common discount factor, \( \beta < 1 \) between periods
Sellers:

\[ U_t^s = x_t - c(y_t) - \begin{cases} 
  k & \text{if the seller incurs high effort} \\
  0 & \text{otherwise} 
\end{cases} \]

- Common discount factor, \( \beta < 1 \) between periods
- Lifetime utility of an individual type \( i = b, s \) is \( \sum_{t=0}^{\infty} \beta^t U_t^i \).
Buyers and sellers direct search to submarkets based on observable characteristics.
Buyers and sellers direct search to submarkets based on observable characteristics.

Sellers cannot commit to contingent prices.
Buyers and sellers direct search to submarkets based on observable characteristics.

Sellers cannot commit to contingent prices.

In any submarket the probability that a buyer gets a trading opportunity is $\alpha(\theta)$.

$\theta$ is the ratio of sellers to buyers who enter the submarket.

The probability that a seller gets a trading opportunity is $\alpha(\theta)/\theta$.

(Produced goods that do not sell rot at the end of the period.)
Buyers and sellers direct search to submarkets based on observable characteristics.

Sellers cannot commit to contingent prices.

In any submarket the probability that a buyer gets a trading opportunity is $\alpha(\theta)$.

$\theta$ is the ratio of sellers to buyers who enter the submarket.
Buyers and sellers direct search to submarkets based on observable characteristics.

Sellers cannot commit to contingent prices.

In any submarket the probability that a buyer gets a trading opportunity is $\alpha(\theta)$.

$\theta$ is the ratio of sellers to buyers who enter the submarket.

The probability that a seller gets a trading opportunity is $\alpha(\theta)/\theta$. 
Buyers and sellers direct search to submarkets based on observable characteristics.

Sellers cannot commit to contingent prices.

In any submarket the probability that a buyer gets a trading opportunity is $\alpha(\theta)$.

$\theta$ is the ratio of sellers to buyers who enter the submarket.

The probability that a seller gets a trading opportunity is $\alpha(\theta)/\theta$.

(Produced goods that do not sell rot at the end of the period.)
Standard requirements: $\alpha(\theta) \leq \min\{1, \theta\}$, $\alpha(0) = 0$, $\alpha'(\theta) > 0$, $\alpha''(\theta) < 0$ and $\lim_{\theta \to \infty} \alpha(\theta) = 1$. 
ENVIRONMENT: Matching (cont.)

- Standard requirements: $\alpha(\theta) \leq \min\{1, \theta\}$, $\alpha(0) = 0$, $\alpha'(\theta) > 0$, $\alpha''(\theta) < 0$ and $\lim_{\theta \to \infty} \alpha(\theta) = 1$.
- c.r.s: $\alpha(\theta) \geq \alpha'(\theta)\theta$
ENVIRONMENT: Matching (cont.)

- Standard requirements: \( \alpha(\theta) \leq \min\{1, \theta\} \), \( \alpha(0) = 0 \), \( \alpha'(\theta) > 0 \), \( \alpha''(\theta) < 0 \) and \( \lim_{\theta \to \infty} \alpha(\theta) = 1 \).
- c.r.s: \( \alpha(\theta) \geq \alpha'(\theta)\theta \)
- To rule out corner solutions: \( \alpha'(0) = 1 \) and \( \alpha''(0) > -\infty \)
Standard requirements: \( \alpha(\theta) \leq \min\{1, \theta\} \), \( \alpha(0) = 0 \), \( \alpha'(\theta) > 0 \), \( \alpha''(\theta) < 0 \) and \( \lim_{\theta \to \infty} \alpha(\theta) = 1 \).

c.r.s: \( \alpha(\theta) \geq \alpha'(\theta)\theta \)

To rule out corner solutions: \( \alpha'(0) = 1 \) and \( \alpha''(0) > -\infty \)

More contentious: the ratio of sellers to matches is convex in the ratio of sellers to buyers,

\[
\frac{d^2}{d\theta^2} \left( \frac{\theta}{\alpha(\theta)} \right) \geq 0.
\]
Standard requirements: \( \alpha(\theta) \leq \min\{1, \theta\} \), \( \alpha(0) = 0 \), \( \alpha'(\theta) > 0 \), \( \alpha''(\theta) < 0 \) and \( \lim_{\theta \to \infty} \alpha(\theta) = 1 \).

c.r.s: \( \alpha(\theta) \geq \alpha'(\theta)\theta \)

To rule out corner solutions: \( \alpha'(0) = 1 \) and \( \alpha''(0) > -\infty \)

More contentious: the ratio of sellers to matches is convex in the ratio of sellers to buyers,

\[
\frac{d^2}{d\theta^2} \left( \frac{\theta}{\alpha(\theta)} \right) \geq 0.
\]

Example: DRWS matching function,

\[
\alpha(\theta) = \frac{\theta}{(1 + \theta^\rho)^{1/\rho}} \quad \text{for} \ 0 < \rho < \infty.
\]
ENVIRONMENT: Matching (cont.)

- Standard requirements: \( \alpha(\theta) \leq \min\{1, \theta\} \), \( \alpha(0) = 0 \), \( \alpha'(\theta) > 0 \), \( \alpha''(\theta) < 0 \) and \( \lim_{\theta \to \infty} \alpha(\theta) = 1 \).
- c.r.s: \( \alpha(\theta) \geq \alpha'(\theta)\theta \)
- To rule out corner solutions: \( \alpha'(0) = 1 \) and \( \alpha''(0) > -\infty \)
- More contentious: the ratio of sellers to matches is convex in the ratio of sellers to buyers,
  \[ \frac{d^2}{d\theta^2} \left( \frac{\theta}{\alpha(\theta)} \right) \geq 0. \]
- Example: DRWS matching function,
  \[ \alpha(\theta) = \frac{\theta}{(1 + \theta^\rho)^{1/\rho}} \quad \text{for} \ 0 < \rho < \infty. \]
- Last assumption requires \( \rho \geq 1 \).
Planner solves

\[(\pi_p, x_p) = \arg \max_{\pi \in [0,1], x \geq 0} W(\pi, x).\]

Where

\[W(\pi, x) = (x - c(x))(1 + \bar{n}) + \alpha(\bar{n}) [\pi \lambda_h + (1 - \pi) \lambda_l] u - \bar{n} \pi k.\]

and \(\pi\) is the share of sellers of the DM good who exert effort.

- \(x_p = x^*\).
Planner solves

\[(\pi_p, x_p) = \arg \max_{\pi \in [0,1], x \geq 0} W(\pi, x).\]

Where

\[W(\pi, x) = (x - c(x))(1 + \bar{n}) + \alpha(\bar{n}) [\pi \lambda_h + (1 - \pi) \lambda_l] u - \bar{n} \pi k.\]

and \(\pi\) is the share of sellers of the DM good who exert effort.

- \(x_p = x^*.\)
- For the Planner to make sellers exert effort,

\[\alpha(\bar{n}) [\lambda_h - \lambda_l] u \geq \bar{n}k.\]
Planner solves

\[(\pi_p, x_p) = \arg \max_{\pi \in [0,1], x \geq 0} W(\pi, x).\]

Where

\[W(\pi, x) = (x - c(x))(1 + \bar{n}) + \alpha(\bar{n}) [\pi \lambda_h + (1 - \pi) \lambda_l] u - \bar{n} \pi k.\]

and \(\pi\) is the share of sellers of the DM good who exert effort.

- \(x_p = x^*\).

- For the Planner to make sellers exert effort,
  \[\alpha(\bar{n}) [\lambda_h - \lambda_l] u \geq \bar{n} k.\]

- If this holds with strict inequality, every seller incurs effort
Planner solves

\[(\pi_p, x_p) = \arg \max_{\pi \in [0,1], x \geq 0} W(\pi, x).\]

Where

\[W(\pi, x) = (x - c(x))(1 + \bar{n}) + \alpha(\bar{n}) [\pi \lambda_h + (1 - \pi) \lambda_l] u - \bar{n}\pi k.\]

and \(\pi\) is the share of sellers of the DM good who exert effort.

- \(x_p = x^*\).
- For the Planner to make sellers exert effort,

\[\alpha(\bar{n}) [\lambda_h - \lambda_l] u \geq \bar{n}k.\]

- If this holds with strict inequality, every seller incurs effort.
- Implementation is through contingent transfers.
Some medium of exchange is essential for trade in the DM.

Buyers can use money or a credit card to purchase goods.

Money is perfectly divisible and agents can hold any non-negative amount.

Aggregate nominal money supply, $M_t$, grows at constant gross rate $\gamma < \beta$ so that $M_{t+1} = \gamma M_t$.

New money is injected (or withdrawn if $\gamma < 1$) by lump-sum transfers (taxes) in the CM.

Transfers go only to buyers

Price of CM goods is normalized to 1 the relative price of money is denoted $\phi_t$.

Let $z_t = \phi_t m_t$ be the real value of an amount of money $m_t$. 
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.

At the beginning of the CM, buyers choose:

- what to produce and consume in the CM, \((x, y)\)
- which DM submarket to enter, \((\hat{z}, \hat{\theta})\)

At the beginning of the CM, sellers choose:

- what to produce and consume in the CM, \((x, y)\)
- the probability of exerting high effort, \(\hat{\pi}\) (contingent on the realized quality choice)
- which DM submarket to enter, \((\hat{z}_i, \hat{\theta}_i)\), \(i = h, l\)
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.

At the beginning of the CM, buyers choose:

- what to produce and consume in the CM, \((x, y)\)
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.

At the beginning of the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- which DM submarket to enter, \((\hat{z}, \hat{\theta})\)
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.

At the beginning of the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- which DM submarket to enter, \((\hat{z}, \hat{\theta})\)

At the beginning of the CM, sellers choose:
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.

At the beginning of the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- which DM submarket to enter, \((\hat{z}, \hat{\theta})\)

At the beginning of the CM, sellers choose:
- what to produce and consume in the CM, \((x, y)\)
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.

At the beginning of the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- which DM submarket to enter, \((\hat{z}, \hat{\theta})\)

At the beginning of the CM, sellers choose:
- what to produce and consume in the CM, \((x, y)\)
- the probability of exerting high effort, \(\hat{\pi}\)
Submarkets of the DM are indexed by \((q, \theta)\), where \(q\) is the real price posted by sellers.

Buyers will never set \(z > q\) so \(q\) can be dropped.

At the beginning of the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- which DM submarket to enter, \((\hat{z}, \hat{\theta})\)

At the beginning of the CM, sellers choose:
- what to produce and consume in the CM, \((x, y)\)
- the probability of exerting high effort, \(\hat{\pi}\)
- (contingent on the realized quality choice) which DM submarket to enter \((\hat{z}_i, \hat{\theta}_i), i = h, l\)
A symmetric equilibrium is a set of active submarkets, $\Gamma \subset R^2_+$, to the DM, a function, $\Lambda(z, \theta)$, that specifies the proportion of high quality goods in submarket $(z, \theta)$, a function, $n(z, \theta)$, that specifies how many sellers enter submarket $(z, \theta)$, and a propensity, $\pi^*$, for sellers to exert high effort such that:

1. Given $\Lambda(. , .)$ every $(z^*, \theta^*) \in \Gamma$ solves the buyers’ problem for $(\hat{z}, \hat{\theta})$.
2. Every $(z^*, \theta^*) \in \Gamma$ solves the sellers’ problem for $(\hat{z}_i, \hat{\theta}_i)$ for $i = h$ or $l$.
3. Rational expectations holds:

$$\pi^* = \int_{\Gamma} \left( \frac{\Lambda(z, \theta) - \lambda_l}{\lambda_h - \lambda_l} \right) \frac{n(z, \theta)}{\bar{n}} \, dz \, d\theta.$$

4. The population constraints for sellers and buyers hold:

$$\int_{\Gamma} n(z, \theta) \, dz \, d\theta = \bar{n}, \quad \int_{\Gamma} \frac{n(z, \theta)}{\theta} \, dz \, d\theta = 1.$$
\[ \Lambda(z, \theta) = \lambda_h \text{ or } \lambda_l \]
CASH ONLY ECONOMY: Results

- \( \Lambda(z, \theta) = \lambda_h \) or \( \lambda_l \)
  - No seller would exert high effort and enter a market where low effort sellers exist.
\[ \Lambda(z, \theta) = \lambda_h \text{ or } \lambda_l \]

- No seller would exert high effort and enter a market where low effort sellers exist.

\[ \Lambda(z, \theta) = \lambda_l, \text{ so } \pi^* = 0 \]
\( \Lambda(z, \theta) = \lambda_h \) or \( \lambda_l \)
- No seller would exert high effort and enter a market where low effort sellers exist.

\( \Lambda(z, \theta) = \lambda_l \), so \( \pi^* = 0 \)
- Any high effort seller can be imitated by low effort sellers
CASH ONLY ECONOMY: Results

- \( \Lambda(z, \theta) = \lambda_h \) or \( \lambda_l \)
  - No seller would exert high effort and enter a market where low effort sellers exist.
- \( \Lambda(z, \theta) = \lambda_l \), so \( \pi^* = 0 \)
  - Any high effort seller can be imitated by low effort sellers.
- There exists a unique equilibrium \((z^*, \theta^*)\) so that \( \theta^* = \bar{n} \).
In every DM, sellers have access to a credit card network.
In every DM, sellers have access to a credit card network.
They post a cash price, $q$, and credit price, $p$. 
In every DM, sellers have access to a credit card network. They post a cash price, $q$, and credit price, $p$. Prices become common knowledge and are used by buyers to direct their search.
If the good is high quality
If the good is high quality
  - the seller receives a cash payment \((1 - \omega)p\) from acquirer

If the good is of low quality,
  - the seller gets nothing
  - the buyer pays nothing

Buyers are fully committed to the payment.

There is a credit card company that sets \(\omega\) and runs the network at zero cost.

Any profits are disbursed to sellers at the beginning of the DM.
CREDIT CARD PURCHASES

- If the good is high quality
  - the seller receives a cash payment $(1 - \omega)p$ from acquirer
  - the buyer pays $p$ to the card issuer in the next CM
CREDIT CARD PURCHASES

- If the good is high quality
  - the seller receives a cash payment \((1 - \omega)p\) from acquirer
  - the buyer pays \(p\) to the card issuer in the next CM

- If the good is of low quality,
If the good is high quality
  - the seller receives a cash payment \((1 - \omega)p\) from acquirer
  - the buyer pays \(p\) to the card issuer in the next CM

If the good is of low quality,
  - the seller gets nothing
If the good is high quality

- the seller receives a cash payment \((1 - \omega)p\) from acquirer
- the buyer pays \(p\) to the card issuer in the next CM

If the good is of low quality,

- the seller gets nothing
- the buyer pays nothing
CREDIT CARD PURCHASES

- If the good is high quality
  - the seller receives a cash payment \((1 - \omega)p\) from acquirer
  - the buyer pays \(p\) to the card issuer in the next CM

- If the good is of low quality,
  - the seller gets nothing
  - the buyer pays nothing

- Buyers are fully committed to the payment.
CREDIT CARD PURCHASES

- If the good is high quality
  - the seller receives a cash payment \((1 - \omega)p\) from acquirer
  - the buyer pays \(p\) to the card issuer in the next CM
- If the good is of low quality,
  - the seller gets nothing
  - the buyer pays nothing
- Buyers are fully committed to the payment.
- There is a credit card company that sets \(\omega\) and runs the network at zero cost.
CREDIT CARD PURCHASES

• If the good is high quality
  • the seller receives a cash payment \((1 - \omega)p\) from acquirer
  • the buyer pays \(p\) to the card issuer in the next CM

• If the good is of low quality,
  • the seller gets nothing
  • the buyer pays nothing

• Buyers are fully committed to the payment.
• There is a credit card company that sets \(\omega\) and runs the network at zero cost.
• Any profits are disbursed to sellers at the beginning of the DM
Buyers and Sellers choose DM submarket to enter

CM

Trade occurs

Buyers receive transfer from central bank

Credit card issuer receives payment from buyers

Sellers produce DM goods

Sellers get transfer from credit card network

Match and Trade

Quality of good realized

Credit Card acquirer pays seller

DM

A. Masters (SUNY Albany)
Seller separated equilibria sought
Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\),
\(\psi\) is the buyer's propensity to carry cash
\(i = h, l\) type of seller in the market

In the CM, buyers choose:
CREDIT/CASH ECONOMY

- Seller separated equilibria sought
- Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\),
- \(\psi\) is the buyer’s propensity to carry cash
- \(i = h, l\) type of seller in the market

In the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
Seller separated equilibria sought
Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\),
\(\psi\) is the buyer’s propensity to carry cash
\(i = h, l\) type of seller in the market

In the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- the probability they bring cash, \(\tilde{\psi}\)
Seller separated equilibria sought
Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\),
\(\psi\) is the buyer's propensity to carry cash
\(i = h, l\) type of seller in the market

In the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- the probability they bring cash, \(\tilde{\psi}\)
- (contingent on cash holding) which DM submarket to enter, \((\hat{p}, \hat{z}, \hat{\theta}, \hat{\psi}, \hat{i})\)
Seller separated equilibria sought

Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\),

\(\psi\) is the buyer's propensity to carry cash

\(i = h, l\) type of seller in the market

In the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- the probability they bring cash, \(\hat{\psi}\)
- (contingent on cash holding) which DM submarket to enter, \((\hat{p}, \hat{z}, \hat{\theta}, \hat{\psi}, \hat{i})\)

At the beginning of the DM, sellers choose:
Seller separated equilibria sought

Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\),

\(\psi\) is the buyer’s propensity to carry cash

\(i = h, l\) type of seller in the market

In the CM, buyers choose:

- what to produce and consume in the CM, \((x, y)\)
- the probability they bring cash, \(\hat{\psi}\)
- (contingent on cash holding) which DM submarket to enter, \((\hat{p}, \hat{z}, \hat{\theta}, \hat{\psi}, \hat{i})\)

At the beginning of the DM, sellers choose:

- what to produce and consume in the CM, \((x, y)\)
Seller separated equilibria sought

Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\),

\(\psi\) is the buyer’s propensity to carry cash

\(i = h, l\) type of seller in the market

In the CM, buyers choose:

- what to produce and consume in the CM, \((x, y)\)
- the probability they bring cash, \(\tilde{\psi}\)
- (contingent on cash holding) which DM submarket to enter, \((\hat{p}, \hat{z}, \hat{\theta}, \hat{\psi}, \hat{i})\)

At the beginning of the DM, sellers choose:

- what to produce and consume in the CM, \((x, y)\)
- the probability of producing a high quality good, \(\hat{\pi}\)
CREDIT/CASH ECONOMY

- Seller separated equilibria sought
- Submarkets of the DM are indexed by \((p, z, \theta, \psi, i)\)
- \(\psi\) is the buyer's propensity to carry cash
- \(i = h, l\) type of seller in the market

In the CM, buyers choose:
- what to produce and consume in the CM, \((x, y)\)
- the probability they bring cash, \(\hat{\psi}\)
- (contingent on cash holding) which DM submarket to enter, \((\hat{p}, \hat{z}, \hat{\theta}, \hat{\psi}, \hat{i})\)

At the beginning of the DM, sellers choose:
- what to produce and consume in the CM, \((x, y)\)
- the probability of producing a high quality good, \(\hat{\pi}\)
- (contingent on the realized quality choice) which DM submarket to enter \((\hat{p}, \hat{z}, \hat{\theta}, \hat{\psi}, \hat{i})\)
A symmetric seller separated equilibrium is a set of active submarkets, \( \Omega \subset \mathbb{R}_+^3 \times [0, 1] \times \{h, l\} \), to each DM, a function, \( n(p, z, \theta, \psi, i) \), that specifies how many sellers enter submarket \((p, z, \theta, \psi, i)\), the aggregate propensity for sellers to exert high effort \( \tilde{\pi}^* \in [0, 1] \), and \( \tilde{\psi}^* \in [0, 1] \), the aggregate propensity for buyers to bring cash to the DM, with:

1. **Separation of sellers:** \((p, z, \theta, \psi, h) \in \Omega \Rightarrow (p, z, \theta, \psi, l) \notin \Omega\)
2. **Individual rationality:** every \((p, z, \theta, \psi, i) \in \Omega\) solves the buyers’ and sellers’ problems
3. **Buyer population constraint:** \(1 = \int_\Omega \frac{n(p, z, \theta, \psi, i)}{\theta} \, dp \, dz \, d\theta \, d\psi \, di\)
4. **Seller population constraint:** \(\bar{n} = \int_\Omega n(p, z, \theta, \psi, i) \, dp \, dz \, d\theta \, d\psi \, di\)
5. **RE:** \(\tilde{\pi}^* = \frac{1}{\bar{n}} \int_\Omega n(p, z, \theta, \psi, h) \, dp \, dz \, d\theta \, d\psi, \quad \tilde{\psi}^* = \int_\Omega \psi \theta n(p, z, \theta, \psi, i) \, dp \, dz \, d\theta \, d\psi \, di\).
CARD versus CASH

- Buyer indifference:

\[ \alpha(\theta_i) \lambda_i p_i = z_i \left[ \frac{\gamma}{\beta} - 1 + \alpha(\theta_i) \right] \quad i = h, l. \]

where \( \theta_i, p_i, z_i \) are the equilibrium values associated with effort level \( i = h, l. \)
Buyer indifference:

$$\alpha(\theta_i)\lambda_ip_i = z_i \left[ \frac{\gamma}{\beta} - 1 + \alpha(\theta_i) \right] \quad i = h, l.$$  

where $\theta_i, p_i, z_i$ are the equilibrium values associated with effort level $i = h, l$.

Sellers indifference

$$\lambda_i(1 - \omega)p_i = z_i \quad i = h, l.$$
CARD versus CASH

- **Buyer indifference:**

  \[ \alpha(\theta_i) \lambda_i p_i = z_i \left[ \frac{\gamma}{\beta} - 1 + \alpha(\theta_i) \right] \quad i = h, l. \]

  where \( \theta_i, p_i, z_i \) are the equilibrium values associated with effort level \( i = h, l. \)

- **Sellers indifference**

  \[ \lambda_i (1 - \omega) p_i = z_i \quad i = h, l. \]

- If \( \omega = 0 \) and \( \gamma = \beta \) they coincide.
For $\gamma > \beta$, the credit card price for buyer indifference exceeds that at which sellers are indifferent.
For $\gamma > \beta$, the credit card price for buyer indifference exceeds that at which sellers are indifferent.

With $\omega = 0$, credit cards have an advantage over cash because they avoid buyers holding idle balances.
For $\gamma > \beta$, the credit card price for buyer indifference exceeds that at which sellers are indifferent.

With $\omega = 0$, credit cards have an advantage over cash because they avoid buyers holding idle balances.

From these, if $\omega < \omega(\gamma, \theta_i)$ where

$$\omega(\gamma, \theta) = \frac{\left(\frac{\gamma}{\beta} - 1\right)}{(\alpha(\theta) + \frac{\gamma}{\beta} - 1)}.$$

no buyers will bring cash to market $i = h, l$. 
For $\gamma > \beta$, the credit card price for buyer indifference exceeds that at which sellers are indifferent.

With $\omega = 0$, credit cards have an advantage over cash because they avoid buyers holding idle balances.

From these, if $\omega < \omega(\gamma, \theta_i)$ where

$$\omega(\gamma, \theta) = \frac{\left(\frac{\gamma}{\beta} - 1\right)}{\left(\alpha(\theta) + \frac{\gamma}{\beta} - 1\right)}.$$ 

no buyers will bring cash to market $i = h, l$.

This also means that as long as $z_i > \lambda_i(1 - \omega)p_i$ the cash price offered is moot.
If $k > u$ we know that no sellers will exert effort.
LOW EFFORT EQUILIBRIUM

- If $k > u$ we know that no sellers will exert effort.
- If $\omega < \omega(\gamma, \bar{n})$ all transactions will be by credit card.
If $k > u$ we know that no sellers will exert effort.

If $\omega < \omega(\gamma, \bar{n})$ all transactions will be by credit card.

Similar logic to that in the cash only economy implies that equilibrium reduces to a unique pair $(p^*, \theta^*)$.
If $k > u$ we know that no sellers will exert effort.

If $\omega < \omega(\gamma, \bar{n})$ all transactions will be by credit card.

Similar logic to that in the cash only economy implies that equilibrium reduces to a unique pair $(p^*, \theta^*)$

$\theta_i^* = \bar{n}$ and

$$p^* = \frac{\gamma \eta(\bar{n}) u}{\beta}.$$  

where $\eta(.)$ is the elasticity of $\alpha(.)$.
If $k > u$ we know that no sellers will exert effort.

If $ω < ω(γ, \bar{n})$ all transactions will be by credit card.

Similar logic to that in the cash only economy implies that equilibrium reduces to a unique pair $(p^*, \theta^*)$

$θ^*_i = \bar{n}$ and

$$p^* = \frac{γη(\bar{n})u}{β}.$$ 

where $η(.)$ is the elasticity of $α(.)$

If $ω > ω(γ, \bar{n})$ all buyers will bring cash and the equilibrium outcome is identical to the cash only economy.
Suppose a time period is one week.
Suppose a time period is one week.

If annual discount factor is 0.96 and an annual money growth rate is 2%, $\gamma/\beta \approx 1.0012$. 
Suppose a time period is one week.

If annual discount factor is 0.96 and an annual money growth rate is 2%, $\gamma/\beta \approx 1.0012$.

The value of $\omega(\gamma, \bar{n})$ depends on $\alpha(\bar{n})$, the chance that a buyer meets a potential trading partner.
Suppose a time period is one week.

If annual discount factor is 0.96 and an annual money growth rate is 2%, $\gamma / \beta \approx 1.0012$.

The value of $\omega(\gamma, \bar{n})$ depends on $\alpha(\bar{n})$, the chance that a buyer meets a potential trading partner.

Inflation means the less likely a trading opportunity the better credit cards become.
Suppose a time period is one week.

If annual discount factor is 0.96 and an annual money growth rate is 2%, $\gamma/\beta \approx 1.0012$.

The value of $\omega(\gamma, \bar{n})$ depends on $\alpha(\bar{n})$, the chance that a buyer meets a potential trading partner.

Inflation means the less likely a trading opportunity the better credit cards become.

For very low values of $\alpha(\bar{n})$, $\omega(\gamma, \bar{n})$ can be arbitrarily close to 1.
Suppose a time period is one week.

If annual discount factor is 0.96 and an annual money growth rate is 2%, \( \gamma / \beta \approx 1.0012 \).

The value of \( \omega(\gamma, \bar{n}) \) depends on \( \alpha(\bar{n}) \), the chance that a buyer meets a potential trading partner.

Inflation means the less likely a trading opportunity the better credit cards become.

For very low values of \( \alpha(\bar{n}) \), \( \omega(\gamma, \bar{n}) \) can be arbitrarily close to 1.

If a buyer has a 50% chance of making the purchase then \( \omega(\gamma, \bar{n}) \approx 0.24\% \).
In a high effort credit only equilibrium (when it exists):

\[ \theta_h = \bar{n} \quad p_h = \frac{u\eta(\bar{n})\gamma}{\beta} \]

\[ z_h > \lambda_h (1 - \omega(\gamma, \bar{n})) p_h \]
In a high effort credit only equilibrium (when it exists):

\[ \theta_h = \bar{n} \quad \quad p_h = \frac{u \eta(\bar{n}) \gamma}{\beta} \]

\[ z_h > \lambda_h (1 - \omega(\gamma, \bar{n})) p_h \]

Incentive constraints:
In a high effort credit only equilibrium (when it exists):

\[ \theta_h = \bar{n} \quad p_h = \frac{u\eta(\bar{n})\gamma}{\beta} \]

\[ z_h > \lambda_h(1 - \omega(\gamma, \bar{n}))p_h \]

Incentive constraints:

1. Sellers would not prefer to offer a viable cash price.
In a high effort credit only equilibrium (when it exists):

\[ \theta_h = \bar{n} \quad p_h = \frac{u\eta(\bar{n})\gamma}{\beta} \]

\[ z_h > \lambda_h (1 - \omega(\gamma, \bar{n})) p_h \]

Incentive constraints:

1. Sellers would not prefer to offer a viable cash price.
2. Sellers will not choose low effort and enter the high effort market.
HIGH EFFORT CREDIT ONLY EQUILIBRIUM

In a high effort credit only equilibrium (when it exists):

\[ \theta_h = \bar{n} \quad p_h = \frac{u\eta(\bar{n})\gamma}{\beta} \]

\[ z_h > \lambda_h(1 - \omega(\gamma, \bar{n}))p_h \]

Incentive constraints:

1. Sellers would not prefer to offer a viable cash price.
2. Sellers will not choose low effort and enter the high effort market.
3. No seller will strictly prefer to open a low effort market.
Sellers would not prefer to offer a viable cash price.

- This will not bind.
Sellers would not prefer to offer a viable cash price.

- This will not bind.
- Offering a viable cash price tells buyers that the seller has not incurred high effort.
Sellers would not prefer to offer a viable cash price.

- This will not bind.
- Offering a viable cash price tells buyers that the seller has not incurred high effort.
- Any attempt to offer $z_h < \lambda_h (1 - \omega(\gamma, \bar{n})) p_h$ would open a new market.
Sellers will not choose low effort and enter the high effort market.

\[
\left( \frac{\alpha(\bar{n})\beta p_h(1 - \omega)}{\bar{n}\gamma} \right) \lambda_h - k \geq \left( \frac{\alpha(\bar{n})\beta p_h(1 - \omega)}{\bar{n}\gamma} \right) \lambda_l
\]
Sellers will not choose low effort and enter the high effort market.

\[
\left( \frac{\alpha(\bar{n})\beta p_h(1 - \omega)}{\bar{n}\gamma} \right) \lambda_h - k \geq \left( \frac{\alpha(\bar{n})\beta p_h(1 - \omega)}{\bar{n}\gamma} \right) \lambda_l
\]

Substituting for \( p_h \) Implies,

\[
\alpha(\bar{n})\eta(\bar{n})(1 - \omega) [\lambda_h - \lambda_l] u \geq \bar{n}k.
\]
Sellers will not choose low effort and enter the high effort market.

\[
\left( \frac{\alpha(\bar{n}) \beta p_h (1 - \omega)}{\bar{n} \gamma} \right) \lambda_h - k \geq \left( \frac{\alpha(\bar{n}) \beta p_h (1 - \omega)}{\bar{n} \gamma} \right) \lambda_l
\]

- Substituting for \( p_h \) Implies,

\[
\alpha(\bar{n}) \eta(\bar{n}) (1 - \omega) [\lambda_h - \lambda_l] u \geq \bar{n} k.
\]

- Planner’s condition was,

\[
\alpha(\bar{n}) [\lambda_h - \lambda_l] u \geq \bar{n} k.
\]
Sellers will not choose low effort and enter the high effort market.

\[
\left( \frac{\alpha(\bar{n}) \beta p_h (1 - \omega)}{\bar{n} \gamma} \right) \lambda_h - k \geq \left( \frac{\alpha(\bar{n}) \beta p_h (1 - \omega)}{\bar{n} \gamma} \right) \lambda_l
\]

- Substituting for \( p_h \) Implies,

\[
\alpha(\bar{n}) \eta(\bar{n}) (1 - \omega) [\lambda_h - \lambda_l] u \geq \bar{n} k.
\]

- Planner’s condition was,

\[
\alpha(\bar{n}) [\lambda_h - \lambda_l] u \geq \bar{n} k.
\]

- \( \eta(.) < 1 \) means even with \( \omega = 0 \), market economy cannot always achieve first-best.
Incentive Constraint 3

No seller will strictly prefer to open a low effort market.

- For low values of \( \omega \),
No seller will strictly prefer to open a low effort market.

- For low values of $\omega$,
  - the low effort market would be all credit
No seller will strictly prefer to open a low effort market.

- For low values of $\omega$,
  - the low effort market would be all credit
  - Incentive constraint 2 will bind first
INCENTIVE CONSTRAINT 3

No seller will strictly prefer to open a low effort market.

- For low values of $\omega$,
  - the low effort market would be all credit
  - Incentive constraint 2 will bind first

- For $\omega > \omega(\gamma, \theta_l)$,
No seller will strictly prefer to open a low effort market.

- For low values of $\omega$,
  - the low effort market would be all credit
  - Incentive constraint 2 will bind first

- For $\omega > \omega(\gamma, \theta_1)$,
  - solving the deviant's problem is required
No seller will strictly prefer to open a low effort market.

- For low values of $\omega$,
  - the low effort market would be all credit
  - Incentive constraint 2 will bind first

- For $\omega > \omega(\gamma, \theta_1)$,
  - solving the deviant’s problem is required
  - It has a unique solution
No seller will strictly prefer to open a low effort market.

- For low values of $\omega$,
  - the low effort market would be all credit
  - Incentive constraint 2 will bind first
- For $\omega > \omega(\gamma, \theta_1)$,
  - solving the deviant’s problem is required
  - It has a unique solution
  - No simple algebraic expression emerges
Functional form:

\[ \alpha(\theta) = \frac{\theta}{(1 + \theta^\rho)^{1/\rho}} \quad \text{for } 1 \leq \rho < \infty. \]
Functional form:

\[ \alpha(\theta) = \frac{\theta}{(1 + \theta^\rho)^{1/\rho}} \quad \text{for} \quad 1 \leq \rho < \infty. \]

If \( \alpha(\bar{n}) = 0.5 \), and \( \bar{n} = 1 \), \( \rho = 1 \).
Functional form:

$$\alpha(\theta) = \frac{\theta}{(1 + \theta^\rho)^{1/\rho}} \quad \text{for } 1 \leq \rho < \infty.$$ 

If $\alpha(\bar{n}) = 0.5$, and $\bar{n} = 1$, $\rho = 1$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\bar{n}$</th>
<th>$u$</th>
<th>$\lambda_h$</th>
<th>$\lambda_l$</th>
<th>$k$</th>
<th>$\gamma$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.99</td>
<td>0.95</td>
<td>0.0095</td>
<td>1.0004</td>
<td>0.9992</td>
</tr>
</tbody>
</table>
Functional form:

\[ \alpha(\theta) = \frac{\theta}{(1 + \theta^\rho)^{1/\rho}} \quad \text{for } 1 \leq \rho < \infty. \]

If \( \alpha(\bar{n}) = 0.5 \), and \( \bar{n} = 1 \), \( \rho = 1 \).

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \bar{n} )</th>
<th>( u )</th>
<th>( \lambda_h )</th>
<th>( \lambda_l )</th>
<th>( k )</th>
<th>( \gamma )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.99</td>
<td>0.95</td>
<td>0.0095</td>
<td>1.0004</td>
<td>0.9992</td>
</tr>
</tbody>
</table>

Maximal value of \( \omega_{\text{max}} = 4.37\% \).